OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Decision Mathematics Computation

4773

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other Materials Required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Thursday 24 June 2010
Morning

Duration: 2 hours 30 minutes

OCE/23400

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 Athos puts £1000 into a deposit account. After a year, interest is added to the account, the amount of interest being 5% of the balance during the year. Athos then draws out £60. After each subsequent year, interest at 5% of the balance is added on, and Athos then withdraws £60.

(i) Let u_n be the amount (in £) in the account after n years, with $u_0 = 1000$. Construct a recurrence relation for u_n in terms of u_{n-1}. **[3]**

(ii) Solve your recurrence relation from part **(i)**, simplifying your answer as far as is possible. **[4]**

(iii) Use your answer to part **(ii)** to find how long Athos can continue to operate the account in this way. **[2]**

Porthos puts £1000 into a deposit account. Every 6 months, interest is added to the account, the amount of interest being $2\frac{1}{2}$ % of the balance over those 6 months. He draws out £60 at the end of 12 months and after each subsequent 12 months.

(iv) Construct a spreadsheet to show how the amount Porthos has in his account varies over time. **[3]**

(v) Use your spreadsheet to find for how long Porthos can operate his account in this way. **[1]**

Aramis puts £1000 into a deposit account. He draws out £30 every 6 months. Every 12 months, interest is added to the account, the amount of interest being 5% of the average balance over those 12 months.

(vi) Construct a spreadsheet to show how the amount Aramis has in his account varies over time. **[4]**

(vii) Use your spreadsheet to find for how long Aramis can operate his account in this way. **[1]**

2 The distance of the point (p, q) from the line $ax + by = c$ is given by $\left|\frac{c - ap - bq}{\sqrt{a^2 + b^2}}\right|$.

For example, when $a = b = 1$ and $c = 10$, the distance of (p, q) from $x + y = 10$ is given by $\left|\frac{10 - p - q}{\sqrt{2}}\right|$.

(i) Find the distance of $(0, 0)$ from the line $x + y = 10$.

Find the distance of $(10, 10)$ from the line $x + y = 10$. **[1]**

Consider the (minimax) LP:

Minimise m
subject to $m \geqslant p$
$m \geqslant -p$
$m \geqslant q$
$m \geqslant -q$
$m \geqslant \frac{10 - p - q}{\sqrt{2}}$
$m \geqslant \frac{p + q - 10}{\sqrt{2}}$

(ii) Rewrite the LP in a form in which it can be submitted to LINDO.
(Approximate $\sqrt{2}$ by 1.414214.) **[4]**

(iii) Run the LP and draw a diagram to explain what it achieves. **[5]**

(iv) Formulate an LP to find the point which is equidistant from the lines $y = 0$, $x + y = 1$ and $x - y = -1$.
(Approximate $\sqrt{2}$ by 1.414214.) **[4]**

(v) Run your LP. **[1]**

(vi) Prove by drawing a diagram and calculating distances that your LP has achieved what was required. **[3]**

[Questions 3 and 4 are printed overleaf]

3 A logistics company has three depots, D1, D2 and D3. On a particular day it has 10 identical containers to collect from each of two supply locations, S1 and S2. These containers then have to be shipped from the depots to four customers, C1, C2, C3 and C4. C1 requires 7 containers, C2 requires 4 containers, C3 requires 6 containers and C4 requires 3 containers. Each depot can handle up to 7 containers.

The transportation costs are shown in the two tables.

	D1	D2	D3
S1	2	3	7
S2	1	8	4

	C1	C2	C3	C4
D1	2	3	9	1
D2	4	7	2	5
D3	1	5	3	6

(i) Formulate an LP to find the cheapest way to transport the containers from the supply locations to the depots. **[3]**

(ii) Run your LP and interpret the results. **[3]**

(iii) Using your answer to part **(ii)**, formulate an LP to find the cheapest way to transport these containers from the depots to the customers. **[3]**

(iv) Run your LP and interpret the results. **[2]**

(v) Formulate an LP to find the cheapest way to transport the containers from the supply locations to the customers via the depots. **[4]**

(vi) Run your LP and interpret and comment on your results. **[3]**

4 Each individual in a population produces either 0, 1 or 2 offspring, the probabilities being 0.1, 0.5 and 0.4 respectively.
The population starts from a single individual (generation 0), whose offspring form generation 1. In turn the offspring from members of the population in generation 1 form generation 2, etc.

(i) Show how to use a spreadsheet command to simulate the offspring produced by a member of the population. **[4]**

(ii) Build a spreadsheet to simulate generations 1 and 2.
Print out the formulae which you used in your spreadsheet. **[9]**

(iii) Run your simulation 20 times and hence estimate the probabilities of there being 0, 1, 2, 3 or 4 individuals in generation 2. **[2]**

A second population has individuals which reproduce according to the same rule, but this population starts with 2 individuals in generation 0.

(iv) Use your simulation model to produce one simulation of the number in this population at generation 2.
Explain how you produced your result. List all possible results. **[3]**