OCR
RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

# MATHEMATICS (MEI) 4777

Numerical Computation

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

**Other Materials Required:**
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

**Monday 28 June 2010**
**Afternoon**

**Duration:** 2 hours 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

## COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**1** The table shows some values of $x$ and $y$ that have been obtained experimentally. The values are assumed to be correct to the numbers of significant figures shown.

| $x$ | 0.09 | 0.93 | 1.91 | 4.10 | 4.91 | 6.04 |
|---|---|---|---|---|---|---|
| $y$ | 1.076 | 0.897 | 0.498 | –0.544 | –0.740 | –0.900 |

**(i)** Estimated values of $y$ are required for various values of $x$. Explain briefly why Newton's divided difference formula might be used here in preference to other methods of interpolation. **[3]**

**(ii)** Use a spreadsheet to obtain a sketch of the data. **[2]**

**(iii)** Set up a spreadsheet, using divided differences, to produce a sequence of estimates, linear, quadratic, cubic and quartic, of $y$ when $x = 3$.

Discuss briefly the likely accuracy of the value of $y$ when $x = 3$. **[14]**

**(iv)** Modify the spreadsheet so that it will estimate $y$ for user-specified values of $x$ near to 3. Hence determine, to 2 decimal places, the value of $x$ for which $y$ is zero. **[5]**

**2** **(i)** The trapezium rule, using $n$ strips of equal width $h$, is used to find an estimate $T_n$ of the integral

$$I = \int_a^b f(x)dx.$$

You are given that the global error in $T_n$ is of the form

$$A_2 h^2 + A_4 h^4 + A_6 h^6 + ...,$$

where the coefficients $A_2, A_4, A_6, \ldots$ are independent of $n$ and $h$.

Show that $T_n^* = \frac{1}{3}(4\,T_{2n} - T_n)$ is an estimate of $I$ with global error of order $h^4$.

Write down, without proof, an expression, $T_n^{**}$, in terms of $T_{2n}^*$ and $T_n^*$, that represents an estimate of $I$ with global error of order $h^6$. **[6]**

**(ii)** Use a spreadsheet to obtain a graph of $y = \ln(1 + \sin x)$ for $0 \leqslant x \leqslant 4.5$. **[2]**

**(iii)** Set up a spreadsheet that uses Romberg's method to find, correct to 5 decimal places, the integral

$$\int_0^{\pi} \ln(1 + \sin x)\, dx.$$ **[11]**

**(iv)** Modify your spreadsheet so that it finds the value of

$$\int_0^{c} \ln(1 + \sin x)\, dx$$

for a user-specified value of $c$. Hence find, correct to 3 decimal places, the value of $c$ for which the integral is zero. **[5]**

**3** The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{1+xy}\ , \text{ with } y = 1 \text{ when } x = 1,$$

is to be solved numerically. When $x = 2$, the value of $y$ is $\alpha$.

**(i)** Use the modified Euler method with $h = 0.1, 0.05, 0.025, \ldots$ to obtain a sequence of estimates of $\alpha$. Show that the convergence of this sequence is second order. Obtain the value of $\alpha$ correct to 4 decimal places. **[12]**

**(ii)** Now set up a predictor-corrector routine to find a sequence of estimates of $\alpha$. Use the Euler method as predictor and the modified Euler method as corrector. Apply the corrector 3 times at each step. As before take $h = 0.1, 0.05, 0.025, \ldots$ until $\alpha$ is secure to 4 decimal places. **[8]**

**(iii)** Compare briefly the computational merits of the methods in parts **(i)** and **(ii)**. **[4]**

**4** The system of linear equations with augmented matrix

$$\left(\begin{array}{cccc|c} 7+\alpha & 6 & 5 & 4 & 1+\beta \\ 6 & 5+\alpha & 4 & 3 & 1 \\ 5 & 4 & 3+\alpha & 2 & 1 \\ 4 & 3 & 2 & 1+\alpha & 1 \end{array}\right)$$

is to be investigated numerically for various values of $\alpha$ and $\beta$.

**(i)** For the case $\alpha = 0.1$ and $\beta = 0$, solve the equations using Gaussian elimination with partial pivoting. Find the magnitude of the determinant of the coefficient matrix. **[14]**

**(ii)** For the case $\alpha = 0.01$, solve the equations for

($A$) $\beta = 0$,

($B$) $\beta = 0.01$,

and find the magnitude of the determinant of the coefficient matrix.

Comment on your results. **[10]**

**THERE ARE NO QUESTIONS ON THIS PAGE**