GCE

MEI Mathematics

Advanced GCE **4769**

Statistics 4

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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Question 1

	$\mathrm{f}(x)=\dfrac{x\mathrm{e}^{-x/\lambda}}{\lambda^2} \quad (x>0)$	
(i)	$\mathrm{E}(X)=\dfrac{1}{\lambda^2}\int_0^\infty x^2\mathrm{e}^{-x/\lambda}\mathrm{d}x$	M1 for integral for E(X) M1 for attempt to integrate by parts
	$=\dfrac{1}{\lambda^2}\left\{\left[-\lambda x^2\mathrm{e}^{-x/\lambda}\right]_0^\infty+\int_0^\infty \lambda.2x\mathrm{e}^{-x/\lambda}\mathrm{d}x\right\}$	For second term: M1 for use of integral of pdf or for integr'g by parts again
	$=\dfrac{1}{\lambda^2}\{[0-0]\}+2\lambda.1 = 2\lambda$.	A1
	$\mathrm{E}(\overline{X})=\mathrm{E}(X) \quad \therefore \mathrm{E}\left(\hat{\lambda}[=\tfrac{1}{2}\overline{X}]\right)=\lambda \quad \therefore \hat{\lambda}$ is unbiased.	M1 A1 E1 **[7]**
(ii)	$\mathrm{Var}(\hat{\lambda})=\dfrac{1}{4}\mathrm{Var}(\overline{X})=\dfrac{1}{4}\dfrac{\mathrm{Var}(X)}{n}$	M1
	$\mathrm{E}(X^2)=\dfrac{1}{\lambda^2}\int_0^\infty x^3\mathrm{e}^{-x/\lambda}\mathrm{d}x$	M1 for use of E(X^2) By parts M1
	$=\dfrac{1}{\lambda^2}\left\{\left[-\lambda x^3\mathrm{e}^{-x/\lambda}\right]_0^\infty+\int_0^\infty 3\lambda x^2\mathrm{e}^{-x/\lambda}\mathrm{d}x\right\}$	
	$=\dfrac{1}{\lambda^2}\{[0-0]\}+3\lambda\mathrm{E}(X) = 6\lambda^2$.	M1 for use of E(X) A1 for $6\lambda^2$
	$\therefore \mathrm{Var}(X)=\mathrm{E}(X^2)-\{\mathrm{E}(X)\}^2=6\lambda^2-4\lambda^2=2\lambda^2$.	A1
	$\therefore \mathrm{Var}(\hat{\lambda})=\dfrac{\lambda^2}{2n}$.	A1 **[7]**
(iii)	Variance of $\hat{\lambda}$ becomes very small as n increases.	E1
	It is unbiased and so becomes increasingly concentrated at the correct value λ.	E1 **[2]**
(iv)	$\mathrm{E}(\tilde{\lambda})=\left(\tfrac{1}{8}+\tfrac{1}{4}+\tfrac{1}{8}\right)2\lambda=\lambda$. $\quad \therefore \tilde{\lambda}$ is unbiased.	$\mathrm{E}(\tilde{\lambda})$: B1; "unbiased": E1
	$\mathrm{Var}(\tilde{\lambda})=\left(\tfrac{1}{64}+\tfrac{1}{16}+\tfrac{1}{64}\right)2\lambda^2=\tfrac{3}{16}\lambda^2$.	M1 A1
	$\therefore$ relative efficiency of $\tilde{\lambda}$ to $\hat{\lambda}$ is $\dfrac{\lambda^2/6}{3\lambda^2/16}=\dfrac{8}{9}$.	M1 any comparison of variances M1 correct comparison A1 for 8/9
	Special case. If done as Var($\tilde{\lambda}$) / Var($\hat{\lambda}$), award 1 out of 2 for the second M1 and the A1 in the scheme.	[Note. This M1M1A1 is allowable in full as FT if everything is plausible.]
	So $\hat{\lambda}$ is preferred.	E1 (FT from above) **[8]**

Question 2

(i)	$\mathrm{G}(t)=\mathrm{E}(t^X)=\sum_{x=0}^{\infty}\frac{e^{-\lambda}(\lambda t)^x}{x!}$ **[M1]** $=e^{-\lambda}\left(1+\lambda t+\frac{\lambda^2 t^2}{2!}+...\right)$ **[A1]** $=\mathrm{e}^{-\lambda}\mathrm{e}^{\lambda t}=\mathrm{e}^{\lambda(t-1)}$ **[A1]** **[Allow omission of previous A1 step and write-down of this for A2 provided opening M1 has been earned (NB answer is given)]**	**[3]**
(ii)	Mean = G'(1) $\mathrm{G}'(t)=\lambda\mathrm{e}^{\lambda(t-1)}$ **[M1]** G'(1) = λ **[A1]** Variance = G''(1) + mean – mean^2 $\mathrm{G}''(t)=\lambda^2\mathrm{e}^{\lambda(t-1)}$ **[M1]** G''(1) = λ^2 **[A1]** $\therefore$ variance = $\lambda^2+\lambda-\lambda^2=\lambda$ **[A1]**	**[5]**
(iii)	$Z=\frac{X-\mu}{\sigma}$: mean 0 **[B1]** variance 1 **[B1]**	**[2]**
(iv)	Mgf of X is $\mathrm{M}(\theta)=\mathrm{G}(\mathrm{e}^\theta)=\mathrm{e}^{\lambda(\mathrm{e}^\theta-1)}$ **[B1]** Linear transformation result is $\mathrm{M}_{aX+b}(\theta)=\mathrm{e}^{b\theta}\mathrm{M}_X(a\theta)$ **[B2 if fully correct, any equivalent form. Allow B1 if either factor correct.]** Use with $a=\frac{1}{\sigma}=\frac{1}{\sqrt{\lambda}}$ and $b=-\frac{\mu}{\sigma}=-\sqrt{\lambda}$ **[M1]** $\mathrm{M}_Z(\theta)=\mathrm{e}^{-\sqrt{\lambda}\theta}\mathrm{e}^{\lambda\left(\mathrm{e}^{\theta/\sqrt{\lambda}}-1\right)}=\mathrm{e}^{\lambda\left(\mathrm{e}^{\theta/\sqrt{\lambda}}-\frac{\theta}{\sqrt{\lambda}}-1\right)}$ **[A1] [A1] [A1] [NB answer is given]**	**[7]**
(v)	Consider $\lambda\left(\mathrm{e}^{\theta/\sqrt{\lambda}}-\frac{\theta}{\sqrt{\lambda}}-1\right)=\lambda\left(1+\frac{\theta}{\sqrt{\lambda}}+\frac{\theta^2}{2!\lambda}+\frac{\theta^3}{3!\lambda^{3/2}}+...-\frac{\theta}{\sqrt{\lambda}}-1\right)$ **[M1]** $=\frac{\theta^2}{2}+$ terms in $\lambda^{-1/2},\lambda^{-1},\lambda^{-3/2},...$ **[A1]** $\rightarrow\frac{\theta^2}{2}$ as $\lambda\rightarrow\infty$ **[M1]** **[some explanation required]** $\therefore\mathrm{M}_Z(\theta)\rightarrow\mathrm{e}^{\theta^2/2}$ as $\lambda\rightarrow\infty$ **[A1] [answer given]**	**[4]**
(vi)	$\mathrm{e}^{\theta^2/2}$ is the mgf of N(0, 1) **[E1]** , and the relationship between distributions and their mgfs is unique **[E1]** . "Unstandardising", X tends to N(μ, σ^2) i.e. N(λ, λ) **[B1, parameters must be given]**.	**[3]**

Question 3

(i) H_0 is accepted if $-1.96 <$ value of test statistic < 1.96	M1 double inequality B1 1.96
i.e. if $-1.96 < \dfrac{(\bar{x}_1 - \bar{x}_2) - (0)}{\sqrt{\dfrac{1.2^2}{8} + \dfrac{1.4^2}{10}}} < 1.96$	M1 num^r of test statistic M1 den^r of test statistic
i.e. if $-1.96 \times 0.6132 < \bar{x}_1 - \bar{x}_2 < 1.96 \times 0.6132$	A1
i.e. if $-1.20(18) < \bar{x}_1 - \bar{x}_2 < 1.20(18)$	A1
Note. Use of $\mu_1 - \mu_2$ instead of $\bar{x}_1 - \bar{x}_2$ can score M1 B1 M0 M1 A0 A0.	**Special case.** Allow 1 out of 2 of the A1 marks if 1.645 used provided all 3 M marks have been earned. **[6]**
(ii) $\bar{x}_1 - \bar{x}_2 = 1.4$	B1 FT if wrong
which is outside the acceptance region	M1 [FT can's acceptance region if reasonable]
so H_0 is rejected.	E1
CI for $\mu_1 - \mu_2$: 1.4 ± (2.576 × 0.6132), i.e. 1.4 ± 1.5796, i.e. (–0.18 [–0.1796] , 2.97[96])	M1 for 1.4 B1 for 2.576 M1 for 0.6132 A1 cao for interval **[7]**
(iii) Wilcoxon rank sum test (or Mann-Whitney form of test)	M1
Ranks are: First 14 13 10 8 6 11 Second 2 12 3 1 4 7 5 9	M1 Combined ranking A1 Correct [allow up to 2 errors; FT provided M1 earned]
W = 14 + 13 + 10 + 8 + 6 + 11 = 62 [or 8 + 8 + 7 + 7 + 6 + 5 = 41 if M-W used]	B1
Refer to $W_{6,8}$ [or $MW_{6,8}$] tables.	M1 No FT if wrong
Lower 2½% critical point is 29 [or 8 if M-W used].	A1 **Special case 1.** If M1 for $W_{6,8}$ has not been awarded (likely to be because rank sum 43 has been used, which should be referred to $W_{8,6}$), the next two M1 marks can be earned but *nothing beyond them*.
Consideration of upper 2½% point is also needed.	M1
Eg: by using symmetry about mean of $\left(\frac{1}{2}\times 6\times 8\right)+\left(\frac{1}{2}\times 6\times 7\right)$ = 45, critical point is 61. [For M-W: mean is $\frac{1}{2}\times 6\times 8$ = 24, hence critical point is 40.]	M1 for any correct method A1 if 61 correct
Result is significant. Seems (population) medians may not be assumed equal.	E1, E1 **Special case 2** (does not apply if Special Case 1 has been invoked). These 2 marks may be earned even if only 1 or 2 of the preceding 3 have been earned. **[11]**

Question 4

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(i) Randomised blocks

Eg:-

WEST	D A C B	C B A D	D C A B	EAST

Plots in strips (blocks)
correctly aligned w.r.t. fertility trend.
Each letter occurs at least once in each block
in a random arrangement.

</td><td>

B1

M1
E1
M1
E1
[5]

</td></tr>
<tr><td>

(ii) μ = population **[B1]** grand mean for whole experiment **[B1]**
α_i = population **[B1]** mean amount by which the *i*th treatment differs from μ **[B1]**

e_{ij} ~ ind N [**B1**, accept "uncorrelated"] (0 **[B1]** , σ^2 **[B1]**)

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4 marks, as shown

3 marks, as shown
[7]

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(ii) Totals are 62.7 65.6 69.0 67.8 all from samples of size 5

Grand total 265.1 "Correction factor" CF = $265.1^2/20$ = 3513.9005

Total SS = 3524.31 – CF = 10.4095

Between varieties SS = $\frac{62.7^2}{5}+\frac{65.6^2}{5}+\frac{69.0^2}{5}+\frac{67.8^2}{5}$ – CF

= 3518.498 – CF = 4.5975

Residual SS (by subtraction) = 10.4095 – 4.5975 = 5.8120

Source of variation	SS	df	MS **[M1]**	MS ratio **[M1]**
Between varieties	4.5975	3 **[B1]**	1.5325	4.22 **[A1 cao]**
Residual	5.8120	16 **[B1]**	0.36325	
Total	10.4095	19		

Refer MS ratio to $F_{3,16}$.

Upper 5% point is 3.24.
Significant.
Seems the mean yields of the varieties are not all the same.

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M1 for attempt to form three sums of squares.
M1 for correct method for any two.
A1 if each calculated SS is correct.

5 marks within the table, as shown

M1 No FT if wrong

A1 No FT if wrong
E1
E1
[12]

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