OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4757

Further Applications of Advanced Mathematics (FP3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

4757

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Option 1: Vectors

1 Four points have coordinates

$$A\,(3,\ 8,\ 27),\quad B\,(5,\ 9,\ 25),\quad C\,(8,\ 0,\ 1)\quad \text{and}\quad D\,(11,\ p,\ p),$$

where p is a constant.

(i) Find the perpendicular distance from C to the line AB. **[5]**

(ii) Find $\overrightarrow{AB} \times \overrightarrow{CD}$ in terms of p, and show that the shortest distance between the lines AB and CD is

$$\frac{21|p-5|}{\sqrt{17p^2 - 2p + 26}}.$$ **[8]**

(iii) Find, in terms of p, the volume of the tetrahedron ABCD. **[4]**

(iv) State the value of p for which the lines AB and CD intersect, and find the coordinates of the point of intersection in this case. **[7]**

Option 2: Multi-variable calculus

2 In this question, L is the straight line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, and $g(x,\ y,\ z) = (xy + z^2)e^{x-2y}$.

(i) Find $\dfrac{\partial g}{\partial x}$, $\dfrac{\partial g}{\partial y}$ and $\dfrac{\partial g}{\partial z}$. **[4]**

(ii) Show that the normal to the surface $g(x,\ y,\ z) = 3$ at the point $(2,\ 1,\ -1)$ is the line L. **[4]**

On the line L, there are two points at which $g(x,\ y,\ z) = 0$.

(iii) Show that one of these points is P $(0,\ 3,\ 0)$, and find the coordinates of the other point Q. **[4]**

(iv) Show that, if $x = -2\mu$, $y = 3 + 2\mu$, $z = \mu$, and μ is small, then

$$g(x,\ y,\ z) \approx -6\mu e^{-6}.$$ **[3]**

You are given that h is a small number.

(v) There is a point on L, close to P, at which $g(x,\ y,\ z) = h$. Show that this point is approximately

$$\left(\tfrac{1}{3}e^6 h,\ 3 - \tfrac{1}{3}e^6 h,\ -\tfrac{1}{6}e^6 h\right).$$ **[2]**

(vi) Find the approximate coordinates of the point on L, close to Q, at which $g(x,\ y,\ z) = h$. **[7]**

Option 3: Differential geometry

3 A curve C has equation $y = x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}$, for $x \geqslant 0$.

(i) Show that the arc of C for which $0 \leqslant x \leqslant a$ has length $a^{\frac{1}{2}} + \frac{1}{3}a^{\frac{3}{2}}$. **[5]**

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant x \leqslant 3$ is rotated through 2π radians about the x-axis. **[5]**

(iii) Find the coordinates of the centre of curvature corresponding to the point $\left(4, -\frac{2}{3}\right)$ on C. **[9]**

The curve C is one member of the family of curves defined by

$$y = p^2 x^{\frac{1}{2}} - \frac{1}{3}p^3 x^{\frac{3}{2}} \quad \text{(for } x \geqslant 0\text{)},$$

where p is a parameter (and $p > 0$).

(iv) Find the equation of the envelope of this family of curves. **[5]**

Option 4: Groups

4 The group $F = \{\mathrm{p, q, r, s, t, u}\}$ consists of the six functions defined by

$$\mathrm{p}(x) = x \qquad \mathrm{q}(x) = 1 - x \qquad \mathrm{r}(x) = \frac{1}{x} \qquad \mathrm{s}(x) = \frac{x-1}{x} \qquad \mathrm{t}(x) = \frac{x}{x-1} \qquad \mathrm{u}(x) = \frac{1}{1-x},$$

the binary operation being composition of functions.

(i) Show that st = r and find ts. **[4]**

(ii) Copy and complete the following composition table for F. **[3]**

	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	u	p	t	s	q
s	s	t	q	u	r	p
t	t	s	u			
u	u	r	t			

(iii) Give the inverse of each element of F. **[3]**

(iv) List all the subgroups of F. **[4]**

The group M consists of $\left\{1, -1, e^{\frac{\pi}{3}j}, e^{-\frac{\pi}{3}j}, e^{\frac{2\pi}{3}j}, e^{-\frac{2\pi}{3}j}\right\}$ with multiplication of complex numbers as its binary operation.

(v) Find the order of each element of M. **[4]**

The group G consists of the positive integers between 1 and 18 inclusive, under multiplication modulo 19.

(vi) Show that G is a cyclic group which can be generated by the element 2. **[3]**

(vii) Explain why G has no subgroup which is isomorphic to F. **[1]**

(viii) Find a subgroup of G which is isomorphic to M. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

An electronic control unit on an aircraft is inspected weekly, replaced if necessary, and is labelled A, B, C or D according to whether it is in its first, second, third or fourth week of service.

In Week 1, the unit is labelled A.

At the start of each subsequent week, the following procedure is carried out.

When the unit is labelled A, B or C, it is tested; if it passes the test it is relabelled B, C or D respectively; if it fails the test it is replaced by a new unit which is labelled A.

When the unit is labelled D, it is replaced by a new unit which is labelled A.

The probability that a unit fails the test is 0.16 when it is labelled A, 0.28 when it is labelled B, and 0.43 when it is labelled C.

This situation is modelled as a Markov chain with four states.

(i) Write down the transition matrix. **[2]**

(ii) In Week 10, find the probability that the unit is labelled C. **[3]**

(iii) Find the week (apart from Week 1) in which the probabilities that the unit is labelled A, B, C, D first form a decreasing sequence. Give the values of these probabilities. **[3]**

(iv) Find the probability that the unit is labelled B in Week 8 and is labelled C in Week 16. **[4]**

(v) Following a week in which the unit is labelled D, find the expected number of consecutive weeks in which the unit is labelled A. **[2]**

(vi) Find the equilibrium probabilities that the unit is labelled A, B, C or D. **[4]**

An airline has 145 of these units installed in its aircraft. They are all subjected to the inspection procedure described above, and may be assumed to behave independently.

(vii) In the long run, find how many of these units are expected to be replaced each week. **[2]**

A different manufacturer has now been chosen to make the units. The inspection procedure remains the same as before, but the probabilities that the unit fails the test have changed. The equilibrium probabilities that the unit is labelled A, B, C or D are now found to be 0.4, 0.25, 0.2 and 0.15 respectively.

(viii) Find the new probabilities that the unit fails the test when it is labelled A, B or C. **[4]**

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