GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Reports on the Units

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Oxford Cambridge and RSA Examinations

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REPORTS ON THE UNITS

Chief Examiners' Report

In this series, as always, the Principal Examiners' reports have tried to give teachers information to help them to evaluate the work of their students in the context of the strengths and weaknesses of the overall entry.

Some weaknesses are commonly mentioned: poor recognition and use of 'technical' language and notation, failure to present methods or reasons clearly and failure to set out work clearly.

Any candidate who does not know the meaning of technical words or notation given in the specification is at a great disadvantage. This is obviously the case when this lack of knowledge prevents the candidate from completely understanding what is required but also, poor or inaccurate use of technical terms or notation can impair a candidate's attempt to comment on an answer or explain a method.

Almost all solutions should include a clear indication of the method used. The rubric for each paper advises candidates that 'an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used'. Of course, when candidates are asked to establish a *given* answer, the detail required may be much greater that when the answer is not known.

Good, clear (and compact) display of working helps a candidate produce a coherent argument and reduces the chance of 'slips'. Candidates of all levels of ability can benefit from presenting their work and ideas well and there is often an association between good layout and high quality of work. It is to be hoped that the introduction of Printed Answer Books will encourage candidates to consider more carefully their setting out of solutions.

There are three matters that have been raised about how candidates should use the Printed Answer Books (that will be scanned). The first is that they should put their answers in the correct sections; the second is that they should not try to erase writing or drawing but should cross it out – the scanning process is sensitive and copies the faint images and marks that often are left after attempts at erasure. Finally it should be noted that the use of additional answer sheets should be unusual, and that sheets of rough working should not be handed in.

Note on accuracy in Statistics modules

The Principal Examiners' reports that follow discuss the candidates' performances on the individual modules. There is one matter that should be discussed in a general way as it applies to all the statistics modules. This is in respect of arithmetical accuracy in intermediate working and in quotation of final answers. Please note that these remarks are specific to the *statistics* modules; they do not necessarily apply to other modules, where it may be natural for somewhat different criteria to be appropriate.

Most candidates are sensible in their arithmetical work, but there is some unease as to exactly what level of accuracy the examiners are expecting. There is no general answer to this! The standard rubric for all the papers sums the situation up by including "final answers should be given to a degree of accuracy appropriate to the context". Three significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting from Normal tables, *some* evidence of interpolation is generally expected and so quotation to four decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice.

Talking now in general terms, the examiners always exercise sensible discretion in cases of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, a candidate is likely to lose an Accuracy mark; but if 4 significant figures are given, there would normally be no penalty. Likewise, answers which are slightly deviant from what is expected in a very minor manner are not penalised (for example, a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected). However, there are increasing numbers of cases where candidates give answers which are *grossly* over- or under-specified, such as insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happens to come off the candidate's calculator. **Such gross over-specification indicates a lack of appreciation of the nature of statistical work and, with effect from the January 2011 examinations, will be penalised by withholding of associated Accuracy marks.**

Candidates must however always be aware of the dangers of premature rounding if there are several steps in a calculation. If, say, a final answer is desired that is correct to 3 decimal places, this can in no way be guaranteed if only 3 decimal places are used in intermediate steps; indeed, it may not be safe to carry out the intermediate work even to 4 decimal places. The issue of over-specification may arise for the final answer but not for intermediate stages of the working.

It is worth repeating that most candidates act sensibly in all these respects, but it is hoped that this note may help those who are perhaps a little less confident in how to proceed.

4751 Introduction to Advanced Mathematics

General comments
Many candidates answered most of the questions in both Sections A and B very competently, with the algebraic problem in question 6 being the question found hardest in section A. In section B, question 12(i) was done particularly well, with most candidates being successful in factorising a cubic expression despite being given no 'help'. Finding the equation of a circle in 11(iv) proved difficult for many, with frequent errors in finding the centre and / or radius, although most did know the general equation of a circle. Solving quadratic inequalities and completing the square remain weaknesses of many candidates. Candidates had sufficient time to complete the paper.

This was the second series of marking this paper online, with candidates using an answer booklet with pre-prepared spaces. Fewer candidates used additional pages than in January, with the most common questions for 'overflow' or second attempts being questions 6, 11(ii) and 11(iv).

Centres are reminded that separate sheets of rough work should not be handed in.

Comments on individual questions

1) Most candidates knew that the gradient of the required line was 3, although a few used 1/3. The value of the constant was usually found successfully, with very few arithmetic errors.

2) There were the usual errors of indices in this question. In the first part the power of *a* was sometimes given as 5 rather than 6 and often candidates failed to multiply 2 by 125 to get 250. The second part was usually correct. There were many correct answers to the last part but also a wide variety of errors; a few failed to complete, especially those who found 4096 first.

3) Rearranging the formula was on the whole well answered, apart from the last step when finding the square root was used instead of squaring or the two terms were squared separately.

4) Solving the linear inequality was usually done well, although those who took the *x* terms to the left did not always cope with the change of inequality when dividing by the negative number. Some candidates correctly multiplied out the brackets, but then made errors in collecting the terms of their inequality.

   Many found –4 and ½ as the limits of the quadratic inequality, but relatively few obtained the correct final answer. Answers such as '$x < -4$ or $x < ½$' were common. Quite a few multiplied out the brackets and then used the quadratic formula to solve the equation they found, or gave up. Those who did a sketch were often more successful in writing the inequality correctly.

5) The majority knew how to deal correctly with the surds in the first part though occasionally $\sqrt{75}$ or $9\sqrt{3}+16\sqrt{3}$ were seen. In the second part, most candidates knew to multiply both numerator and denominator by $3+\sqrt{2}$ but attempted this with mixed success; surprisingly it was the numerator that caused more problems.

6) Fully correct answers were seen from only about 30% of the candidates. Many picked up marks by substituting 3 in the cubic, although it was disappointing to see that so many thought that 3^3 is 9. Many also picked up one mark for $5x^3 + 2kx^3$, often seen as part of a whole expansion, with candidates often not knowing how to proceed from there. However sometimes no marks were scored. Those who tried to divide by $(x – 3)$ rarely had any success.

7) A fair number managed to get to the correct answer, usually using the binomial expansion, but occasionally by expanding the brackets 'by hand'. However the majority made errors of one sort or another. The most common was to fail to raise ½ to the powers 2, 3, 4. Most at least picked up 1 mark for 1, 4, 6, 4, 1.

8) In this question on completing the square, most could find $a = 5$, but b varied, with the majority recognising it as 2 but many using 4 or 10. Many candidates did not find c correctly, failing to realise the part that the factor of 5 played in this term. Checking that their answer gave the correct quadratic could have helped them to realise their error – most who did this were successful.

9) Most realised that both –5 and 5 satisfy $x^2 = 25$ and often gained both marks on this question. A few weak candidates substituted other numbers into the equations as their only attempt.

10) (i) Most candidates factorised the given quadratic correctly, though of these a small minority omitted to solve the equation. An error sometimes seen was x = 2/3 instead of 3/2. Many of those who factorised incorrectly showed that they were able to obtain roots from factors and obtained the final mark. Some used the quadratic formula despite the instruction ‘Solve, by factorising...’ and some of these obtained the final mark if their roots were correct.

(ii) Most candidates obtained full marks for this part, where there were follow through marks from part (i) for incorrect roots. A minority of candidates lost marks through not labelling their intersections or scales. Although most candidates knew the general shape of a quadratic graph, some distorted the symmetry by treating the intersection with the y-axis as the minimum, and some had a poor shape as y increased.

(iii) Most candidates knew to use the discriminant and calculated it correctly, obtaining both marks – this part was particularly well done.

(iv) A high proportion of candidates obtained 3 out of 4 marks on this part. Almost all equated correctly for the first mark, and most of those made no more than one error in rearranging to zero, and so obtained the second mark. A small minority did not know how to proceed. Those who attempted subtraction to eliminate y tended to make more errors. Most candidates knew the quadratic formula correctly and substituted correctly to obtain the third mark. Subsequent arithmetic errors were quite common, and disappointingly few were able to complete and give their answer in the form asked for, failing to simplify $\sqrt{68}$ to $2\sqrt{17}$ or ignoring the surd completely when dividing by 2. Some candidates used completing the square instead of the quadratic formula, with varying success.

11) (i) The first part was generally done well with very many candidates scoring full marks. The most common error was to give the gradient with the wrong sign, sometimes following a correct attempt. One or two candidates calculated the change in *x* divided by the change in *y*. The calculation of the constant term in the equation did involve working with fractions and a negative, which resulted in an arithmetic error for some candidates. A few errors were made by transposing the values of *x* and *y* when putting the coordinate values into one of the general forms of the equation of a straight line. A number of candidates did not simplify the value of the gradient and worked with $-\frac{2}{6}$, but this did not hinder their progress. Most candidates left the final answer in the form "*y* = ", involving fractions, which was acceptable, but many multiplied through by the appropriate value to give an equation involving integers.

(ii) Most candidates understood what was required in this part and used the result of part (i) to determine where their line crossed the axes. Because of the nature of the fractions involved, it was a help to the candidates that the target answer was given in improper form rather than as a mixed number. A few candidates tried to work back from the answer, but the dependency of the method marks prevented them gaining undeserved credit. Some candidates realised here that their answer in part (i) did not give them the required answer and sensibly checked back and found their error in that part. A few candidates thought it appropriate to use Pythagoras' Theorem and there were also one or two unsuccessful attempts at using '*½ ab* sin*C'.*

(iii) In showing the result for the equation of the perpendicular bisector, candidates invariably worked with the correct gradient, but a significant number of the candidates failed to justify the value used for the gradient, which is necessary when they are given a final answer to work towards. For a number of candidates this was the only mark dropped in the whole paper. Some candidates made errors in using the condition for perpendicularity. Only a small number of candidates failed to determine and work with the midpoint.

(iv) Only a few candidates managed to answer this question correctly. This was caused by one, or both, of two fundamental errors. The first was the failure to determine the centre of the circle. Most candidates, although recognising the need to use $x = 3$, substituted into the equation of the line AB, rather than into the equation of the perpendicular bisector. Others assumed that the midpoint of AB would be the centre. A few candidates, rather than using the perpendicular bisector, used the condition that the centre would be equidistant from the points A and B. This led to more complicated working involving Pythagoras which often resulted in failing to reach the correct solution. The second fundamental error was to assume that AB was the diameter of the circle which often yielded the apparently correct value based on the incorrect calculation that $\sqrt{40} \div 2 = \sqrt{20}$. However many candidates made some attempt at using the general equation of a circle as well as an attempt to use Pythagoras to determine what they thought would be the radius and gained some credit for doing this. This part discriminated between those candidates who had understood the geometry of the problem and those who tried to apply some standard procedures without really understanding what they were doing.

12) (i) Only a small number of candidates failed to attempt this question. The vast majority used the factor theorem successfully to find the first factor, usually $x - 2$. A great many were then able to complete the factorisation (mainly by first dividing, or using inspection to find the relevant quadratic factor, although some used the factor theorem again to find further factors). Most candidates seemed to realise that they needed to try factors of 30. A few candidates reached the correct factorisation but did not show evidence of using the factor theorem and did not earn the first two marks.

(ii) The graphs of the cubic were generally good, with only occasional marks lost. As in 10(ii), the number of candidates who failed to show the y-intersection seemed to be fewer than previous years.

(iii) A pleasing number of candidates gained all 3 marks here, although some had no idea of how to start. The most profitable method was to start from the revised factors and expand these to obtain the required function. Some who chose to replace x with $(x - 1)$ in the original equation came unstuck, more often by expanding $-(x - 1)$ wrongly than in expanding the cubic term.

4752 Concepts for Advanced Mathematics

General comments

A full range of marks was achieved on this paper; however, very few obtained full marks or even close to full marks. Some candidates lost easy marks because they were unable to use algebra (eg solving simultaneous equations) or arithmetic (eg dealing with index numbers) expected of GCSE candidates correctly, even though they understood the ideas from the Core 2 course. Similarly, what is essentially GCSE work on transformations of curves seldom earned full marks. Some candidates used degrees when radians were required, although very few used radians when degrees were required. Failure to show adequate working cost marks for many – a small number of candidates relied on a "clever" calculator to produce the answer - and even some strong candidates failed to appreciate what is required when asked to show a given result.

Comments on individual questions

1) Almost half the candidates scored full marks on this question. A few lost a mark through poor arithmetic or for failing to express all three answers as fractions. However, a considerable number of candidates were not able to cope with the notation, and treated it as a *substitution* instead of appreciating that it was a *recurrence relation,* so $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ and $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ were common (incorrect) answers.

2) (i) Approximately three quarters of candidates obtained full marks on this question. Some candidates made mistakes with the arithmetic, but only a few did not understand the meaning of sigma. In some cases no working was shown, but full credit could still be earned.

2) (ii) Approximately one quarter of candidates failed to score on this question. $a = 5$ and $f(r) = (r + 1)(r + 2)$ was sometimes seen, but more frequent errors were to use other letters (frequently n or a) instead of r, or to confuse $f(r) = r(r + 1)$ with $f \times (r + 1)$. Another common error was $r \times r + 1$ instead of $r(r + 1)$.

3) (i) Almost everyone scored full marks. Only a few made slips such as "+ c" or "- $6x$" or "$2x^2$".

3) (ii) Over three quarters of candidates obtained full marks. Nearly everyone appreciated the need to set $\frac{dy}{dx}$ equal to zero. A few made sign errors in factorising, or slipped up in using the quadratic formula or completing the square.

4) Just over 40% of candidates achieved full marks, but approximately a quarter failed to score at all. Those candidates who tried to construct the equations for $f(2x)$ or $3f(x)$ generally found the approach too challenging. The majority used transformations of $y = f(x)$, and were usually successful in sketching $y = 3f(x)$, although a few candidates lost marks by labelling the vertex incorrectly. A common error was to sketch $y = f(\frac{1}{2}x)$ instead of $y = f(2x)$.

5) Most candidates understood that integration was required; only a small minority differentiated or substituted the values into the original function. The majority obtained the first term correctly, although $\frac{1^2}{2}$ was seen surprisingly often. The second term caused more difficulty. Sign errors were common, but some candidates made more significant errors such as $x^{\frac{6}{3}}$ or $6x \div \frac{x^4}{4}$. Most candidates scored the second M1 by evaluating their "F(5) – F(2)"; only a few calculated F(5) + F(2). Over 50% of candidates scored full marks. A very small number of candidates presented the correct numerical value, but showed no working. This earned no marks.

6) More than half of the candidates scored full marks on this question, and many others earned at least the first three marks. However, approximately one quarter of candidates failed to score. Some candidates misunderstood what was required, and went straight into "$y = mx + c$" with $m = 6x^2 + 12x^{½}$, thus failing to score. A small number differentiated first, thinking this would give them the gradient function.

7) This question was well done, with almost 70% of candidates obtaining both marks. Of those that didn't, more than half obtained M1, usually for $3\log x$. A surprising number made the basic error $3\log x + ½\log x = \frac{3}{2}\log x$, or a version of this. A small minority wrote $3\log x + ½\log x = 3½\log x^2$, and $\sqrt{x} = x^{-1}$ or $x^{-½}$ were sometimes seen.

8) Most candidates realised that $1-\sin^2\theta$ or $1-\cos^2\theta$ needed to be substituted, many had problems rearranging the equation in a suitable form. Many who obtained, say, $5\cos^2\theta = 1$ failed to take the square root and found $\cos^{-1}0.2$ – a few of these then went on to square root the angle. Approximately one third of candidates did take the square root correctly at this stage; of these approximately one quarter dealt with the negative root. Some candidates failed to score at all, either because they substituted $\cos^2\theta - 1$ or because they divided through by $\cos\theta$ and fudged the algebra.

9) The majority of candidates struggled with this question: many thought that *n* was the gradient of the line joining (2,6) to (3,18). In general they then formed only one correct log equation, and failed to score any marks. Of those who realised that two equations needed to be formed and solved simultaneously, a surprisingly high proportion made basic errors. $\log(ax^n) = n\log(ax)$ was frequently seen, and many made basic errors in attempting to eliminate one of the variables. A small number of candidates obtained the correct numerical values for *n* and *a*, but failed to give the answers to the required degree of precision.

10) (i) More than half of candidates scored full marks on this question. Of the rest, the majority found *y* correctly, and most appreciated the need to differentiate. A significant minority of candidates found $\frac{dy}{dx}$ and evaluated it correctly, but went on to use m = $-\frac{1}{32}$.

10) (ii) More than 60% of candidates obtained full marks; only a few candidates lost a mark for premature rounding. A small number of candidates evaluated $\frac{dy}{dx}$ at the endpoints of the chord, found the mean and came up with an answer of 34.5 to 1 d.p. This did not score. A few candidates calculated $\frac{x_{\text{step}}}{y_{\text{step}}}$ and also failed to score.

10) (iii) The majority of candidates obtained full marks in part (A). A few candidates who used the binomial expansion lost marks because they failed to simplify one or more of the terms. Those who expanded the brackets one by one often made errors. Some candidates wrote down the answer $16 + h^4$.
Part (B) was generally well done, with many candidates obtaining both marks on a follow through basis. A few candidates just reduced the power of h in one term only.
Many candidates failed to understand what was required in part (C), approximately 80% of candidates failed to score. Very few managed to earn both marks.

11) (a) The majority of candidates appreciated the need to use the cosine rule to find QR, and by and large they were successful. The sine rule (and occasionally the cosine rule) was often correctly applied to find either angle Q or angle R. A surprising number of candidates needlessly changed the labelling to A or B, and sometimes lost marks when putting their answers back in context. The response often broke down at this stage, but many who did proceed correctly lost the final mark by using rounded answers and then giving the bearing to an unreasonable level of precision which fell outside the acceptable range: 174.8° was a common (incorrect) answer.

11) (b) Part (i) was very well done: approximately 70% of candidates earned both marks. A few used the correct method and then converted to degrees, and others worked in degrees from the start. A small number of candidates carelessly used $r\theta$ or $r\theta^2$.
In part (ii) approximately half of the candidates failed to score. Generally this was because they attempted to show the requested result, but only gave one step, and didn't attempt to find the area. Those who did attempt to calculate the area were generally successful – but some lost marks by using the wrong angle in $\frac{1}{2}dc\sin A$ or by obtaining an incorrect value for AD before using ½ AD × DC.
Some candidates generally appreciated the steps involved here, and often benefitted from the follow through mark. However, a significant proportion made the mistake of thinking that the area of the sector minus the area of the quarter circle gave the area of the rudder. A surprising number of candidates found the area of triangle ADC when responding to part (iii) instead of part (ii).

12) (i) More than half of candidates obtained both marks in part (*A*). A minority used a formula connected with arithmetic progressions, but the most common wrong answer was 512.
Approximately one quarter of candidates earned both marks in part (*B*) – the overwhelming majority failed to score. Common incorrect answers were 2046, 1024 and 1023 obtained from $\frac{1-2^{10}}{1-2}$.

12) (ii) Very few candidates obtained both marks in part (*A*), and approximately 80% of candidates were awarded zero or failed to respond. Most candidates opted for a verification of the formula by substitution of 1, 2, 3… or gave wordy explanations – neither approach earned any marks.
The modal mark for part (*B*) was zero, but a reasonable number of candidates managed to extract the least value of n correctly. A surprising number failed to appreciate that n had to be an integer: 14.8 was seen frequently, as was $n > 15$.
Some candidates obtained the first method mark. Thereafter various mistakes were presented. $7 \times 2^n = 14^n$ was frequently seen, as was $\log(2^n - 1) = \log 2^n - \log 1 = \log 2^n$.
Some candidates worked with = or < instead of > and usually failed to recover. A surprising proportion decimalised their working and obtained 2 marks out of 4 in total.

4753 Methods for Advanced Mathematics (Written Examination)

General comments

The paper attracted the full range of responses, and proved to be accessible to all candidates who were suitably prepared. Many excellent scripts, gaining marks of 65 and over, were seen and relatively few candidates scored less than 20. Virtually all candidates had time to attempt all the questions. The standard of presentation of work was variable.

There was evidence that some topics which in the past have caused difficulties for candidates, such as function notation, inverse trigonometric functions, and implicit differentiation, seem to be being answered with greater success than usual.

The concept of 'proof' is of course central to mathematics, and responses to questions such as 9(iv) and 9(v) suggest that more emphasis could be placed upon how to present a proof convincingly. Many candidates work simultaneously with both sides of the equality to be proved, without using $\Leftrightarrow$ signs or appreciating the directional character of logical implications. In this regard, offering the following 'proof' that 1 = 2, and inviting students to discuss and find the flaw, may be instructive:

	1 = 2
$\Rightarrow$	2 = 1
$\Rightarrow$ (adding)	3 = 3, true so 1 = 2!

The responses to question 7 suggest that many candidates have not grasped the difference in status between example and proof and counter-example and disproof. Many candidates proffered single examples as proofs of (i) and (ii); and the counter-examples offered for 7(iii) implied that many, indeed most, cannot distinguish between a rational and an irrational number.

Comments on individual questions

1) This proved to be a straightforward first question for which the majority of candidates scored three marks. Those who failed to achieve full marks usually mixed up integrals with differentials, or failed to multiply by $\frac{1}{3}$.

2) Results for this question were variable, but many candidates scored full marks. Occasionally, 'fg' and 'gf' were the wrong way round. As for the graphs, we required the intercepts with the axes to be indicated. Some candidates only sketched gf for positive values of x, misunderstanding the effect of the modulus.

3) (i) This proved to be a straightforward test of the chain rule, with almost all candidates gaining full marks.

3) (ii) Most candidates used their result from part (i) in a product rule and scored M1 A1; however, quite a few then failed to express this as a single fraction with denominator $\sqrt{(1 + 3x^2)}$.

4) Solutions to this 'related rates of change' question were quite variable in standard. Many completed the question without difficulty. Of those who did not solve it, most recognised the need to use a chain rule, but made errors in differentiating $100/x$, often getting $100 \ln x$.

5) The implicit differentiation was often well done, including handling the xy term on the right. Some candidates persist in prefacing their implicit differentiation by writing dy/dx = …. This was condoned provided it was not pursued, but would recommend they start d/dx (LHS) = d/dx (RHS), etc. There was some evidence of working backwards from the answer to correct errors.

The second part was less successful. Of those who missed the solution given in the mark scheme, many substituted $x = 1/8$ into the implicit equation to obtain a cubic in y which they then failed to solve, although verifications that $y = ¼$ was a root of this were accepted.

6) Finding the inverse function was well done, with the commonest error mishandling the step from $\sin 3y = (x – 1)/2$ to get $y = \arcsin (x – 1)/6$. However, domains and ranges often seem to baffle candidates, and the domain given here was often incorrect.

7) Many candidates got the correct answers, possibly by guesswork. However, their attempts to find a counter-example to prove (iii) false, by using fractions, suggest that most candidates are not aware of what an irrational number actually is.

8) This question tested differentiation and integration techniques as applied to a logarithmic function.

8) (i) Virtually all candidates verified that when $x = 1$, $y = 0$. Those who did not tried to solve the equation $0 = 3 \ln x + x – x^2$.

8) (ii) Many candidates scored the full nine marks here, or missed the final mark through a slip in their working. The key was to get the derivative right – admittedly not a difficult task, which most did correctly. Occasionally, we saw $1/3x$ instead of $3/x$. Re-arranging into a quadratic and solving this was usually done correctly. Candidates who wrote down the solution $x = 1.5$ without working were awarded a special case 1, and lost 1 or 2 marks depending on whether they re-arranged to obtain the quadratic or not.

The second derivative was also well done, as was substituting $x = 1.5$ into this to get $–10/3$ to prove the maximum. However, candidates were required to comment that d^2y/dx^2 was negative.

8) (iii) Using integration by parts to integrate ln x was done well by many candidates. Others who quoted the correct result were awarded full marks. They were able to score 'follow-through' marks for substituting their result into the correct integral. However, many candidates just used ln x here rather than the given function.

9) This question applied calculus to a quotient function with exponentials, and applied some work on functions to explore the point symmetry of the given curve about P. In general, parts (i) – (iii) were well answered, and (iv) and (v) less so.

9) (i) This was an easy mark for everyone, though we did penalise P = ½, or (1/2, 0), or $x = ½$.

9) (ii) The quotient rule was usually correctly applied, with only an occasional $u\mathrm{d}v - v\mathrm{d}u$. However, quite a few candidates made algebraic errors in simplifying the numerator, such as $e^{2x}.e^{2x} = e^{2x\wedge 2}$.

9) (iii) Well-prepared candidates wrote down the correct integral by inspection, substituted appropriate limits and gained the five marks readily. Those who used substitution were less successful, often not substituting $\mathrm{d}u = 2e^{2x}\,\mathrm{d}x$ correctly.

9) (iv) A lot of candidates failed to convince with their ‘proofs’ by not finishing the argument satisfactorily. For example, just writing expressions for $-g(x)$ and $g(-x)$ without showing convincingly they were equal did not gain the ‘E’ mark. A common misconception was that, for odd functions, $g(-x) \neq -g(x)$.

Many candidates offered graphs in order to ‘interpret graphically’ – what was required here was a comment on the symmetry of odd functions, which needed ‘180°’, ‘rotation’, and ‘centre O’ to gain the mark.

9) (v) (*A*) was poorly done: those who combined the fractions correctly often failed to see the step of multiplying top and bottom by e^x.

(*B*) was more successful –the term ‘translation’ was required before the ‘E’ mark was awarded.

The final mark for (*C*) was the preserve of the best candidates.

4754 Applications of Advanced Mathematics

General comments
The standard of work on this paper was generally good. The paper provided questions that were accessible to all while at the same time providing sufficient questions to challenge the more able candidates. In general candidates were well prepared for this paper and there were, consequently, relatively few low scores.

Most candidates achieved good marks on the Comprehension although there were few very high marks. Questions 4 and 5 were not answered particularly well as either insufficient mathematics was used or answers were too vague.

Candidates should be advised to
- give full explanations when establishing given answers
- answer questions as required in radians or degrees
- avoid prematurely approximating their working
- use the rules of logarithms correctly- especially those involving constants of integration

Centres should be reminded that candidates' scripts for Paper A and Paper B are to be attached to one another before being sent to examiners.

Comments on individual questions

Paper A

1) Almost all candidates understood the method to add two fractions together. The most common approach was to add the fractions first. Those that then proceeded to factorise the numerator usually obtained all three marks. Many failed to do this and candidates commonly scored either one or three marks. The approach from the difference of two squares was less common but usually successful.

2) (i) The trapezium rule was well understood and most candidates scored marks in this part.

2) (ii) Many candidates failed to explain why this graph would give rise to an over-estimate when using the trapezium rule. A lot gave the impression that, for all curves, the more strips you use with the trapezium rule the less the result. Most, however, could explain why an over-estimate would lead to Jenny's calculations being incorrect.

2) (iii) The marks for finding the volume of revolution were almost always obtained.

3) This was the least successfully answered question. A wide variety of methods were seen. Some candidates used $\sin 2\theta = 2\sin\theta\cos\theta$ and substituted $2y = \sin 2\theta$ in $\sin^2 2\theta + \cos^2 2\theta = 1$. More commonly, others chose longer methods involving changing $\cos 2\theta$ to $\cos^2\theta - \sin^2\theta$, squaring, and adding to $4\sin^2\theta\cos^2\theta$. Often this work was inaccurate. $(\cos^2\theta - \sin^2\theta)^2 = \cos^4\theta + \sin^4\theta$ and similar errors when squaring were seen. For those with this approach, many failed to proceed further instead of using $\sin^2\theta + \cos^2\theta = 1$ and substituting. Many attempts involved many false starts at trigonometrical equations without making real progress.

Relatively few candidates drew an ellipse. This part was frequently omitted. Some drew a circle (which was allowed if the axes had different scales) or part of an ellipse and there were a wide variety of other incorrect graphs.

4) The factor of 2 was usually factorised out correctly but was also often incorrectly given as 4 or ½. Most candidates used a binomial expansion for the power ½, though in some cases using *x* instead of (*x*/4). A common mistake was to correctly factorise the 2 at the start of the question, but to forget to include it again after the expansion.
The validity was attempted more often than in some previous years' papers. The most common error was to include equality signs.

5) (i) The partial fractions were almost always correct.

5) (ii) The majority of candidates correctly separated the variables in order to integrate. Some candidates dealt efficiently with the factor 3 by including a 3 or a 1/3 on both sides. Many candidates failed to realise this was necessary and obtained ln(*y*-2) – ln(*y*+1) = $x^3/3 + c$. Only a few omitted the constant of integration. At the next stage the common error was, when taking exponentials of both sides, to find $\frac{(y-2)}{(y+1)} = e^{x^3} + e^c$ instead of e^{x^3+c}. As the answer was given, the process of taking exponentials needed to be seen in order to obtain the marks.

6) Most candidates correctly used the double angle formula for tan(θ + 45°) then multiplied up to achieve a one line equation. A common error was the sign of 2tan²θ i.e. (1 − 2tanθ)(1 − tanθ) = 1 − 3tanθ − 2tan²θ. Most found a quadratic to solve although many errors were made in the process. Also, 2tanθ(tanθ − 2) = 0, tanθ = 2, θ = 63.43° followed the common error of cancelling out the tanθ = 0 term.
Overall the question was answered well with the errors usually in the algebra rather than the trigonometry.

7) (i) Most candidates correctly verified the given result and found the length of the pipeline. A few omitted the length.

7) (ii) Most candidates were successful in finding the vector equation of the line AB- although '*r* = ' was often omitted.
For those that realised that the vertical direction was of the form $\lambda\begin{pmatrix}0\\0\\1\end{pmatrix}$, the correct angle was usually found. Some incorrectly used the position and direction vectors from the line AB. A few used the *y*-direction which was north, not vertically upwards. This gave the same answer, but from an incorrect method.
There were a few candidates who chose to use trigonometry rather than the scalar product in order to find the angle. In these cases, the candidates needed to make it clear that they were finding the angle with the vertical in order to obtain the marks.

7) (iii) These co-ordinates, where the pipeline meets the layer of rock, were usually found correctly.

7) (iv) The scalar product was usually used successfully here. The commonest error was finding the angle between the line AB and the normal but then not to use this to find the required angle at which the pipeline cut through the layer. It was unclear if this was overlooked or whether candidates felt that the angles were the same. In some cases the first angle was subtracted from 180° instead of using 90° − their angle.

8) (i) Some candidates were successful here but there were many poor answers and all or part of this was often omitted. Candidates often confused *x* and θ, giving such answers as *x* = π/2, '*B* (π,−4)' and *x* = 2.141… or, for *A* , stating θ = 90° and then following this with *x* = 180° − 1° = 179°.

8) (ii) This part was very successful. Candidates seemed to know the method for finding dy/dx from the parametric equations. The most common error was to substitute incorrectly for θ from the first part, often using $\pi - 1$.

8) (iii) The majority scored both of these marks provided they had correctly obtained dy/dx in the previous part.

8) (iv) Most candidates scored well with the 'R' method. The angle should have been given in radians here. The question concerns the track of a roller-coaster with $x = 2\theta - \sin\theta$ and there was differentiation in part (ii) but still some candidates used degrees. This was condoned for the angle α but not for the subsequent angles for θ. Most candidates were able to obtain the method mark for invcos(2/√17) but few went on to find both angles correctly.

Paper B: The Comprehension

1) This was well answered. Most candidates obtained full marks. A few used 307km and 300 km as if they were miles and were awarded one mark.

2) The majority of candidates differentiated. Those who did not calculate but used trial and error or the graph obtained no marks (as indicated in the question). The majority differentiated correctly, equated this to zero and solved for x. A few differentiated 10^{-4} as -4×10^{-5}. Very few candidates continued to justify that their $x = 100$ was a minimum. Many overlooked the necessity to find the value of the emissions, y, at x = 100 km per hour. Those that did find it were usually correct.

3) (i) The majority of candidates obtained the two marks available here for substitution and evaluation. Those that left the answer as 3694.625 (kg) did not gain the E mark.

3 (ii) The figure for the extra emissions was often found correctly – usually by substituting 251 in the formula and subtracting. Some failed to realise that by working to only 1dp and finding the difference as 0.4 they had not fully justified that the difference was less than 0.5kg.

4) (i) This was a low scoring question. Many did correctly draw a step graph, but in these cases the first step was often the same length as all the others instead of being twice as long. The scales on the axes were also often omitted. Some drew a histogram, but a common answer was a continuous curve through, approximately, the mid-points or end points of where the steps would have been.

4) (ii) Here, instead of saying that there were 10 trains a day for up to a million yearly passengers and then an extra daily train each time the number of yearly passengers increased by half a million, common answers were of the form 'As the daily number of trains increases, the annual number of passengers also increases. It is directly proportional.' Often they were said to be positively correlated or, worse, that when the number of trains increased by one this caused the number of passengers to increase by half a million.

5) This was successful for many, but many were also confused about units often not knowing how many metres there are in a kilometre or centimetres in a metre. Some performed no calculations at all and merely stated, for example, that millimetres were too great a level of accuracy to state for a distance of 100 miles or alternatively that millimetres were the best as they represented a more precise degree of accuracy. In general, they did not use sufficient mathematics in these cases.

4755 Further Concepts for Advanced Mathematics

General comments

Candidates' overall performance on the paper was consistent with previous papers. There was a wide range of performance, but overall most candidates produced some good work. The paper contained a full breadth of topics from the syllabus which enabled all candidates to show at least some of what they had learned.

Comments on individual questions

1) This was generally well done. Almost all candidates found A correctly, but some made numerical errors in calculating B and C following their expansions.

2) Again most did well on this. The main problem was candidates not using their inverse matrix to solve the equations. Since the method was specified in the question, alternative methods scored no marks.

3) Some candidates used the sum and product of roots to find k. Those using alternative methods involving substituting in a root or expanding factors often made errors in their algebra.

4) This was better attempted than question 3, with many candidates using sums and products of roots successfully, and many others using the $x = w - 1$ substitution method to good effect. Many lost a mark for forgetting '= 0' in their cubic equation.

5) This was generally well done. Most got as far as cancelling and earned the first four marks. Quite a number forgot the 1/5 factor or used 5 instead. Few candidates earned full marks. Several used r rather than n throughout, so lost at least one mark.

6) Candidates found this the most difficult question. Only the very best candidates scored all eight marks. The first three marks were earned by most candidates, but many did not know what they were assuming to be true when $n = k$. The majority had clearly expected a sum and tried to add the 'next term', even though there wasn't one! Even some of those who did complete the inductive step algebraically could not state the logic correctly to fully complete the proof by induction.

7) Parts (i), (ii) and (iii) were done well by most. In part (iv) many candidates had difficulty showing that the curve crosses the horizontal asymptote at (1, 2). Most multiplied the inequality through by $(x - 3)(x - 2)$, so making their workings invalid. However, most identified $2 < x < 3$ as part of the solution and many also found $x < 1$.

8) In part (i) a surprisingly large number of candidates could not find modulus and argument correctly. The argument of 3j caused particular difficulty. Most candidates were able to multiply and divide the complex numbers correctly in part (ii) and the Argand diagram in part (iii) was usually well done.

9) Only a few scored all four marks for the description of transformations P and Q in part (i). Many gave incomplete descriptions of the rotation, often omitting the centre or the sense, and many described the stretch as an enlargement. The main problem in (ii) was applying the matrices in the wrong order. Part (iii) was generally well done, with candidates earning follow through marks if they had made an error in (ii). A surprising number could not find the matrix for reflection in $y = -x$ in part (iv). There were some good answers to the last part; a variety of different methods were seen, some using the points themselves, others inverting matrices. Again the order of applying the matrices often led to errors.

4756 Further Methods for Advanced Mathematics

General comments
The paper discriminated well and provided opportunities for candidates across the range of ability to show what they knew, understood and could do. About a quarter of candidates scored 60 or more marks, while only about 6% scored fewer than 20 marks.

Although the overall standard of work was impressive, there was still some evidence of carelessness, or incompetence, with basic algebra. For example, in Question 1(b), $(\pi + \theta)^2$ was frequently $\pi^2 + \theta^2$; then $\frac{1}{\pi^2+\theta^2}$ became $\frac{1}{\pi^2}+\frac{1}{\theta^2}$. In Question 3(a)(iii), the vast majority of candidates knew exactly what to do, but were defeated by elementary errors when rearranging their formulae. Presentation was generally good although, as always, there were candidates who split questions up and scattered them around the paper, and others who used up to three eight-page answer books. It was pleasing that so few candidates used supplementary graph sheets to produce their sketch graphs.

Question 1 was the best done question, followed by Questions 3 and 4, with Question 2 the worst done by some margin. Question 5 was attempted by fewer than 2% of candidates. There was little evidence of time trouble, although as always some candidates were slowed down by the use of inefficient or inappropriate methods. Often these could have been avoided by a more careful reading of the question: parts labelled (i), (ii), (iii) are linked, while "write down" means what it says.

Comments on individual questions

1) *(Maclaurin series, polar curves, calculus of inverse trigonometric functions)*
This question was done well, with a mean mark of about 15 out of 19.

1) (a) Most candidates were able to find the correct result in the formula book for $f'(t)$ in part (i). This was all that was required for the first mark: some candidates felt they had to derive the result, even though the instruction in the question was "Write down". Then the chain rule was usually applied efficiently to obtain the given result for $f''(t)$, although some used the quotient rule. For part (ii), some candidates showed minimal working. When results are given it is expected that candidates' working leading to those results is complete and transparent. Just stating the given results will attract no credit. The most common error in finding the term in x^2 in the Maclaurin series was failure to divide $f''(0)$ by 2.

1) (b) There were many fully correct spirals. A very few were absolutely determined to draw a "standard" polar curve, such as a cardioid. The area of the region bounded by the polar curve was, by some margin, the worst-done part of this question. Even those who recognised that $\int_0^{\pi} \frac{\pi^2 a^2}{2(\pi+\theta)^2}\,\mathrm{d}\theta$ was required often failed to make much progress, succumbing to the type of algebraic errors referred to in the general comments above or producing integrals involving ln or powers of –3. Generally, only the strongest candidates gained the full four marks for this area.

1) (c) This arctan integral was usually very efficiently done and full marks were often scored, although there were some errors with one of the constants. Usually candidates used the standard result, although an explicit tan substitution was quite common.

2) *(Complex numbers)*
The mean mark for this question was just over 8 out of 16. In general, candidates coped well with (b) (i), and much less well with (a) and (b) (ii).

2) (a) The first mark, for expressing $z^n \pm \frac{1}{z^n}$ in simplified trigonometric form, was often scored, although there were many cases of missing *n*s and *j*s. Many candidates spent a considerable amount of time (and paper) deriving these results. Then they needed to express $\sin^5\theta$ in terms of sines of multiple angles. A surprisingly large number of candidates did not seem to be familiar with this process, and were determined to use De Moivre's theorem instead. Much working was produced, usually to no avail although it was possible to obtain the required answer by this method, and some credit was given even though the question included the instruction "Hence". Those who did know the process often lost a 2 somewhere in their working, and *j*s sometimes appeared and disappeared almost randomly.

2) (b) In part (i), the four fourth roots of $-9j$ were often obtained correctly and efficiently, and there were many correct Argand diagrams, although they were sometimes spoiled by carelessness. The argument was less often correct than the modulus, mainly due to a belief that the argument of $-9j$ was $\frac{\pi}{2}$; in a few cases, r was given as $-\sqrt{3}$.

Arguments in the range $-\pi$ to π were accepted on this occasion.
It was reasonably rare for the full five marks to be awarded in part (ii), although there were some pleasingly succinct perfect solutions. Even those who took a geometrical approach frequently made errors in both the modulus and argument of the fourth root of w, while confusion of w and its fourth root was common, with w being marked on the Argand diagram at a midpoint.

3) *(Matrices)*
The mean mark for this question was about 13 out of 19. (a) (i), (a) (iii) and (b) were done well, while (a) (ii) proved more of a challenge.

3) (a) In part (i), most candidates were able to provide a convincing explanation that 2 was an eigenvalue. The vast majority went on to find the other two eigenvalues correctly. Part (ii) was much less well done. A number of candidates missed this part out altogether. In general, candidates did not recognise this part as being about the properties of eigenvectors: lots of general 3 × 3 matrices appeared to no avail, although the first mark was scored reasonably often.
Most candidates recognised part (iii) as being an application of the Cayley-Hamilton Theorem and were able to score well, although M3A0 was extremely common due to algebraic slips.

3) (b) There were very many fully correct solutions to this part. Many solved the problem successfully by using simultaneous equations rather than the $\mathbf{N} = \mathbf{PDP}^{-1}$ method. Sometimes $\mathbf{N} = \mathbf{P}^{-1}\mathbf{DP}$ appeared, and slips in finding $\mathbf{P}^{-1}$ or in the matrix multiplication were reasonably common.

4) *(Hyperbolic functions)*
The mean mark for this question was about 12 out of 18.

4) (i) The proof of the identity from exponential definitions was done well by most candidates. The second part required candidates to differentiate both sides of the given identity with respect to x. As in the January examination, many went back to the exponential definitions and differentiated or otherwise manipulated those: those who did attempt to use the hyperbolic functions often lost a 2 somewhere, although the product rule was often applied correctly to the right-hand side.

4) (ii) The graph of $y = \cosh x - 1$ was well done. Calculating the volume of revolution caused many more problems. Those who failed to square y ceased to score immediately. Many did try to find an integral involving y^2 but the multiple was often 2π or $\frac{1}{2}$, rather than π. The next hurdle was the integration of $\cosh^2 x$. Many candidates realised that they needed to connect this with $\cosh 2x$, but were unable to do so correctly. A fair number coped successfully with all the calculus, but were unable to correctly substitute the limits to get the final answer and there were some rounding errors. A number of candidates converted their hyperbolic functions to exponential functions: this was more or less successful depending at what stage the conversion took place, but a number obtained the correct final answer by this method.

4) (iii) Candidates began by differentiating the equation of the curve: this was often done successfully, but again a 2 was liable to appear in the wrong place, or disappear altogether. In this part, those who changed to exponential functions found themselves with a quartic expression to factorise and little further progress was made. Those who used the linked parts of the equation and substituted the identity in (i) often made rapid progress to a fully correct solution, although again there were errors involving lost 2s. Candidates were required to show that there was exactly one stationary point, so $\cosh x = 0$ needed to be repudiated. Arsinh$(-\frac{1}{4})$ was very often correctly evaluated in logarithmic form, usually via the form given in the formula book.

5) *(Investigations of Curves)*
There were only a small number of attempts at this question, so it is difficult to make general comments. It appeared that some of these candidates were working without a graphical calculator; one is needed if candidates are to make a serious attempt. Some candidates did score well.

4757 Further Applications of Advanced Mathematics

General comments

The work on this paper was again of a high standard and about 40% of the candidates scored 60 marks or more (out of 72). Many candidates produced substantially correct solutions to all three of their questions. The most popular combination of questions was questions 1, 2 and 4. Overall, questions 1 and 2 were each attempted by about 80% of the candidates, question 4 by about 60% and questions 3 and 5 were each attempted by about 40% of the candidates.

Comments on individual questions

1) *(Vectors)*

There were very many good answers to this question. Most candidates used efficient methods to answer the four parts and applied the techniques competently, but arithmetic and algebraic slips were fairly frequent. In part (i), a common error was to use a scalar product instead of the vector product in the formula $\left|\overrightarrow{\mathbf{AC}}\times\overrightarrow{\mathbf{AB}}\right| / \left|\overrightarrow{\mathbf{AB}}\right|$. In part (iv), the point of intersection was almost always obtained correctly although it was not immediately obvious to all candidates, from the given result in part (ii), that the lines intersect when $p=5$.

2) *(Multi-variable calculus)*

The partial differentiation was usually done accurately in part (i) then applied correctly to find the normal line in part (ii). Part (iii) was also answered well. In part (iv) it was expected that the partial derivatives at P would be used to find the approximate small change in g. However, most candidates substituted in to obtain g in terms of μ, and could then write down the linear approximation; this was perfectly acceptable, and a similar method could be used in part (vi). Most could see how part (v) followed from part (iv). Part (vi) invited candidates to repeat the work done in parts (iv) and (v) using Q instead of P, but a substantial number were unable to make any progress here.

3) *(Differential geometry)*

Most candidates could find the arc length in part (i) and the curved surface area in part (ii). The method for finding the centre of curvature in part (iii) was generally well understood, but the correct answer was quite rare. As well as arithmetic slips, sign errors were common, particularly going in the wrong direction along the normal. Finding a unit normal vector also caused some difficulty. In part (iv), most candidates were able to find the envelope correctly.

4) *(Groups)*

This question was answered very well indeed, and it was only the final part (viii) which caused any problems. Despite having expressed all the elements of *G* as powers of the generator 2 in part (vi), most candidates were unable to pick out the subgroup of order 6.

5) *(Markov chains)*
This was found to be more difficult than the corresponding question last year. The techniques were generally well understood and calculators were used competently; parts (i), (ii), (vi), (vii) and (viii) were all answered very well. Many candidates were not sufficiently careful in part (iii), for example having found that $\mathbf{P}^4$ is the appropriate power, giving the answer as week 4 instead of week 5. Most could not answer part (iv) correctly; the usual error was to use elements from $\mathbf{P}^7$ and $\mathbf{P}^{15}$ instead of from $\mathbf{P}^7$ and $\mathbf{P}^8$. In part (v), many candidates used the formula $p/(1-p)$ forgetting that, in this case, the first day of the run is to be included in the expected run length.

4758 Differential Equations (Written Examination)

General comments

The standard of work was generally good, with the majority of candidates demonstrating a clear understanding of the techniques required. Almost all candidates opted for Questions 1, 2 and 4 and their solutions gained most of the available marks in many cases. The minority who chose to attempt Question 3 rarely achieved more than half of the marks available on this question.

There was a slight improvement in the accuracy of the arithmetic and algebra, compared with recent series. The main loss of marks was due to an inability to interpret solutions which had been successfully obtained.

Comments on individual questions

1) *(Second order differential equation)*

1) (i) The method required here was well known and many candidates scored full marks. The main source of error was due to an incorrect choice of particular integral, of the form Ax^2 rather than the form $Ax^2 + Bx + C$.

1) (ii) Most candidates earned full marks.

1) (iii) A description involving oscillatory motion with increasing amplitude was expected here. Most candidates simply stated that the curve became very large for large negative values of *x*.

1) (iv) Most candidates were able to comment correctly on the behaviour of the complementary function for large positive values of *x.*

1) (v) The graphs were variable in quality. Candidates were expected to use the information that they were given together with the knowledge that they had gained in the earlier parts of the question. Marks were scored for showing a minimum point at the origin, oscillatory behaviour with increasing amplitude for large negative values of *x* and parabolic behaviour for large positive values of *x*.

1) (vi) Candidates were expected to use $\frac{dy}{dx} = 0$ in the original differential equation to deduce the sign of the second derivative. Those who worked with their solution from part (ii) rarely scored any marks.

2) *(First order differential equations)*

2) (a)(i) Candidates were almost always successful in gaining full marks.

2) (a)(ii) There were two methods available here. The majority of candidates opted to find a complementary function and a particular integral, but often tried to work with a simpler form of particular integral than was required. The minority of candidates, who used an integrating factor method, were usually successful, though the presence of the constant carried through from part (i) was a source of confusion for some when they applied the initial conditions.

2) (b)(i)(ii) Almost all candidates gained full marks.

2) (b) (iii) It was pleasing to see that a significant number of candidates were able to complete the trigonometrical manipulation required here.

3) *(First order differential equations)*
This was the least popular question. Only a minority of candidates attempted it and of these attempts few earned more than 50% of the available marks.

3) (i) The method of solution, by separation of variables, was identified by most candidates, but several marks were often lost through inaccurate arithmetic. The sketch of the solution curve was rarely correct.

3) (ii) Again, any loss of marks was due to arithmetical slips.

3) (iii) This was a straightforward application of Euler's method and most candidates earned full marks.

3) (iv) Very few candidates scored any marks. They did not seem to know how to begin the process of setting up a differential equation from the given information.

4) *(Simultaneous differential equations)*
This question was extremely well answered by the vast majority of candidates.

4) (i) Solutions were almost always fully correct, exhibiting a sound knowledge of the method and accurate execution.

4) (ii) The complementary function and particular integral were found successfully by most candidates.

4) (iii) The method required was well known, but some solutions were marred by errors in algebraic manipulation.

4) (iv) Again, the method was known and the majority of candidates scored full marks.

4) (v) It was pleasing to see that many candidates were able to display a good working knowledge of the behaviour of the exponential terms obtained in the solutions in part (iv). A convincing argument involving both the behaviour of exponential terms for large values of t and the change in the constants obtained in part (iv) was expected. Statements such as "the behaviour is different" only earned marks when fully supported by a convincing argument.

4761 Mechanics 1

General comments

As usual there were many candidates who made good attempts at every question and quite a few scored very high marks. It was a pleasure to see many efficient solutions to Q5(ii), Q6(ii), Q7(iv) and Q8(iii), (iv) and (v).

There were more candidates this series who seemed unable to engage properly with several questions; sometimes it was lack of familiarity with the technical language of the topics (eg see the comments on Q3(ii) below).

Candidates seemed to have few obvious problems using the Printed Answer Book, no doubt thanks to sufficient practice being provided by centres. However, some candidates did put their work in the wrong boxes and they should not do that.

The work of many candidates would benefit from the use of clear, accurate diagrams not only when considering forces but also when attempting kinematics problems. Q1 is a good example of the need for clear diagrams even when dealing with apparently simple problems in kinematics. Such diagrams should include the direction taken to be positive; if this is done in Q1, then when substitution is made into $v^2 = u^2 + 2as$, it is much easier to see that a and s are either both positive or are both negative.

Comments on individual questions

1) *(An egg falling)*
Most of the candidates knew what to do. Unfortunately, many of them made sign errors or forgot to convert from centimetres to metres. A number of candidates worked via t, adding to their work, and some stopped there. Please see the general comments above.

2) *(Resolution of forces and tension in a rope)*

(i) Most of the candidates correctly took the tension in the rope to be 250 N, but some took it to be 250g N and a few did calculations indicating that they didn't realise that the tension was the same throughout the rope. Most candidates resolved properly and many candidates obtained the correct values; quite a few confused sine and cosine and so gave the wrong directions and a few didn't commit themselves to which directions their components were in.

(ii) A lot of candidates thought that the tension in the rope was still 250 N and quite a few attempted elaborate wrong calculations, suggesting that they had not seen this situation before.

3 *(Kinematics with constant acceleration using 3D vectors and finding speed)*
My impression was that more candidates used vectors appropriately than had done so in similar problems set in recent series, including correctly giving the answer to (i) as a vector not a scalar.

(i) This part was done very well by many candidates. The manipulation of the vectors was generally done well but there were quite a few errors with the signs. A fairly common error was to forget the mass when using Newton's second law. Few candidates gave a scalar as the answer.

(ii) Many candidates found the velocity correctly; of these quite a few correctly went on to find the speed as its modulus. However, many candidates seemed not to know what speed was in this context (one said, 'I thought it was the same as velocity'). Some left the calculation out, others gave a vector whose components were the moduli of the components of the velocity vector, others used the modulus of each term of $\mathbf{v} = \mathbf{u} + t\mathbf{a}$ and quite a few others calculated the vector $\mathbf{s} = \mathbf{u}t + \frac{1}{2} t^2\mathbf{a}$.

4) *(Newton's second law applied to boxes descending vertically supported by a parachute)*
There were many completely correct solutions to this problem, but many candidates could not draw the diagrams and/or did not know that they should use Newton's second law.

(i) Many candidates departed from the simple rules of entering all the known forces with their positive directions arrowed, labelling quantities with the known same magnitude with the same label and using different labels for things that could have different magnitudes. One commonly seen error was to show the 1030 N force acting on each box; another was to omit the force acting on box P due to the tension in the wire.

(ii) Most candidates knew that they should apply Newton's second law and did so at least quite well. The most common error was to omit one or more forces or introduce extra ones (such as the weight of box P being taken to be the weight of P and Q combined). Many candidates clearly did not use their own diagrams and many attempts at the equations of motion were more accurate than the diagrams.

(iii) A few candidates got lost but most who had obtained two equations in T and a solved them efficiently. Some candidates at this stage produced a 3rd equation for the 'overall' motion and solved this with one they had already found; there was nothing wrong with this but it might have taken a little longer. As is always the case with situations of this type, candidates who omit the weight of each box still obtain the same value for the tension in the wire – full marks were not awarded for 257.5 N found this way.

5) *(The direction of a vector)*

(i) Most candidates applied a method that obtained 56.3° or 33.7°, but rather fewer ended up with the correct bearing. However, it seemed that more were successful doing this than in recent series. Quite a few candidates wasted time going via arcsine or arccosine instead of using arctangent.

(ii) Quite a few (but a minority) of candidates knew exactly what to do and did it well. Some used a ratio method and some of these went via the angle of the vector; the common error made by those using these approaches was to use, say, $\frac{7}{9}$ instead of $-\frac{7}{9}$. Others set up simultaneous equations and others used trial and error (effective because k was a small integer). Many candidates did not know how to get started and large numbers equated the two vectors that were parallel instead of equating one to a multiple of the other.

6) *(A ball kicked off the end of a jetty that falls into the sea)*

(i) Most candidates were able to deal with this part. The most common error was to give the height above the sea instead of the height above the jetty.

(ii) A few candidates did not know how to get started, but most had a plan and many of them executed it efficiently. A number of methods were seen. By far the most widely used was to equate the height above the jetty to –3.6, solve the quadratic equation for t and then write down the distance as $12t$. Generally this was done very well but some candidates made substitution or sign errors. A method used by a few candidates was to obtain the equation of the trajectory, equate the height to –3.6 and solve the quadratic equation for horizontal distance; those who tried this usually did well. The method that led to most errors was that of dividing the motion into sections; many candidates who adopted this approach missed out a section or used wrong heights or velocities in their calculations. Some candidates used the formula they had remembered for range on a horizontal plane, but few of them considered the rest of the motion.

7 *(Interpretation of a displacement-time graph for motion in a straight line, the use of calculus and finding an expression in terms of time for a displacement)*
Quite a few candidates made mistakes in part (i) that seemed to be caused either by unfamiliarity with the exact meaning of the words or by not being clear exactly how information could be obtained from the given displacement-time graph.

(i) (A) Many candidates thought the answer was 12 m.

(i) (B) This part had more correct answers than (i) but 12 m was again the most common wrong answer.

(i) (C) Many candidates realised that the boundaries were 1s and 3.5 s but many of them did not give the strict inequality $1 < t < 3.5$. A mark was awarded to candidates who gave their answer as 2.5 s.

(i) (D) This part was answered better than the others in (i). Common mistakes were to give the times when $y = 0$, or to include them with the correct answers.

(ii) This was one of the best answered parts of any question on the paper. This may have been because of the instruction to use calculus; this instruction was ignored by some candidates, but none of them had a plan for answering the question without calculus. The most common mistakes were either to integrate twice or to integrate for one expression and differentiate for the other.

(iii) Most candidates know what to do; many of them found $t = 0.5$ but omitted $t = 1.5$.

(iv) Some candidates integrated; the most common error seen was omission of one or more constants of integration. Some candidates tried to use $s = ut + \frac{1}{2}at^2$; the most common error being for candidates to use t instead of $t - 3$. A very small number of candidates tried to fit a quadratic curve through the known points; their most common error was to use an expression of the form $(t - 3)(t - 4)$ instead of $k(t - 3)(t - 4)$.

Most candidates made some progress and there were many neat answers. Some candidates used the graph to see that $y = -4$ and $v = 0$ when $t = 3.5$ and correctly used these conditions instead of the velocity when $t = 3$.

8) *(Static equilibrium and acceleration of a tub on a horizontal floor. Some situations involving friction)*

(i) As always, candidates find this simple application of Newton's second law easy to do and there were few wrong answers.

(ii) This was answered well by many candidates, but some made slips. One quite common error was to try to incorporate the weight of the tub in the value of the friction. Most candidates indicated in some way that the frictional force was in the opposite direction to the applied force of 150 N.

(iii) Many candidates did not seem to understand what to do; they did not realise that the resultant being in the **i** direction meant that the components perpendicular to the **i** direction must have a zero resultant and instead tried to find some condition to be met by the components in the **i** direction. Credit was given for accurate resolution even when the underlying method was not sound. Candidates did not receive credit in this part if they simply stated the value of the resultant deduced from part (iv).

Of course, many of the candidates who adopted a sound strategy produced very short and efficient correct solutions to this problem; most of these went on to obtain full marks in parts (iv) and (v).

(iv) This part was done quite well by many of the stronger candidates. A common mistake was to write down the acceleration as distance ÷ time instead of using proper reasoning.

(v) This part was done well by quite a few candidates. Most who attempted it clearly knew what they had to do.

4762 Mechanics 2

General comments
Many good responses to this paper were seen and the majority of candidates could attempt at least some part of every question and gain credit for their efforts. The standard of presentation was variable and some candidates did not appreciate that poor use of mathematical terms and failure to state the principles or processes being employed could lead to avoidable errors and to loss of marks. As in previous series, those parts of the questions that required a candidate to *explain or show* a given answer were the least well done. Many candidates did not give enough detail in either case. As has also happened in previous series, those candidates who appreciated the value of a good diagram were generally more successful than those who either omitted a diagram or included a poorly drawn one.

There was evidence to suggest that some candidates ran out of time but this usually seemed to be because they used inefficient methods of solution.

Comments on individual questions

1) This question on impulse and momentum was a high scoring question for many candidates with almost all showing an understanding of the principles involved. Those candidates who drew diagrams were generally more successful than those who failed to do so.

(i) The majority of candidates obtained at least 3 out of the 4 marks for this part. Those that lost a mark usually did so because they had failed to give the direction of the velocities.

(ii) Almost all of the candidates showed understanding of the principle of conservation of momentum and Newton's experimental law. However, once again, many did not give any indication of the direction of the velocity of Q. It was pleasing to see fewer sign errors in applying Newton's experimental law than have been seen in recent series.

(iii) Many candidates could calculate the speed of the object but few went on to find the speed of separation of the object from the sledge.

(iv) It was encouraging to see that the majority of candidates obtained the right answer to this part.

2) Candidates seemed to understand the principles to be employed in this question on centre of mass and could obtain significant credit for their efforts.

(i) Very few candidates had difficulty with this part and it was common to see completely correct answers.

(ii) The majority of candidates could show the given results for this part but some wasted time by not employing the answer from the previous part and instead restarting the calculation from scratch.

(iii) It was very pleasing to see that a high proportion of candidates appreciated what was required for this part of the question. Most of them opted to find the 'angle of tilt' for each edge, very few took the easier route of considering the distance of the *x*-co-ordinate of the centre of mass to each of the edges under investigation.

(iv) This part caused problems for many candidates. Most realised that moments about RS should be attempted but then failed to include the moments of both components of the tension. Others used the mass of the stand rather than the weight.

3) Many excellent answers were seen to this question on frameworks with almost all of the candidates showing some understanding of the processes required.

(i) Many candidates obtained full marks for this part but a significant minority did not give sufficiently detailed answers to show the given result for *T* and how the values for *Y* and *X* had been obtained.

(ii) As has happened in previous series, the quality of the diagrams offered was very variable. Many candidates failed to adequately label the internal forces; others omitted the external forces.

(iii) The vast majority of candidates understood that they had to resolve at the pin-joints to obtain the internal forces in the rods. The candidates who were most successful were those who had drawn a clear diagram for the previous part and then *used* it to find the forces required. A significant minority of candidates produced equations that were inconsistent with their diagram and sign errors were common. A small number of candidates assumed that the angles at A and C were 45° even though the dimensions given were not consistent with this.

(iv) This part was not done well by many candidates. Most stated that the only rod with an internal force that would change would be BD but could not give a coherent reason for their answer. Many stated without any explanation that the force in BD would have the same magnitude but be in the opposite sense to that already found.

4) Most candidates made some progress worthy of credit on this work-energy question appearing to understand the methods to be employed.

(i) The majority of the candidates gained most of the marks for this part. However as in other series, many candidates appeared to think that $F = \mu R$ in all circumstances and hence did not state that, in this particular case, friction was at its maximum value.

(ii) This part posed few problems to the majority of candidates but a small minority did not appreciate that they had to resolve the applied force in the direction of motion to obtain the work done.

(iii) Many fully correct and full solutions using the constant acceleration formulae were seen to this part of the question.

(iv) This part of the question was found difficult by the majority of the candidates. Some, having given a good explanation in the previous part as to why the acceleration was not constant, proceeded to use the constant acceleration equations along with Newton's second law. Others failed to realise that the frictional force would change from that in part (i) whilst others mixed energy with forces in an attempt at a work-energy equation.

4763 Mechanics 3

General comments

This paper was found to be relatively straightforward; there was a lot of very competent work and the marks were generally high. About half the candidates scored 60 marks or more (out of 72), and very few scored less than 30. Nevertheless, the questions on circular motion and simple harmonic motion caused problems for a substantial proportion of the candidates.

Comments on individual questions

1) *(Elasticity and dimensional analysis)*
This question was very well answered and the average mark was about 16 (out of 18). Approximately 40% of the candidates scored full marks.

(a)(i) This was well understood, and most candidates were able to show that the system is in equilibrium.

(a)(ii) The elastic energy stored in the string was almost always calculated correctly.

(a)(iii) The use of conservation of energy was well understood and only a few failed to include all of gravitational, elastic and kinetic energy in their equation. Sign errors were also quite rare, although a substantial number (perhaps 40% of the candidates) forgot to double the elastic energy found in part (ii) to account for the two strings. There were some who simply found the point at which the strings just became slack.

(b)(i) The dimensions were usually given correctly, although there were some errors with stiffness.

(b)(ii) The method for finding the powers was very well known, and it was almost always carried out accurately.

2) *(Circular motion)*
This was found to be the most difficult question, but even so about 20% of the candidates scored full marks. The average mark was about 12.
Many candidates used an angle θ in their work without indicating (on their answer script) which angle this was and this often led to confusion and errors. In these comments, θ is the angle between OB and the vertical.

(i) Most candidates found the normal reaction correctly, but a fairly common error was to resolve in the direction of OB ($R = mg\cos\theta$) instead of vertically ($R\cos\theta = mg$).

(ii) The use of v^2/r for the acceleration towards the centre was well understood, although a few used $r = 2.5$ instead of $r = 1.5$.

(iii) This was answered quite well using conservation of energy and considering the vertical forces at the lowest point. Most candidates worked numerically, but a few produced elegant algebraic solutions.

(iv) This was the most challenging item on the question paper. Candidates needed to obtain two equations involving v and θ, and solve them simultaneously. The first equation, from forces towards the centre ($2mg - mg\cos\theta = mv^2/r$), was usually correct, although failure to resolve the weight properly was a fairly common error. However, the second equation, from conservation of energy ($\frac{1}{2}mv^2 = mgr\cos\theta$), was very often omitted or incorrect. Many candidates thought they could just resolve vertically ($2mg\cos\theta = mg$) and obtained $\theta = 60°$.

(v) A good proportion of the candidates knew that the tangential acceleration was $g\sin\theta$ (although quite a few gave it as $mg\sin\theta$), but most had not been able to find the value of θ.

3) *(Centres of mass)*
This was the best answered question, with an average mark of about 16. Approximately 45% of the candidates scored full marks, and most of the errors were minor slips such as failing to give answers in an exact form.

(i) The method for finding the centre of mass of a solid of revolution was well understood and most candidates carried it out accurately.

(ii) Most candidates found the centre of mass of the lamina correctly. Only a few made errors in evaluating the definite integrals, but a factor of ½ was sometimes omitted from the *y*-coordinate.

(iii) The great majority of candidates realised that they could simply interchange the coordinates of the centre of mass of R_1. A few did not use the symmetry, and worked it all out from scratch.

(iv) Finding the centre of mass of the combined lamina was generally done well, although some just added up the coordinates of the three parts and divided by three.

4) *(Simple harmonic motion)*
The average mark on this question was about 14, and about 25% of the candidates scored full marks.

(i) Almost every candidate proved the given result satisfactorily.

(ii) Most candidates understood how to find *B* and ω, although $B = 16$ (instead of $B = -16$) and $\omega = 4$ (instead of $\omega = 0.25$) were fairly common errors. Many used complicated methods to find *A*, such as $v^2 = \omega^2(A^2 + B^2 - x^2)$, rather than simply differentiating to find *v* and substituting $t = 0$.

(iii) Many answered this efficiently, calculating $a = \sqrt{A^2 + B^2}$, ωa and $\omega^2 a$ to find the maximum displacement, speed and acceleration. More complicated (but correct) methods were very often used, such as finding the value of *t* when $v = 0$ to calculate the maximum displacement.

(iv) Almost all candidates differentiated and substituted $t = 15$, although there were some slips in the differentiation, and a large number had their calculators in degree mode.

(v) A fair proportion answered this correctly, often supporting their argument with a clear displacement-time graph. However, the majority did not consider carefully enough how P had moved and changed direction. Again the use of degree mode prevented many from obtaining the correct answer.

4764 Mechanics 4

General comments

The standard of work seen was, in general, very high. Many candidates found the work on equilibrium difficult.

Comments on individual questions

1) *(Variable mass – Rockets)*

1) (i) Generally well answered, though the majority of candidates failed to account properly for the signs of their δm or $|\delta m|$ terms.

1) (ii) The technique of separating variables and integrating was well understood and executed by most candidates, though a common error in some candidates was to treat *m* as a constant.

2) *(Variable force)*

2) (i) Again, the process of separation of variables was well understood. This was well answered, with only small errors in signs or manipulation.

2) (ii) Most candidates knew that they had to replace *v* with $\frac{\mathrm{d}x}{\mathrm{d}t}$ and integrate again. Errors were mostly in signs and manipulation as in part (i).

2) (iii) Candidates who had answered parts (i) and (ii) usually gave good answers here, though many discussed only the velocity *or* displacement. The mark for the velocity required candidates to state that the velocity of the particle was tending to zero, not just "slowing down".

3) *(Equilibrium)*

3) (i) This was almost universally done correctly. Some candidates found the GPE relative to the horizontal rail, which gave the correct value for V', but not for V.

3) (ii) This question caused many candidates problems. Though almost all knew how to find the positions of equilibrium and their nature, they were not able to follow through the process, particularly with regards to the position where $\theta = \sin^{-1}\frac{2mg}{9\lambda}$.

(A) The first part was usually well answered by most candidates, with both values of θ found and the correct expression for the second derivative. Some candidates included extra values for θ. Most candidates knew that they had to find the sign of the second derivative for their value of θ. In general the work for $\theta = \pi/2$ was good, but many struggled to make any progress with $\theta = \sin^{-1}\frac{2mg}{9\lambda}$, often because they did not rewrite their expression for $V``$ in terms of $\sin\theta$.

(B) Almost all candidates that got this far through the question realised that the condition resulted in only one position of equilibrium and then successfully determined the nature of that position.

(C) Again many candidates found that only one position of equilibrium existed and that $V'' = 0$ at that point. However, very few realised that this resulted in an ambiguity that could be resolved; most labelled this position as a point of inflection and were unsure how to proceed.

4) *(Rotation - Moments of inertia and dynamics of a rotating body)*

4) (i) This part was not well answered in general. Many candidates used an expression of the form $\rho\pi r^2\delta r$ and those that gave the correct expression for δm in terms of y and δx often failed to change variables accurately, or used $I_{\text{slice}} = \frac{1}{2}Mx^2$. As a result, most candidates did not get the required expression for the moment of inertia of the cone; many then attempted to reverse engineer an extra constant multiple rather than find their error.
Some candidates added unnecessary work by deriving the mass of the cone from first principles instead of using the formula provided.

4) (ii) This part was usually done correctly. Many candidates did far more work than was necessary by finding the density of the cone and the volumes of each solid to find their masses instead of using proportionality.

4) (iii) Almost universally correct.

4) (iv) The concept was often well understood, but many of the candidates did not find the centres of mass of the cones relative to the same point

4) (v) This part was not answered well, or at all, by many candidates. The essential concept that moment of impulse is equal to the change in angular momentum was often quoted, but not used correctly.

4766 Statistics 1

General comments
The level of difficulty of the paper appeared to be entirely appropriate for the candidates with a good range of marks obtained. Question 4 proved to be the most challenging question on the paper and question 7 the easiest. Very low scores were rare and very few candidates seemed totally unprepared. There were, on the other hand, a good number of almost completely or completely correct scripts. There seemed to be no trouble in completing the paper within the time allowed.

Most candidates supported their numerical answers with appropriate explanations and working although some rounding errors were noted particularly in questions 5 and 6. Arithmetic accuracy was generally good although there is still evidence of candidates not being proficient or sensible in their use of calculators. In particular the simplest method of doing question 5(i) is by use of the statistical functions on a calculator, but few candidates used this approach. Amongst some candidates, there was evidence of incorrect use of point probabilities instead of tail probabilities in question 6 and of a totally wrong method to establish outliers in question 1.

The scripts were invariably well presented and legible with the use of a pre-printed answer book not appearing to constrict candidates' work; most candidates were able to answer in the space provided in the answer book, and only a few used additional sheets.

Comments on individual questions

1) (i) Most candidates gave the correct answer of positive skewness although a few thought that the skewness was negative; the occasional response of 'skewed to the right' was not acceptable.

1) (ii) The answer of 2.3 for the IQR was obtained by most candidates. Wrong answers included (17.6 – 7)/4 = 2.65 and errors in stating the value of the upper quartile. Many candidates made mistakes in finding the boundaries for outliers with the use of median ± 1.5 × IQR being very common. Those who used the quartiles occasionally combined the values with multiples of 1, 2 or even 3 of the IQR. The use of the limits to establish the presence of outliers, or otherwise, was good although a number of candidates used a value of 18 rather than 17.6. This error was treated generously. Some candidates curiously tried to argue in terms of the standard deviation.

1) (iii) There were many sensible and complete answers with the most common including 'an error in the data', 'no lower outlier due to the minimum wage' and 'the outlier being a manager or supervisor'. Some candidates only gave one reason or just concentrated on one end of the data. A very few candidates just repeated the information about outliers given in part (ii).

2) (i) This was very often correct but a number of candidates stopped when they had worked out the first two terms. Some candidates tried to sum the terms without k or the k became an afterthought after the summation was completed.

2) (ii) The calculation of E(X) and Var(X) was well executed on the whole. There were still some candidates who mistakenly divided E(X) and/or Var(X) by 4. Some forgot to square E(X) in the calculation of the variance. A few candidates thought that they could attempt this question without using any probabilities.

3) (i) Most candidates knew that they had to find frequency density and on the whole were very successful. Occasionally-seen errors were attempts to multiply frequency by width or divide by mid-interval or divide width by frequency. However the most common error was to use the given frequencies as the heights. Labelling was not always successful, and although a pleasing number of candidates knew that the label should be frequency density, some gave it simply as frequency, and those using a non-unitary class width as standard often had difficulty indicating this correctly on the graph. The vertical linear scale was usually correct (and sensible!). On the horizontal scale the majority of candidates were able to get the width of the bars correct, but a number of candidates thought that they should number their scale with inequalities rather than giving a correct linear scale. Very few candidates mistakenly left gaps between the bars. Use of rulers is to be encouraged to produce a clearer diagram.

3) (ii) This was usually well done although some candidates seemed to think the frequency was 100. Although candidates should have been trying to find the 45.5th value many were looking for the 45th value; this error was not penalised. Many candidates failed to indicate that it was the 45th value not just 45 that was in the correct interval.

4) (i) This was the least well done question on the whole paper. The majority of candidates had 0.2×0.2 in some form as their answer.

4) (ii) This part was done slightly better, but even so the correct answer of 0.0384 was fairly unusual. Common errors included $({}^{1}/_{5})^{5}$ and ${}^{1}/_{5!}$ whilst some candidates tried to use some form of binomial probability.

4) (iii) Most candidates managed to subtract their answer to part (ii) from 1 although some made arithmetical errors whilst others did not attempt this part at all.

5) (i) This question was answered more successfully than in the past. There were many wholly correct solutions, usually showing full working but occasionally by use of calculator. The vast majority of candidates found the mean correctly, although a number of incorrect answers were seen including ${}^{75}/_{59}$, ${}^{59}/_{6}$ or ${}^{59}/_{5}$. Some lost a mark because of inappropriate rounding of their answer. Many candidates found the standard deviation correctly but there was a wide variety of wrong methods including finding $(fx)^2$ or xf^2 instead of fx^2. A few candidates correctly found fx^2 but then forgot to subtract $(\sum x)^2/n$ or used 59, 58 or 6 rather than 75 as the value of n. Only a few candidates divided by 75 and thus found the root mean square deviation and only a few forgot to square root their variance. Candidates who obtained ridiculously large answers often did not seem to realise that their answers could not possibly be correct.

5) (ii) Most candidates found the new mean successfully. However many stated that the standard deviation would not change. Units were often missing or only given for the mean. A number of candidates gave the new mean as 0.82p rather than 82 pence or £0.82. Some candidates did not realise that they could just multiply their answers to part (i) by 1.04 and instead multiplied the numbers of loaves by 1.04 then recalculated the new mean and standard deviation.

6) (i)(A) The vast majority of candidates found the correct value of 0.2385, with most preferring to use a binomial expression rather than tables. Occasionally an answer of $P(X \leq 2) = 0.7338$ was seen.

6) (i)(B) Candidates were less successful in this part, with mistakes occurring due to rounding errors when using the point probability approach, the omission of a term such as $P(X = 0)$ when using point probabilities, misuse of tables, or answers such as 1 – 0.2835, 1 – 0.9018 or 1 – 0.4503 rather than 1 – 0.7338.

6) (i)(C) This was very well answered although a significant number of candidates rounded to 2 or even 1, losing a mark.

6 (ii) Many candidates constructed the hypotheses correctly although a few used "equals" for H_1. The main loss of marks came from poor notation such as $P(X) = 0.1$, $H_0 = 0.1$, X=0.1, P(0.1), etc. However many candidates still failed to define p as the probability that a randomly selected tile is faulty. Virtually all adequately explained why H_1 took the form it did.

6) (iii) Few candidates confidently scored full marks in this part. Some candidates often had little idea as to where to begin; other candidates used point probabilities and even those who used the correct probabilities of 0.0982 and 0.0282 with a comparison of 0.05 often started the critical region at 4 rather than 5. Occasionally the critical region was given as 'from 5 to 8' rather than 'from 5 to 18'. Some candidates failed to show necessary working; an answer along the lines of 'the first value in tables above 0.95 is 4, so critical region is 5 to 18' did not score full marks. A very small number of candidates thought their comparison should be with 0.025 rather than 0.05. As has been stressed in past examiners' reports, candidates must quote specific probabilities in finding critical regions and then explicitly compare these probabilities to the significance level. If they do not do this they may not get any marks.

Although it is given in the mark scheme, it is worth repeating here the recommended method for comparing the probabilities with the significance level. Candidates should find the two upper tail (in this case) cumulative probabilities which straddle the significance level.

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9018$ or $0.0982 > 5\%$

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9718$ or $0.0282 < 5\%$

6) (iv) This was poorly answered with very few using their critical region and stating that 4 was outside it. Most successful answers started again with 0.0982>0.05; often those using this approach wrote down 0.0282<0.05 and wrongly rejected H_0. The other major error was to finish by saying 'insufficient evidence to reject H_0' and then making no reference to the context. The use of point probabilities was again frequent, even by some candidates who had successfully used cumulative probabilities in part (iii).

7) (i) This question provoked a variety of responses. Those who read the question carefully often gained high marks but there were many candidates who got off on the wrong foot by either having too many sets of branches on their tree diagram eg (0900, 1000, 1100, 1200) or having the 1000 branches labelled with probabilities of 0.5 instead of the correct 0.95 and 0.05. Some candidates omitted some or all of the labels for 'on time' and 'late'.

7) (ii)(A) Whether using their tree diagram or not, this was well answered and most candidates gained both of the marks.

7) (ii)(B) Most candidates were able to trace their way through the tree diagram to achieve the correct response of 0.0325. A generous follow through was in place for those who may have made an error in one or more of their probabilities.

7) (ii)(C) This was again well answered by most candidates, and once again a generous follow through was in place. Candidates should be reminded that total accuracy in intermediate working is important when dealing with probabilities eg 0.857375 should not be rounded to 0.86. The cumulative effect of 4 prematurely rounded probabilities caused some candidates to have their final answer outside the required range.

7) (iii) Many candidates realised they had to evaluate the 1000, 1100 and 1200 on time and the 1000 on time, 1100 late and 1200 on time to reach 0.885875 or its equivalent on follow through. Often they stopped at this point, not realising the conditional probability requirement of the question. The more discerning candidates used their answer from part (ii) (C) to complete the question successfully.

4767 Statistics 2

General comments

A strong overall performance from candidates yet again. The vast majority demonstrated a good working knowledge and could convey their understanding to a reasonable degree. Many candidates were less than convincing when clear explanations were required.

Comments on individual questions

1) (i) Well answered, with most earning full marks. Odd slips with ranking meant some candidates lost a mark. Some lost the final M1A1 through errors in applying the Spearman's rank correlation formula; typically, neglecting to include "1 – ". A small number failed to rank the data and were awarded 0/5.

1) (ii) Well answered; the majority of candidates picked up 5 out of the 6 marks available. The hypotheses needed to be stated in words and refer to the "association between the judges' scores", or equivalent, to earn full credit. Many candidates lost a mark for failing to include the word "positive" in their alternative hypothesis, despite subsequently carrying out one-tailed tests. Very few candidates were awarded the mark for indicating that the test was for association between "X and Y in the population". The majority of candidates identified 0.6429 as the correct critical value and went on to compare their test statistic and make a suitable conclusion to earn the final 3 marks. Alternative critical values included pmcc critical values, incorrect Spearman's critical values (2-tail or incorrect n used) and, in a few cases, values from the t distribution.

1) (iii) Well answered. Awareness that the underlying distribution required to carry out a test based on the pmcc is "bivariate Normal" is still not wide amongst candidates. However, most candidates scored 4 out of the 5 available marks for sketching a scatter diagram and using it to comment on the validity of the distributional assumption. Candidates should be aware that their explanations should leave no doubt as to their meaning; answers which require examiners to refer to the question then deduce the meaning are not deemed worthy of credit. For example, stating "ellipse, so yes" is not considered to be a discussion.

2) (i) Again, candidates found it difficult to express what they wanted to say here. Examiners were looking for comments relating to the counts occurring independently and with a uniform average rate. Comments referring to the "data" or "results" or "values" or "variables" occurring independently, were seen frequently. Many comments suggested that the rate of decay was constant.

2) (ii) Well answered on the whole, but some doubt was introduced by those writing their answers as $\lambda = 3.4$, or Po(3.4). Examiners needed to be left in no doubt that the candidates answers showed that the variance was 3.4 to award the mark.

2) (iii)(A) Well answered.

2) (iii)(B) Well answered. A few made mistakes in using tables; typically, looking up $P(X \le 3)$ then finding $1 - P(X \le 3)$.

2) (iv) Well answered. Including some attempts to use the Normal distribution.

2) (v) Most candidates managed to score at least 3 out of the 5 available marks, and many earned 5/5. The most common mistake was in using a continuity correction; some used 40.5 and others neglected to use one at all. A small proportion of candidates were unsure what value to use for the standard deviation, with 3.4 seen frequently. It was not uncommon to see answers where candidates had found $P(X \leq 40)$; those making a sketch to show which tail they were using tended not to make this mistake.

2) (vi) Well answered, with most earning full marks. A few candidates found the overall mean of 4.8 but then did not know what to do with it.

3) (i)(A) Well answered. A few lost a mark through inaccurate use of the Normal probability tables; rounding of z values to 2d.p. and/or neglecting to use the difference column of the Normal distribution tables. A small number ended up finding $P(X > 65)$. Many candidates erroneously applied continuity corrections and were consequently penalised.

3) (i)(B) Well answered.

3) (ii) Well answered. A small number of candidates multiplied their previous answer by 5. Some found the probability that only one of the 5 riders recorded times between 60 and 65 minutes.

3) (iii) Well answered. Most candidates scored all three marks. Using +1.645, leading to an answer of 71.544 minutes, was seen frequently.

3) (iv) Well answered. Very few candidates were awarded the mark for defining μ as the population mean time on the new course. Otherwise, most of the remaining marks were usually given. In this hypothesis test, candidates were expected to write their hypotheses in terms of μ; other symbols were accepted only if defined as the population mean. Most candidates managed to correctly obtain the test statistic of –0.968, compare it with –1.645 and make a suitable conclusion. Alternative methods were allowed; for example, finding a p-value of 0.1665 and comparing it with 0.05. Some candidates revealed their lack of understanding by making inappropriate comparisons such as "–0.968 < +1.645" or "0.968 > 0.05" or "0.1665 < 0.968" and so on. For the final mark, candidates were required to answer in context.

4)(i) Most candidates scored full marks in this part of the question. The hypotheses needed to be written in words and to include reference to "category and type of runner". In calculating X^2, a small number of candidates lost marks for inaccurate working either through premature rounding or through odd slips. Some neglected to provide "a table showing the contributions of each cell to the test statistic" despite being requested to do so. Most candidates were awarded the final four marks in this part of the question, but some did not specify the number of degrees of freedom and gained three marks. Those using anything other than 4 degrees of freedom were awarded no marks here.

4) (ii) In this part of the question there were two marks available for comments relating to each of the three categories of runner. Candidates were required to refer to their table of contributions then make a judgement regarding the level of association between category of runner and type of running. To score full marks, candidates had to make clear, accurate comments; however, most found this difficult. Poorly-worded comments could achieve no more that one mark out of the two available for each category of runner. From the comments provided it was often difficult to tell whether the table of contributions had been considered; many made simple statements about whether there were more (or fewer) runners observed than expected, regardless of the level of association. From this it was difficult to conclude whether or not the candidates were aware that a very small contribution indicated that the observed results were “as expected”. Ambiguous answers such as “the Junior track is higher than expected” were not uncommon. Some candidates thought that if there were no association then there would be an even spread of results. Those candidates who made general statements giving reasons why “juniors prefer running on tracks” rather than commenting on the statistics were given no credit.

4768 Statistics 3

General comments
There were 428 candidates from 83 centres (June 2009: 371 from 77) for this sitting of the paper. The overall standard of the scripts seen was very pleasing: the increase in the size of the entry seemed to correspond to an improvement in quality. There were many occasions when questions were answered completely, or almost completely, correctly. It was pleasing to note that most candidates remembered to state the hypotheses in Questions 2 and 3 despite not having been instructed to do so.
However, in those parts of Questions 1, 2 and 3 where discussion or comment was called for, very few candidates showed little, if any, statistical insight.
Yet again candidates continue to show poor regard for clear and accurate notation in their work, and for the need for accurate computation. On a number of occasions the work contained glaring errors of a kind that one simply would not normally expect to see at this level. Furthermore, despite the remarks made in recent reports on this unit concerning the quality of the language used in the conclusions to hypothesis tests, there remains much room for improvement in this respect.

Invariably all four questions were attempted. With few exceptions there was no evidence to suggest that candidates found themselves unable to complete the paper in the available time.

Comments on individual questions
1) *(Sampling; Combinations of Normal distributions. Salesmen and their expenses.)*

(i) While most, but by no means all, could name "Systematic sampling" correctly, far fewer were able to identify and fix the weakness without resorting to a completely different method, usually simple random sampling, which missed the point.

(ii) It was very rare indeed for anyone to not get this part right.

(iii) The correct probability was obtained here most of the time, but, unsurprisingly, a largish minority of candidates found the variance of $3X$ instead of $X_1 + X_2 + X_3$. One way or another most candidates identified the need for independence, but then far fewer were able to supply a convincing explanation that focused on the independence of one month from another. Seasonal variation was a popular but incorrect suggestion.

(iv) There were the inevitable errors with the variance in this part (usually neglecting to square either or both of the factors 0.45 and 0.1), but on the whole the work seen was impressive. Allowing for these errors, candidates worked through the calculation to a creditable conclusion and many scored full marks.

2) *(The t distribution: test and confidence interval for a population mean. The specific gravity of a brewing mixture.)*

(i) Some candidates gave clear and correct answers to this part but for many others their responses were woolly and vague.

(ii) There were many good answers to this part. However, in many cases there was room for improvement in all aspects of it: stating the hypotheses concisely, accurate calculation of the test statistic from the data and an appropriately worded non-assertive conclusion. In the calculation of the test statistic considerable tolerance was allowed, but candidates are expected to know how to use their calculators efficiently and they should be able to obtain an accurate result.

(iii) Most candidates knew how to calculate the required confidence interval, but as in the past there were too many candidates who, for some reason, chose to switch to the Normal distribution instead of staying with the *t* distribution that they had used for the test in part (ii). While many correct well-rehearsed explanations of the meaning of a confidence interval were seen, there was a fairly widespread lack of understanding of this concept.

3) *(Wilcoxon paired sample test. Effectiveness of a vaccination programme. Chi-squared test of goodness of fit. Benford's Law.)*

(a)(i) Hardly any candidates could explain why the use of paired data was appropriate. A common response was that there were differences between the two sets of data which the pairing would eliminate thus allowing the differences to be investigated. ("Within" instead of "between" would have resolved the matter satisfactorily.)

(ii) The hypotheses here were very poorly expressed. In many cases it seemed that candidates thought that they were testing a mean rather than a median while in many others there was no mention of a parameter at all. Furthermore, references to difference and/or population were often missing.
The calculation of the test statistic, on the other hand, was almost always correct. There were occasional issues with the conduct of the test but, by and large, apart from assertive conclusions, it was done well.

(b) The test statistic was usually obtained correctly and, as above, apart from some assertive conclusions, the test was conducted as required. Unusually, however, there were a few instances of candidates using the critical value chosen from the left-hand tail of the distribution.

4) *(Continuous random variables; the Central Limit Theorem. The lifetime of domestic appliances.)*

(i) In this part some candidates were careful to ensure that their work was complete and convincing, including stating that $e^{-\lambda x} \to 0$ as $x \to \infty$. Others were probably reliant on the stated result to keep them on the straight and narrow.
The quality of the sketches seen was very mixed: axes drawn without a ruler and not labelled; curves that were not drawn carefully and sometimes went into the second quadrant.

(ii) In this part many candidates did not seem to think carefully enough about the relationship between the given result and the integrals needed to find the mean and the variance. The consequence was that a factor of λ was often missing, resulting in a mean of $\frac{1}{\lambda^2}$ and a variance of $\frac{2\lambda - 1}{\lambda^4}$, the latter having potentially disastrous consequences later on. Thankfully very few ignored the given result and attempted to integrate by parts. There was some evidence that a few candidates were able to quote the results for the exponential distribution.

(iii) This part was either answered very well or badly. There was a clear split between candidates who realised that this part was all about the Central Limit Theorem and those who appeared unfamiliar with it. This was heightened further by the apparent lack of appreciation that the variance of the sample mean was needed. Many did not realise that, since they were given that $\mu = 6$, they could find λ and hence the variance. Furthermore, with the exception of perhaps just one or two candidates, of those who made the mistake described in part (ii) above none realised that they were ending up with a negative variance.

(iv) Candidates who answered part (iii) correctly usually had little difficulty in this part. The simplest approach was to consider the usual criterion for an outlier and to set out the evidence that 7.8 was indeed an outlier. However a worrying aspect of many attempts was that candidates did not seem to recognise what they were supposed to have written down in part (iii), ie the distribution, including the variance, of the sample mean. Consequently many of the calculations seen were inappropriate. Meanwhile, attempts that did not contain hard evidence were deemed unacceptable.

4769 Statistics 4

General comments
There were 31 candidates from 12 centres (plus 4 more centres, each of whose candidates were absent). While this is obviously a small entry, it is pleasing that it is holding up. It is only slightly down on last year and is a noticeable and welcome increase from the year before last.

It is also pleasing to report that there was much very good work – for the paper as a whole and for each individual question. Sadly there was also some poor work, but the good work was very much in the majority.

As usual, the paper consisted of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted, and all candidates obeyed this. Question 4, on design and analysis of experiments, was very much the least popular question, with only a handful of attempts – not a feature that has occurred in previous years. The other three questions were equally popular.

Comments on individual questions

1) This was on the "estimation" option. It was mainly about investigating two unbiased estimators and comparing their variances.

The question involved integration of functions of the form $x^n e^{-x}$ for fairly small integer values of n. Candidates, even those who were successful, seemed to make fairly heavy weather of this. Many candidates did much more work than they needed by not seeing that the integration by parts in part (i) of the question re-created the pdf of the original random variable whose integral could be written down as 1; and then again by not seeing that the integration by parts in part (ii) re-created the integral that had already been found in part (i).

Part (iii) sought an explanation of two desirable features of the estimator – its variance becomes very small as n increases and so, being unbiased, it becomes increasingly concentrated at the correct value of the parameter. Most explanations more-or-less made these points, but sometimes not very securely. It was pleasing to see that some candidates were familiar with the correct technical term "consistent".

The second estimator was introduced in part (iv). Candidates generally knew that it should be compared with the first in terms of their variances. Some candidates had the relative efficiency definition "upside down", though they still generally knew how to use the result.

2) This was on the "generating functions" option and explored the Normal approximation to the Poisson distribution.

Many intermediate answers were given in this question, partly for the comfort of candidates as they successfully worked through it and partly so that candidates who could not derive a result could nevertheless use it in the sequel. The usual point has to be made that, where an answer is given, candidates have to be *convincing* in their derivations of it. In fact most candidates were, except in the limiting result in part (v).

Part (i) asked for the probability generating function of the Poisson distribution and part (ii) sought derivation of the mean and variance. These parts were usually done without difficulty.

In part (iii), it was remarkable that many candidates were unable *simply* to write down the mean (zero) and variance (one) of the standardised variable.

In part (iv), most candidates knew that the moment generating function is of the same form as the probability generating function, merely with a change of variable; and most candidates could write down the linear transformation result without much ado. Using this to obtain the moment generating function of the "standardised Poisson" required some care in algebra, but mostly this was done successfully.

Part (v) was where more than a few candidates had problems, not knowing how to handle the limiting process. However, several candidates were fully successful here.

In part (vi), candidates mostly realised the importance of the uniqueness of the relation between a distribution and its moment generating function, though this was not always explicitly stated. The "unstandardising" was usually understood, except that several candidates did not seem to appreciate that the unstandardised mean and variance were both λ (the parameter of the original Poisson distribution) – which is a key feature of this Normal approximation.

3) This question was on the "inference" option.

Part (i) asked, fairly formally, for the acceptance region to be set up for an unpaired Normal test. Many candidates knew what to do and correctly obtained it (even if not necessarily following the explicit steps that are set out in the published mark scheme), but there were a number of errors here. The worst error, and it is a serious mistake and especially sad to see in candidates working at Statistics 4 level, was to express the acceptance region as a double inequality on the difference in the *population* means, not the sample means. The logical absurdity of this seemed to escape these candidates. Other rather bad errors occurred in the denominator of the test statistic, where a number of incorrect forms appeared.

Part (ii) opened by giving a numerical value of the difference in sample means and asking what was the result of the test. All that had to be done here was to note that this value was outside the acceptance region and therefore the null hypothesis is rejected. While several candidates did this, many actually performed the full significance test – this of course leads to the correct answer but, as well as being a waste of time and effort, suggests that these candidates did not understand the force of the general concept of an acceptance region.

Part (ii) concluded by asking for a confidence interval. This was usually done well.

Part (iii) moved on to the Wilcoxon rank sum test. A few candidates made the error of thinking that "the smaller sample" (from which the rank sum is obtained) means the one which contains the numerically smaller values rather than the one whose size is smallest. This matters, because published tables are always drawn up on the basis of the rank sum coming from the sample of smaller size. It was however particularly pleasing that most candidates realised that, in this case, they needed to consider the upper tail points as well as the tabulated lower tail points in order to establish whether the result was significant; and, further, most of these candidates used a valid method based on symmetry to obtain the upper point.

4) This was on the "design and analysis of experiments" option. As already mentioned, there were very few attempts at this question, so these notes are only very brief so as to avoid the danger of accidental identification of individual candidates.

Candidates knew that the required design was randomised blocks and gave good descriptions of this design. They were generally sound with the modelling, and they were able to construct and interpret the required one-way analysis of variance.

4771 Decision Mathematics 1

General comments
The mean score on this paper was lower than was the case in 2009 (46 compared to 50). Performance at the lower end was comparable to 2009, with a grade E boundary of 36 (also 36 in 2009). Marks at the top end were harder to obtain (grade A boundary 55 against 63 in 2009). There was one mark which was recognised in advance as being difficult, Q3(iii).

Some candidates made mistakes in producing diagrams, rubbed their initial effort out, and then redrew an improved version on top of this. (This was seen most often on questions 6 and 3.) This caused significant problems. The scanning process is sensitive and what appears on screen is often the original diagram superimposed on the new one making it very difficult to read or follow. Candidates should instead cross out the old working and draw a new diagram, using an additional sheet of paper if necessary.

Comments on individual questions

1) *(Networks)*
This question on Dijkstra proved to be an easy starter, although few scored the single mark in part (ii).

2) *(Algorithms)*
This was also an easy question. Most candidates were successful with parts (i) and (ii), but some failed with part (iii). “Lowest Common Multiple” was the commonest error. Candidates should note that an algorithm describes a set of rules that need to be followed exactly. A minority of candidates failed to delete rows when asked, forfeiting marks.

3) *(Graph Theory)*
A large proportion of candidates failed with part (i). Most succeeded with part (ii). Hardly anyone collected the single mark for part (iii).

There were hints within the question about part (iii). In part (i) candidates were required to show that the graph was planar. In part (ii) they were told that the graph was planar. Few candidates were able to see that if this modelling was correct, then we could do without flyovers, since flows could be organised not to intersect in two dimensions. We know this is not the case, so something must be amiss. It can be seen, for instance, that the inevitable “loop around the outside” is not possible in reality because of incoming and outgoing traffic flows, but only a very few candidates could see that.

4) *(Linear Programming)*

Compared with recent LP questions, answers to this were disappointing.

In part (i) it was enormously disappointing to see just how few candidates could cope with the area computations which were required. One would really have expected that AS candidates would have been able to give the area covered by a large tile, the area covered by a box of small tiles, and the area of the wall. This was not the case for the majority of candidates.

Most managed the easier inequality in part (ii). Few were able correctly to sort out the area issues for the second inequality; if they could not manage part (i) then part (ii) would clearly be closed to them.

There were plenty of marks available in part (iii), with lots of follow-through marks, including for $x \le y$. However, the majority of candidates did not manage to achieve any sensible optimisation, which is de rigueur for this question. Few obtained the correct objective function. A disappointing number of students failed even to draw their own linear functions accurately, and many failed to shade accurately.

The question in part (iv) was very specific; it asked for two other factors which would *HAVE* to be taken into account in deciding how many of each tile to purchase. Only answers which related to wastage or design were deemed to be acceptable; others were regarded as flights of fancy, and some were very fanciful indeed! Future candidates would do well to appreciate that this sort of question aims to elicit a comment on the modelling; they are not intended to test the candidates' powers of imagination.

5) **(***Simulation)*

Part (i) created more of a problem than had been expected. Candidates were asked how to simulate the classic "drunkard's walk". Almost to a person they described how to simulate a step, but neglected to accumulate those steps into a walk.

The majority then negotiated part (ii) successfully.

In part (iii) many then displayed a similar inadequacy to that seen in part (i). Readers are reminded that the question was how to estimate the probability of the drunkard falling into the canal within six steps. The often seen answer can be paraphrased as "Repeat the simulation and work out the probability from the repetitions".

Parts (iv) and (v) were generally good sources of marks.

6) *(Critical Path Analysis)*

There were the inevitable few who produced activity-on-node digraphs, or activity-on-node digraphs which masqueraded as activity-on-arc, or activity-on-arc masquerading as activity-on-node, or graphs. But most candidates did very well in parts (i) and (ii). Note that activity-on-node scored zero in part (i) and a maximum of two in part (ii).

Hardly any correct answers were seen to part (iii). Again it had been anticipated that these three marks would be difficult and discriminating, but not to the extent which they proved.

Finally, it was anticipated that the last two marks, in part (iv), would be found to be easy. Indeed more candidates scored them than scored those in part (iii), but again, far fewer than expected.

4772 Decision Mathematics 2

General comments
Candidates found last year's paper relatively easy. This paper reverted to the norm. The mean mark was 42 compared to 49 in 2009 and 43 in 2008.

Comments on individual questions

1) *(Logic)*

(a) Most candidates scored 5 or 4 marks out of the 5 available in part (a). Most chose to give a full truth table for (a)(ii) rather than just one appropriate line. Not all of those candidates indicated which line/lines was/were relevant.

(b) The majority of candidates scored very badly in part (b). The modal mark was 0 out of 11. As last year, combinatorial circuits proved to be a stumbling block. Many candidates were unable to make a start.
There were many good candidates who did score well and some of the proofs seen in (b)(iii) were very good indeed.

2) *(Networks)*

(i) A very large number of flights of fancy were seen on this question, some very fanciful indeed. This comment mirrors one made on the 4771 report. Future candidates would do well to appreciate that this sort of question aims to elicit a comment on the modelling; they are not intended to test the candidates' powers of imagination. Yes, there might well have been some awful creature sitting somewhere on the direct track from vertex **2** to vertex **3**, causing delay. But what was looked for was an appreciation of the fact that times are not bound to follow the triangle inequality, any more than distances are. So prosaic answers, eg "The direct route might wend its way around flower beds" were much more satisfying.

(ii) (iii) These parts were well done. In part (iii) some candidates used the matrices from part (i) instead of the final matrices.

(iv) Few candidates converted the route they obtained from the shortest time matrix into a route through the garden, even though nearly all had explained how to do this in part (iii).

(v) Only about 50% of candidates succeeded in this straightforward application of syllabus material.

(vi) This was also disappointing for what was a very straightforward question.

3) *(Decision Analysis)*
There have been series in which, for many candidates, this question has proved to be the bedrock of their scripts. That was seldom the case for candidates in this series. For many candidates the stumbling block was their inability to distinguish between decision nodes and chance nodes.

(i) The structure of this problem was perceived to be easy; it was defined carefully in paragraphs 1 and 3 of the question. But many candidates produced complicated decision trees which failed to model the given information.

(ii) It was recognised that the computations were intricate, and that many candidates would not get them fully correct under examination conditions. The mark scheme was carefully constructed to allow for this. The most common error was in failing to have the pension paid each year.

(iii) Again, the computations were numerically intricate, but many marks were available for candidates who made slips. However, some deserted good answers from part (ii), failing to replicate EMV calculations to produce expected utilities.
To re-emphasise the point made in paragraph 2 above, many candidates gave courses of action which indicated when Ken should retire AND whether or not he should obtain part-time work!

4) *(Linear Programming - Simplex)*

(i) Most candidates succeeded with the formulation.

(ii) The basic simplex algorithm was well understood and executed by most candidates. The "explain" part of the question was meant to provide a check as well as to test understanding. Most had obtained the correct answer, but few gave a decent *explanation of how* to interpret the final tableau, which is rather a different task from interpreting it.

(iii) A majority of candidates saw through this, and gave a fortnightly production plan.

(iv) Very few candidates noted that alternative optimal solutions could exist.

(v) Most had a good idea of 2-stage or big-M, though not all could execute the procedures accurately.

(vi) The essence of this question is to note that the given solution also gives a weekly profit of £1260, and that the solution lies on the line joining the earlier alternative optimal solutions. Very few candidates noted this.

4773 Decision Mathematics Computation

General comments
Again, there were fewer candidates than the paper deserves.

Performances were very similar to those seen last year.

The paper really does demand modelling skills, sometimes in a "real world" context (see questions 1, 3 and 4) and sometimes within mathematics (question 2).

Comments on individual questions

1) *(Recurrence Relations)*

The three musketeers, with three different investment plans ... the first simple enough to model using a recurrence relation, and the other two needing Excel.

Athos The recurrence relation was straightforward – linear first order. Not many candidates succeeded in manipulating it correctly.

Porthos This scheme should have been very easy to model in Excel, but many mistakes were made, usually with the timing out of kilter.

Aramis The modelling here was more difficult, with the mean annual balance being needed for computing the interest. Few candidates were able to manage this.

2) *(LP Modelling)*

Candidates were often prepared to invest a very great deal of writing and effort for the 1 mark in part (i). It really was easy!

It was expected that virtually all candidates would be able to succeed with part (ii), but regrettably that was not the case. Those that did succeed could run the LP, but few could draw the picture to interpret the result.

The remaining parts repeated the work, but with the modelling required from the candidate instead of being supplied. Not all candidates succeeded with this.

3) *(Networks)*

Parts (i) and (ii) were very straightforward, and a good number of candidates succeeded with them.

In parts (iii) and (iv) there was an added complication in that the answer to part (ii) was required in the formulation. This tripped up several candidates.

Finally, the modelling in the last two parts was more difficult, in that equations were required linking the flows into and out of the depots. Only some candidates succeeded with this. These candidates invariably noticed that the sub-optimisation in the first two problems did not together deliver the global optimum found by the final model.

4) *(Simulation)*

This question involved the simulation modelling of a Galton-Watson branching process. Most candidates were able to deal with part (i).

It was pleasing that many candidates were also able to produce satisfactory answers to parts (ii) and (iii), although it was often very difficult to disentangle what they were doing either from their attempted explanations, or from their code.

Most candidates had run out of steam by part (iv).

4776 Numerical Methods (Written Examination)

General comments
There was a lot of good work seen, but as ever there were some candidates who appeared to be unready for the examination. Routine numerical calculations were generally carried out accurately, though it is yet again disappointing that so many candidates set their work out badly. This is an algorithmic subject and good work will reflect that. A poor layout is difficult to follow for the examiner – and difficult for the candidate to check.

Interpretation is still a weak area, with quite a number of candidates simply omitting such parts of questions – or writing vaguely and at length in the hope that they will produce something worthy of a mark.

Comments on individual questions

1) *(Fixed point iteration to solve an equation)*
This proved an easy starter for most candidates. There were many solutions gaining full marks, though a significant minority failed to find a convergent iteration in part (ii).

2) *(Numerical differentiation)*
The first part of this question proved easy for most candidates. The second part, however, was a little more challenging for some. A curious error, seen quite a few times, involved saying that if 0.9996 is correct to 4 decimal places then its maximum possible value is 0.99964. Presumably the reasoning was that 0.99965 would round to 0.9997. This is of course incorrect.

3) *(Relative error)*
The relationship $X = x(1 + r)$ proved troublesome once again. Candidates are just not happy with errors analysed this way. For certain sorts of problem – such as the one in this question – it is by far the easiest approach. In part (ii), candidates were asked to 'state, in similar terms, a relationship …'. An algebraic result without a suitable form of words did not gain full marks.

4) *(Numerical approximation)*
The first four marks were obtained easily by most candidates with only a few making errors in the numerical work or the signs. (There are two conventions for the meaning of the word 'absolute' in the term 'absolute error'. Some books take 'absolute' to be a contrast with 'relative'; others take it to mean the positive value. Either interpretation, used consistently, is acceptable.) Part (ii) was not done well. In quite a number of cases the idea was understood well enough, but the calculation of k involved algebraic errors. Many candidates appeared to ignore the information that k is an integer.

5) (*Lagrange's interpolation formula)*
This was an easy source of marks for many, but as usual some got the x and $f(x)$ values muddled. There were some algebraic errors in the simplification, but perhaps fewer than usual.

6) *(Numerical integration)*
Part (i) was an easy source of marks for most, though there was a lot of inefficient work poorly set out. In part (ii), the correct approach is to compare the Simpson's rule estimates and, noting how small the change is between the second and third values, to conclude that 1.56895 is justified. In part (iii), candidates were mostly able to calculate the errors in the mid-point and trapezium rules. The interpretation of those errors was less well done however, with a lot of rather vague statements being made.

7) *(Newton-Raphson and secant methods)*
In part (i), the root was generally found successfully using the Newton-Raphson method. Candidates were then required to find differences and ratios of differences to assess the rate of convergence. Quite a number of candidates said that because the ratios of differences are not constant the process is faster than first order. This was not enough: they needed to say that ratios of differences are decreasing (fast).

In part (ii), some candidates seemed less secure in their use of the secant method. The conclusion about the rate of convergence was handled much as in part (i).

4777 Numerical Computation

General comments
A usual, the entry for this paper was small. However, candidates were mostly well prepared and there was some excellent work seen.

Comments on individual questions

1) *(Interpolation; divided differences)*
This was, perhaps surprisingly, found a little more difficult than the other questions. The approach required is one in which a sequence of estimates is examined in order to assess accuracy. Individual values do not provide any indication of accuracy.

2) *(Integration; Romberg's method)*
This was handled well by most candidates. The method does, of course, involve formulae of higher order than the T^* and T^{**} explicitly mentioned. The approach of iterating T^{**} to convergence without using higher order formulae is not as efficient.

3) *(First order differential equation; modified Euler and predictor-corrector methods)*
Again, this question was done well by most candidates who tackled it. The comparison asked for at the end of the question should have led to candidates saying that, in this case at least, modified Euler appears to deliver about the same accuracy as predictor-corrector with rather less programming effort.

4) *(System of linear equations; Gaussian elimination)*
Most, but not all, candidates clearly understood the Gaussian elimination method. Equally importantly, they could implement it on a spreadsheet. In the final part, candidates were expected to notice that a small change in one of the coefficients produces a large change in the solution and that the determinant is very small in relation to the size of the coefficients. The latter is a good indicator of ill-conditioning.

Coursework

Administration

Administration difficulties caused by the failure of centres to adhere to instructions was significantly reduced this series. Most coursework arrived on time, there were few clerical errors and the vast majority enclosed the Authentication Form, CCS160. This all made the process of external moderation very much easier.

Centres are once again reminded that it is also a great help to have the cover sheets filled in properly. This means

- Full candidate name and candidate number,
- Marks given by criteria rather than domain,
- Comments to help the external moderator determine which marks have been awarded and which have been withheld,
- An oral communication report,
- The criteria marks summed correctly to give a final total which is correctly transferred onto the MS1 and which is the mark submitted to OCR.

It is also helpful to have the work annotated, particularly in places where the work has been checked. Assessors are asked not to tick work that they have not checked.

The marks of candidates in most centres were appropriate and acknowledgement is made of the amount of work that is involved to mark and internally moderate. The unit specific comments are offered for the sake of centres that have had their marks adjusted for some reason.

Teachers should note that all the comments offered have been made before. These reports should provide a valuable aid to the marking process and we would urge all Heads of Department to ensure that these reports are read by all those involved in the assessment of coursework.

Methods for Advanced Mathematics - 4753

The marking scheme for this component is very prescriptive. However, there are a significant number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

Change of Sign
- A graph of the function being used does not constitute an illustration of the method.
- Trivial equations should not be used to demonstrate failure.
- If a table of values actually finds the root then the method has not failed.
- Graphs which candidates claim cross the axis or just touch but don't should be checked.
- The answer is given as an interval rather than an actual value with error bounds.

Newton–Raphson
- Candidates who use equations with only one root should not be credited the second mark.
- Iterates should be given for success and failure.
- A print out from "Autograph" is not sufficient. Candidates should derive the formula by differentiation and algebra. This should be for the equation being used rather than theory.
- Poor illustrations (for example, an "Autograph" generated tangent with no annotation or just a single tangent) should not be awarded the full mark.
- The graph used to demonstrate the method should match the iterates.
- Error bounds should be established by a change of sign.
- Failure should be demonstrated "despite a starting value close to the root". If the starting value is too far away from the root or too artificial then the mark should not be awarded.
- The roots in this domain should be given to 5 significant figures. Teachers should note that the default position of "Autograph" is only 4 significant figures so candidates need to make some adjustment before the use of the software is satisfactory.

Rearrangement
- Incorrect rearrangements are often not spotted and marked as correct.
- Graphs do not match iterates.
- The method should be explained by a graph which may need to be annotated.
- Weak discussions of $g'(x)$ are often given. Candidates should not just quote the criterion without linking it to their function.

Comparison
- The methods need to be compared by finding the same root of the same equation with the same starting value to the same degree of accuracy.
- The discussions are often thin and omit references to the software actually being used.

Notation
- Equations, functions, expressions still cause confusion. Candidates who assert that they are going solve $y = x^3 + x + 7$ or that they are going to solve $x^3 + x + 7$ should be penalised.

Differential Equations - 4758

As is the usual pattern of entry, only a small number of centres submitted work this series. Therefore any generalisations may be a little misleading.

Whilst there were specific problems which are largely dealt with by reports to individual centres, the following general points may be of value.

The narrative and flow of the coursework is important and there should not be an assumption that the marker or moderator is familiar with the task. The introduction, especially when considering modelling task such as 'Aeroplane Landing' or 'Interacting Species', should introduce the task and discuss the nature and origin of the data.

Curve fitting, although tempting, should be resisted since it does not rely upon the formulation and later modification of the assumptions. A particular example of this is assuming that the flow of water in cascades is proportional to some power of its height in the container, then finding which particular power produces the best fit. The choice of parameters, based on guesswork or shape of the data also often seems to occur in 'Interacting Species'.

The essential function of the coursework element of this module is to test the candidates' ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

There was the usual problem of discarding the initial model for 'Aeroplane Landing' after only considering the first 9 seconds of motion, although the proportion of candidates doing this seems to be declining. Assessors are reminded that any model should involve the whole of the data provided. In Aeroplane Landing candidates are usually successful in identifying that there are two phases to the motion. Their model should cover both phases.

Finally, when using the experimental / modelling cycle care must be taken to avoid circular arguments. That is, the data used to produce the parameters should not then be used to compare with the predicted values. It is better, where possible, to to calculate the parameters from one set of data and compare the predictions with another set.

Numerical Methods - 4776

There continue to be cases where incorrect work had been ticked. Assessors are requested not to tick work unless it has been checked thoroughly.

The most popular task is to find the value of an integral numerically. The following comments are offered – it is to be hoped that those teaching and assessing will take note so that the problems do not continue to occur with such regularity.

Domain 1
Not all candidates fulfil the basic requirement of a formal statement of the problem.

Domain 2
Most candidates describe what method they are to use but fail to say why – this is part of the criteria for this domain.

Domain 3
Finding numerical values for one of the methods up to at least 64 strips is a requirement for a substantial application.

Domain 4
It is not enough to state what software is being used. A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout.

Domain 5
It is accepted that candidates might use a function that they are unable to integrate (because of where they are in the course) but which is integrable. However, it is not then appropriate to state a value found by direct integration.
Many candidates will state the value to which the ratio of differences is converging without justification from their values. Indeed, some candidates use the "theoretical" value regardless of the values they are getting (or not if they do not work the ratio of differences) far too early giving inaccurate solutions.

Domain 6
Most of the marks in this domain are dependent on satisfactory work in the error analysis domain and so often a rather generous assessment of that domain led also to a rather generous assessment here as well. Teachers should note that comments justifying the accuracy of the solution are appropriate here, but comments on the limitations of Excel are not usually creditworthy.

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