GCE

Mathematics (MEI)

Advanced GCE **4757**

Further Applications of Advanced Mathematics (FP3)

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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1 (i)	$\overrightarrow{AC}\times\overrightarrow{AB}=\begin{pmatrix}5\\-8\\-26\end{pmatrix}\times\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}42\\-42\\21\end{pmatrix}$	B2	Give B1 for one component correct
	Perpendicular distance is $\dfrac{\lvert\overrightarrow{AC}\times\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}$	M1	
	$=\dfrac{\sqrt{42^2+42^2+21^2}}{\sqrt{2^2+1^2+2^2}}=\dfrac{63}{3}$	M1	Calculating magnitude of a vector product
	$=21$	A1 **5**	www
	OR $\left[\begin{pmatrix}3+2\lambda\\8+\lambda\\27-2\lambda\end{pmatrix}-\begin{pmatrix}8\\0\\1\end{pmatrix}\right].\begin{pmatrix}2\\1\\-2\end{pmatrix}=0$ M1 A1 $2(2\lambda-5)+(\lambda+8)-2(-2\lambda+26)=0$ A1 ft $\lambda=6$ [F is (15, 14, 15)] $\text{CF}=\sqrt{7^2+14^2+14^2}=21$ M1A1		Appropriate scalar product
(ii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}2\\1\\-2\end{pmatrix}\times\begin{pmatrix}3\\p\\p-1\end{pmatrix}=\begin{pmatrix}3p-1\\-2p-4\\2p-3\end{pmatrix}$	B1 B1 B1	
	$\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})=\begin{pmatrix}5\\-8\\-26\end{pmatrix}.\begin{pmatrix}3p-1\\-2p-4\\2p-3\end{pmatrix}$	M1	
	$=5(3p-1)-8(-2p-4)-26(2p-3)$ [$=-21p+105$]	A1 ft	
	$\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert=\sqrt{(3p-1)^2+(-2p-4)^2+(2p-3)^2}$ $=\sqrt{17p^2-2p+26}$	B1 ft	
	Distance is $\dfrac{\lvert\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})\rvert}{\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert}=\dfrac{21\lvert p-5\rvert}{\sqrt{17p^2-2p+26}}$	M1A1 (ag) **8**	Correctly obtained
(iii)	$V=(\pm)\ \frac{1}{6}(\overrightarrow{AC}\times\overrightarrow{AB}).\overrightarrow{AD}=(\pm)\ \frac{1}{6}\begin{pmatrix}42\\-42\\21\end{pmatrix}.\begin{pmatrix}8\\p-8\\p-27\end{pmatrix}$	M1 A1 ft	Appropriate scalar triple product In any form
	$=(\pm)\ 56-7(p-8)+\frac{7}{2}(p-27)$	M1	Evaluation of scalar triple product *Dependent on previous M1*
	$=(\pm)\ \frac{35}{2}-\frac{7}{2}p$ $=\frac{7}{2}\lvert p-5\rvert$	A1 **4**	$\frac{1}{6}(105-21p)$ or better
(iv)	Intersect when $p=5$	B1	
	$\begin{pmatrix}3\\8\\27\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}8\\0\\1\end{pmatrix}+\mu\begin{pmatrix}3\\5\\4\end{pmatrix}$	B1 ft M1	Equations of both lines *(may involve p)* Equation for intersection *(must have different parameters)*
	$3+2\lambda=8+3\mu$ $8+\lambda=5\mu$ [$8+\lambda=p\mu$] $27-2\lambda=1+4\mu$ [$27-2\lambda=1+(p-1)\mu$]	A1 ft A1 ft	Equation involving λ and μ Second equation involving λ and μ *or* Two equations in $\lambda,\ \mu,\ p$
	$\lambda=7,\quad\mu=3$	M1	Obtaining λ or μ
	Point of intersection is $(17,\ 15,\ 13)$	A1 **7**	

2 (i)	$\frac{\partial g}{\partial x} = (y + xy + z^2)e^{x-2y}$ $\frac{\partial g}{\partial y} = (x - 2xy - 2z^2)e^{x-2y}$ $\frac{\partial g}{\partial z} = 2z\,e^{x-2y}$	M1 A1 A1 A1 **4**	Partial differentiation
(ii)	At $(2, 1, -1)$, $\frac{\partial g}{\partial x} = 4$, $\frac{\partial g}{\partial y} = -4$, $\frac{\partial g}{\partial z} = -2$ Normal has direction $\begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix}$ *L* passes through $(2, 1, -1)$ and has this direction	M1 A1 M1 A1 (ag) **4**	
(iii)	When $g = 0$, $xy + z^2 = 0$ $(2 - 2\lambda)(1 + 2\lambda) + (-1 + \lambda)^2 = 0$ $3 - 3\lambda^2 = 0$ $\lambda = \pm 1$ $\lambda = 1$ gives $P(0, 3, 0)$ $\lambda = -1$ gives $Q(4, -1, -2)$	M1 M1 A1 (ag) A1 **4**	Obtaining a value of λ Or B1 for verifying $g(0, 3, 0) = 0$ *and* showing that P is on *L*
(iv)	At P, $\frac{\partial g}{\partial x} = 3e^{-6}$, $\frac{\partial g}{\partial y} = 0$, $\frac{\partial g}{\partial z} = 0$ $\delta g \approx \frac{\partial g}{\partial x}\delta x + \frac{\partial g}{\partial y}\delta y + \frac{\partial g}{\partial z}\delta z$ $= 3e^{-6}(-2\mu) + 0 + 0 = -6\mu e^{-6}$	M1 M1 A1 (ag) **3**	OR give M2 A1 www for $g(-2\mu,\ 3 + 2\mu,\ \mu)$ $= (-3\mu^2 - 6\mu)e^{-6\mu - 6} \approx -6\mu e^{-6}$
(v)	When $-6\mu e^{-6} \approx h$, $\mu \approx -\frac{1}{6}e^6 h$ Point $(-2\mu,\ 3 + 2\mu,\ \mu)$ is approximately $(\frac{1}{3}e^6 h,\ 3 - \frac{1}{3}e^6 h,\ -\frac{1}{6}e^6 h)$	M1 A1 (ag) **2**	
(vi)	At Q, $\frac{\partial g}{\partial x} = -e^6$, $\frac{\partial g}{\partial y} = 4e^6$, $\frac{\partial g}{\partial z} = -4e^6$ When $x = 4 - 2\mu$, $y = -1 + 2\mu$, $z = -2 + \mu$ $\delta g \approx (-e^6)(-2\mu) + (4e^6)(2\mu) + (-4e^6)(\mu)$ $= 6\mu e^6$ If $6\mu e^6 \approx h$, then $\mu \approx \frac{1}{6}e^{-6}h$ Point is approximately $(4 - \frac{1}{3}e^{-6}h,\ -1 + \frac{1}{3}e^{-6}h,\ -2 + \frac{1}{6}e^{-6}h)$	M1 M1 M1A1 M1 A2 **7**	OR give M1 M2 A1 www for $g(4 - 2\mu,\ -1 + 2\mu,\ -2 + \mu)$ $= (-3\mu^2 + 6\mu)e^{-6\mu + 6} \approx 6\mu e^6$ Give A1 for one coordinate correct *If partial derivatives are not evaluated at Q, max mark is M0M1M0M0*

3 (i)	$\dfrac{dy}{dx}=\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{1/2}$	B1	
	$1+\left(\dfrac{dy}{dx}\right)^2=1+(\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{1/2})^2$	M1	
	$=1+\frac{1}{4}x^{-1}-\frac{1}{2}+\frac{1}{4}x=\frac{1}{4}x^{-1}+\frac{1}{2}+\frac{1}{4}x$		
	$=(\frac{1}{2}x^{-1/2}+\frac{1}{2}x^{1/2})^2$	A1	
	Arc length is $\int_0^a(\frac{1}{2}x^{-1/2}+\frac{1}{2}x^{1/2})\,dx$		
	$=\left[x^{1/2}+\frac{1}{3}x^{3/2}\right]_0^a$	M1	
	$=a^{1/2}+\frac{1}{3}a^{3/2}$	A1 (ag) **5**	
(ii)	Curved surface area is $\int 2\pi y\,ds$	M1	For $\int y\,ds$
	$=\int_0^3 2\pi(x^{1/2}-\frac{1}{3}x^{3/2})(\frac{1}{2}x^{-1/2}+\frac{1}{2}x^{1/2})\,dx$	A1	Correct integral form *including limits*
	$=2\pi\int_0^3(\frac{1}{2}+\frac{1}{3}x-\frac{1}{6}x^2)\,dx$		
	$=2\pi\left[\frac{1}{2}x+\frac{1}{6}x^2-\frac{1}{18}x^3\right]_0^3$	M1A1	For $\frac{1}{2}x+\frac{1}{6}x^2-\frac{1}{18}x^3$
	$=3\pi$	A1 **5**	
(iii)	When $x=4$, $\dfrac{dy}{dx}=-\dfrac{3}{4}$	B1	
	Unit normal vector is $\begin{pmatrix}-\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$	M1 A1 ft	Finding a normal vector Correct unit normal (either direction)
	$\dfrac{d^2y}{dx^2}=-\frac{1}{4}x^{-3/2}-\frac{1}{4}x^{-1/2}\quad(=-\dfrac{5}{32})$	B1	
	$\rho=\dfrac{\{1+(-\frac{3}{4})^2\}^{3/2}}{(-)\frac{5}{32}}\quad(=\dfrac{125/64}{5/32}=\dfrac{25}{2})$	M1 A1 ft	Applying formula for ρ or κ
	$\mathbf{c}=\begin{pmatrix}4\\-\frac{2}{3}\end{pmatrix}+\dfrac{25}{2}\begin{pmatrix}-\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$	M1	
	$=\begin{pmatrix}-3\frac{1}{2}\\-10\frac{2}{3}\end{pmatrix}$	A1 A1 **9**	
(iv)	Differentiating partially w.r.t. p	M1	
	$0=2p\,x^{1/2}-p^2x^{3/2}$	A1	
	$p=\dfrac{2}{x}$		
	Envelope is $y=\dfrac{4}{x^2}x^{1/2}-\dfrac{1}{3}\dfrac{8}{x^3}x^{3/2}$	M1 A1	
	$y=\dfrac{4}{3}x^{-3/2}$	A1 **5**	

4 (i)	$\mathrm{s\,t}(x)=\mathrm{s}\left(\frac{x}{x-1}\right)=\frac{\frac{x}{x-1}-1}{\frac{x}{x-1}}$ $=\frac{x-(x-1)}{x}=\frac{1}{x}=\mathrm{r}(x)$ $\mathrm{t\,s}(x)=\mathrm{t}\left(\frac{x-1}{x}\right)=\frac{\frac{x-1}{x}}{\frac{x-1}{x}-1}$ $=\frac{x-1}{(x-1)-x}=1-x=\mathrm{q}(x)$	M1 A1 (ag) M1 A1 **4**	
(ii)	$\begin{array}{c\|cccccc} & p & q & r & s & t & u \\ \hline p & p & q & r & s & t & u \\ q & q & p & s & r & u & t \\ r & r & u & p & t & s & q \\ s & s & t & q & u & r & p \\ t & t & s & u & \mathbf{q} & \mathbf{p} & \mathbf{r} \\ u & u & r & t & \mathbf{p} & \mathbf{q} & \mathbf{s} \end{array}$	B3 **3**	Give B2 for 4 correct, B1 for 2 correct
(iii)	Element: p, q, r, s, t, u Inverse: p, q, r, u, t, s	B3 **3**	Give B2 for 4 correct, B1 for 2 correct
(iv)	$\{\,\mathrm{p}\,\},\ F$ $\{\,\mathrm{p},\ \mathrm{q}\,\},\ \{\,\mathrm{p},\ \mathrm{r}\,\},\ \{\,\mathrm{p},\ \mathrm{t}\,\}$ $\{\,\mathrm{p},\ \mathrm{s},\ \mathrm{u}\,\}$	*Ignore these in the marking* B1B1B1 B1 **4**	Deduct one mark for each non-trivial subgroup in excess of four
(v)	Element: 1, -1, $\mathrm{e}^{\frac{\pi}{3}\mathrm{j}}$, $\mathrm{e}^{-\frac{\pi}{3}\mathrm{j}}$, $\mathrm{e}^{\frac{2\pi}{3}\mathrm{j}}$, $\mathrm{e}^{-\frac{2\pi}{3}\mathrm{j}}$ Order: 1, 2, 6, 6, 3, 3	B4 **4**	Give B3 for 4 correct, B2 for 3 correct B1 for 2 correct
(vi)	$2^1=2,\ 2^2=4,\ 2^3=8,\ 2^4=16,\ 2^5=13,\ 2^6=\mathbf{7}$ $2^7=14,\ 2^8=9,\ 2^9=\mathbf{18},\ 2^{10}=17,\ 2^{11}=15,\ 2^{12}=11$ $2^{13}=3,\ 2^{14}=6,\ 2^{15}=12,\ 2^{16}=5,\ 2^{17}=10,\ 2^{18}=1$ Hence 2 has order 18	M1 A1 A1 **3**	Finding (at least two) powers of 2 For $2^6=7$ and $2^9=18$ Correctly shown *All powers listed implies final A1*
(vii)	*G* is abelian (so all its subgroups are abelian) *F* is not abelian	B1 **1**	*Can have 'cyclic' instead of 'abelian'*
(viii)	Subgroup of order 6 is $\{1,\ 2^3,\ 2^6,\ 2^9,\ 2^{12},\ 2^{15}\}$ i.e. $\{1,\ 7,\ 8,\ 11,\ 12,\ 18\}$	M1 A1 **2**	or B2

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.16 & 0.28 & 0.43 & 1\\ 0.84 & 0 & 0 & 0\\ 0 & 0.72 & 0 & 0\\ 0 & 0 & 0.57 & 0\end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two columns correct
(ii)	$\mathbf{P}^9\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.3349\\0.3243\\0.2231\\0.1177\end{pmatrix}$ Prob$(C)=0.2231$	M2 A1 **3**	Using $\mathbf{P}^9$ Give M1 for using $\mathbf{P}^{10}$
(iii)	Week 5 $\mathbf{P}^4\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.5020\\0.2851\\0.1577\\0.0552\end{pmatrix}$	B1 M1 A1 **3**	First column of a power of **P** *SC* Give B0M1A1 for Week 9 and 0.3860 0.3098 0.2066 0.0976
(iv)	$\mathbf{P}^7=\begin{pmatrix}. & . & . & .\\ 0.2869 & . & . & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ $\mathbf{P}^8=\begin{pmatrix}. & . & . & .\\ . & . & . & .\\ . & 0.2262 & . & .\\ . & . & . & .\end{pmatrix}$ Probability is 0.2869×0.2262 $=0.0649$	M1M1 M1 A1 **4**	Elements from $\mathbf{P}^7$ and $\mathbf{P}^8$ Multiplying appropriate probabilities
(v)	Expected run length is $\dfrac{1}{1-0.16}=1.19$ (3 sf)	M1 A1 **2**	Allow 1.2
(vi)	$\mathbf{P}^n \to \begin{pmatrix}0.3585 & 0.3585 & 0.3585 & 0.3585\\ 0.3011 & 0.3011 & 0.3011 & 0.3011\\ 0.2168 & 0.2168 & 0.2168 & 0.2168\\ 0.1236 & 0.1236 & 0.1236 & 0.1236\end{pmatrix}$ A: 0.3585 B: 0.3011 C: 0.2168 D: 0.1236	M1 M1 A2 **4**	Evaluating $\mathbf{P}^n$ with $n\geq 10$ *or Obtaining (at least) 3 equations from* $\mathbf{P}\mathbf{p}=\mathbf{p}$ Limiting matrix with equal columns *or Solving to obtain one equilib prob* Give A1 for two correct
(vii)	Expected number is 145×0.3585 ≈ 52	M1 A1 ft **2**	
(viii)	$\begin{pmatrix}a & b & c & 1\\ 1-a & 0 & 0 & 0\\ 0 & 1-b & 0 & 0\\ 0 & 0 & 1-c & 0\end{pmatrix}\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}=\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}$ $0.4a+0.25b+0.2c+0.15=0.4$ $0.4(1-a)=0.25$ $0.25(1-b)=0.2$ $0.2(1-c)=0.15$ $a=0.375,\quad b=0.2,\quad c=0.25$	M1 A1 M1 A1 **4**	Transition matrix and $\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}$ Forming at least one equation *Dependent on previous M1*

Post-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.16 & 0.84 & 0 & 0\\ 0.28 & 0 & 0.72 & 0\\ 0.43 & 0 & 0 & 0.57\\ 1 & 0 & 0 & 0\end{pmatrix}$	B2	**2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two rows correct
(ii)	$\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}\mathbf{P}^9$ $=\begin{pmatrix}0.3349 & 0.3243 & 0.2231 & 0.1177\end{pmatrix}$ $\text{Prob}(C)=0.2231$	M2 A1	**3**	Using $\mathbf{P}^9$ Give M1 for using $\mathbf{P}^{10}$
(iii)	Week 5 $\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}\mathbf{P}^4$ $=\begin{pmatrix}0.5020 & 0.2851 & 0.1577 & 0.0552\end{pmatrix}$	B1 M1 A1	**3**	First row of a power of **P** *SC* Give B0M1A1 for Week 9 and 0.3860 0.3098 0.2066 0.0976
(iv)	$\mathbf{P}^7=\begin{pmatrix}. & 0.2869 & . & .\\ . & . & . & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ $\mathbf{P}^8=\begin{pmatrix}. & . & . & .\\ . & . & 0.2262 & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ Probability is 0.2869×0.2262 $=0.0649$	M1M1 M1 A1	**4**	Elements from $\mathbf{P}^7$ and $\mathbf{P}^8$ Multiplying appropriate probabilities
(v)	Expected run length is $\dfrac{1}{1-0.16}=1.19$ (3 sf)	M1 A1	**2**	Allow 1.2
(vi)	$\mathbf{P}^n \rightarrow \begin{pmatrix}0.3585 & 0.3011 & 0.2168 & 0.1236\\ 0.3585 & 0.3011 & 0.2168 & 0.1236\\ 0.3585 & 0.3011 & 0.2168 & 0.1236\\ 0.3585 & 0.3011 & 0.2168 & 0.1236\end{pmatrix}$ A: 0.3585 B: 0.3011 C: 0.2168 D: 0.1236	M1 M1 A2	**4**	Evaluating $\mathbf{P}^n$ with $n\geq 10$ *or Obtaining (at least) 3 equations from* $\mathbf{p}\mathbf{P}=\mathbf{p}$ Limiting matrix with equal rows *or Solving to obtain one equilib prob* Give A1 for two correct
(vii)	Expected number is 145×0.3585 ≈ 52	M1 A1 ft	**2**	
(viii)	$\begin{pmatrix}0.4 & 0.25 & 0.2 & 0.15\end{pmatrix}\begin{pmatrix}a & 1-a & 0 & 0\\ b & 0 & 1-b & 0\\ c & 0 & 0 & 1-c\\ 1 & 0 & 0 & 0\end{pmatrix}$ $=\begin{pmatrix}0.4 & 0.25 & 0.2 & 0.15\end{pmatrix}$ $0.4a+0.25b+0.2c+0.15=0.4$ $0.4(1-a)=0.25$ $0.25(1-b)=0.2$ $0.2(1-c)=0.15$ $a=0.375,\quad b=0.2,\quad c=0.25$	M1 A1 M1 A1	**4**	Transition matrix and $\begin{pmatrix}0.4 & 0.25 & 0.2 & 0.15\end{pmatrix}$ Forming at least one equation *Dependent on previous M1*

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk