OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \cos 3x \, dx$. **[3]**

2 Given that $f(x) = |x|$ and $g(x) = x + 1$, sketch the graphs of the composite functions $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. **[4]**

3 **(i)** Differentiate $\sqrt{1 + 3x^2}$. **[3]**

(ii) Hence show that the derivative of $x\sqrt{1 + 3x^2}$ is $\dfrac{1 + 6x^2}{\sqrt{1 + 3x^2}}$. **[4]**

4 A piston can slide inside a tube which is closed at one end and encloses a quantity of gas (see Fig. 4).

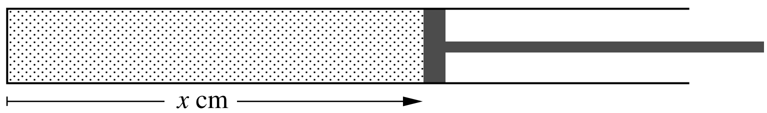

Fig. 4

The pressure of the gas in atmospheric units is given by $p = \dfrac{100}{x}$, where x cm is the distance of the piston from the closed end. At a certain moment, $x = 50$, and the piston is being pulled away from the closed end at 10 cm per minute. At what rate is the pressure changing at that time? **[6]**

5 Given that $y^3 = xy - x^2$, show that $\dfrac{dy}{dx} = \dfrac{y - 2x}{3y^2 - x}$.

Hence show that the curve $y^3 = xy - x^2$ has a stationary point when $x = \frac{1}{8}$. **[7]**

6 The function $f(x)$ is defined by

$$f(x) = 1 + 2\sin 3x, \quad -\frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{6}.$$

You are given that this function has an inverse, $f^{-1}(x)$.

Find $f^{-1}(x)$ and its domain. **[6]**

7 State whether the following statements are true or false; if false, provide a counter-example.

(i) If a is rational and b is rational, then $a + b$ is rational.

(ii) If a is rational and b is irrational, then $a + b$ is irrational.

(iii) If a is irrational and b is irrational, then $a + b$ is irrational. **[3]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = 3\ln x + x - x^2$.

The curve crosses the x-axis at P and Q, and has a turning point at R. The x-coordinate of Q is approximately 2.05.

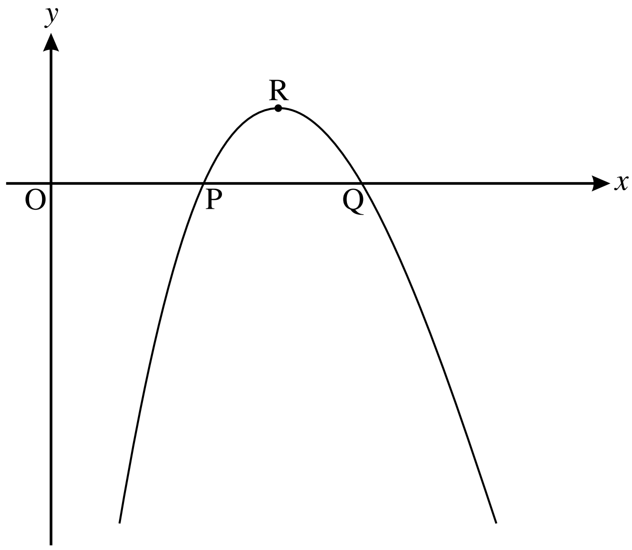

Fig. 8

(i) Verify that the coordinates of P are (1, 0). **[1]**

(ii) Find the coordinates of R, giving the y-coordinate correct to 3 significant figures.

Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, and use this to verify that R is a maximum point. **[9]**

(iii) Find $\int \ln x \, \mathrm{d}x$.

Hence calculate the area of the region enclosed by the curve and the x-axis between P and Q, giving your answer to 2 significant figures. **[7]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\mathrm{e}^{2x}}{1 + \mathrm{e}^{2x}}$. The curve crosses the y-axis at P.

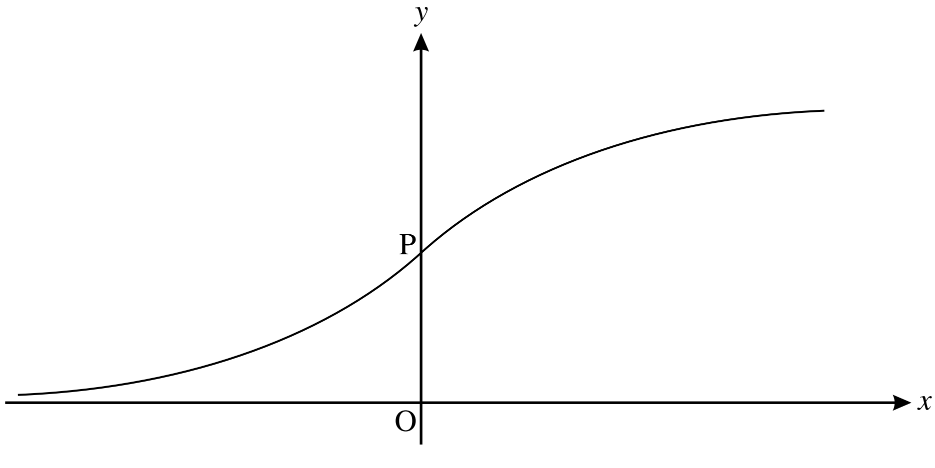

Fig. 9

(i) Find the coordinates of P. **[1]**

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, simplifying your answer.

Hence calculate the gradient of the curve at P. **[4]**

(iii) Show that the area of the region enclosed by $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = 1$ is $\frac{1}{2}\ln\left(\dfrac{1 + \mathrm{e}^2}{2}\right)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \frac{1}{2}\left(\dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}\right)$.

(iv) Prove algebraically that $\mathrm{g}(x)$ is an odd function.

Interpret this result graphically. **[3]**

(v) (A) Show that $\mathrm{g}(x) + \frac{1}{2} = \mathrm{f}(x)$.

(B) Describe the transformation which maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{f}(x)$.

(C) What can you conclude about the symmetry of the curve $y = \mathrm{f}(x)$? **[6]**