GCE

Mathematics (MEI)

Advanced GCE **4756**

Further Methods for Advanced Mathematics (FP2)

Mark Scheme for June 2010

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

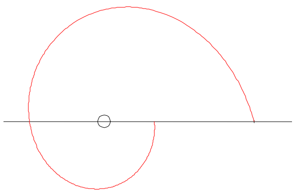
Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

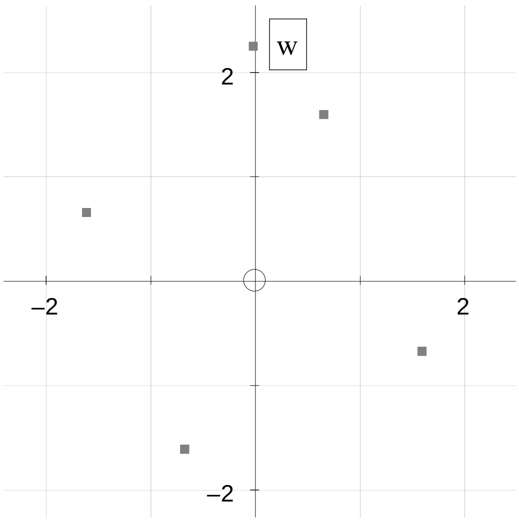
OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

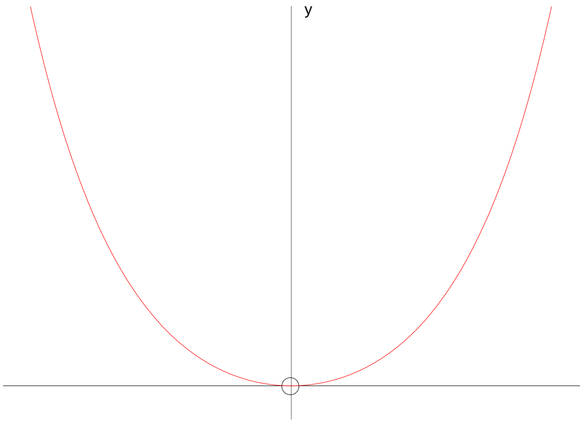
OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

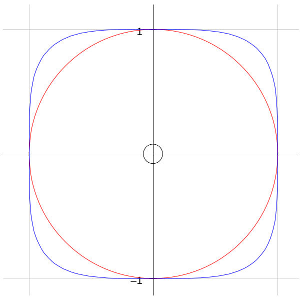
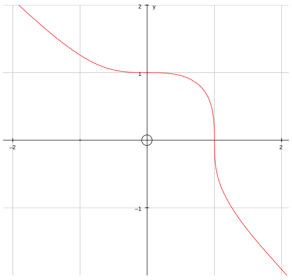
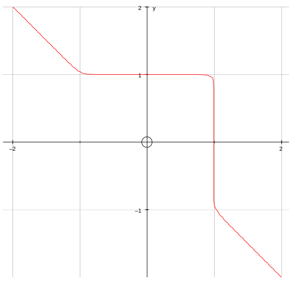
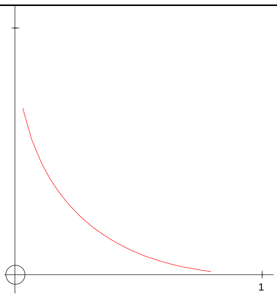
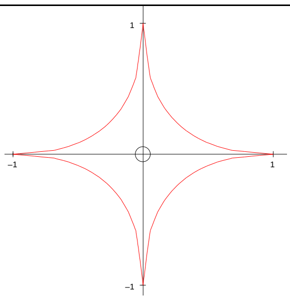
Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1 (a)(i)	$f(t) = \arcsin t$ $\Rightarrow \quad f'(t) = \dfrac{1}{\sqrt{1-t^2}} = \left(1-t^2\right)^{-\frac{1}{2}}$ $\Rightarrow \quad f''(t) = -\frac{1}{2}\left(1-t^2\right)^{-\frac{3}{2}} \times -2t$ $= \dfrac{t}{\left(1-t^2\right)^{\frac{3}{2}}}$	B1 M1 A1 (ag) **3**	Any form Using Chain Rule
(ii)	$f(x) = \arcsin(x + ½)$ $\Rightarrow \quad f(0) = \arcsin(½) = \dfrac{\pi}{6}$ $f'(0) = \left(1-\left(\frac{1}{2}\right)^2\right)^{-\frac{1}{2}} = \dfrac{2}{\sqrt{3}}$ and $f''(0) = \dfrac{\frac{1}{2}}{\left(1-\left(\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} = \dfrac{4\sqrt{3}}{9}$ $f(x) = f(0) + x f'(0) + \dfrac{x^2}{2} f''(0) + \ldots$ $\Rightarrow$ term in x^2 is $\dfrac{2\sqrt{3}}{9}x^2$	B1 (ag) M1 A1 (ag) M1 A1 **5**	$\dfrac{\pi}{6}$ obtained clearly from $f(0)$ www Clear substitution of $x = 0$ or $t = ½$ Evaluating $f''(0)$ and dividing by 2 Accept $0.385x^2$ or better
(b)	 Area $= \displaystyle\int_0^{\pi} \tfrac{1}{2} r^2 \, d\theta$ $= \displaystyle\int_0^{\pi} \frac{\pi^2 a^2}{2(\pi+\theta)^2} d\theta = \frac{\pi^2 a^2}{2}\int_0^{\pi} \frac{1}{(\pi+\theta)^2} d\theta$ $= \dfrac{\pi^2 a^2}{2}\left[\dfrac{-1}{\pi+\theta}\right]_0^{\pi}$ $= \dfrac{\pi^2 a^2}{2}\left(\dfrac{-1}{2\pi} + \dfrac{1}{\pi}\right)$ $= \frac{1}{4}\pi a^2$	G1 G1 M1 A1 M1 A1 **6**	Complete spiral with $r(2\pi) < r(0)$ $r(0) = a$, $r(2\pi) = a/3$ indicated or $r(0) > r(\pi/2) > r(\pi) > r(3\pi/2) > r(2\pi)$ Dep. on G1 above Max. G1 if not fully correct Integral expression involving r^2 Correct result of integration with correct limits Substituting limits into an expression of the form $\dfrac{k}{\pi+\theta}$. Dep. on M1 above
(c)	$\displaystyle\int_0^{\frac{3}{2}} \frac{1}{9+4x^2} dx = \frac{1}{4}\int_0^{\frac{3}{2}} \frac{1}{\frac{9}{4}+x^2} dx = \frac{1}{4} \times \left[\frac{2}{3}\arctan\frac{2x}{3}\right]_0^{\frac{3}{2}}$ $= \dfrac{1}{6}\arctan 1$ $= \dfrac{\pi}{24}$	M1 A1A1 M1 A1 **5**	arctan $\dfrac{1}{4} \times \dfrac{2}{3}$ and $\dfrac{2x}{3}$ Substituting limits. Dep. on M1 above Evaluated in terms of π **19**

2 (a)	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2j\sin n\theta$	B1	Both
	$\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$	M1	Expanding $\left(z - \frac{1}{z}\right)^5$
	$= z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$		
	$\Rightarrow \quad 32j\sin^5\theta = 2j\sin 5\theta - 10j\sin 3\theta + 20j\sin\theta$	M1 A1	Introducing sines (and possibly cosines) of multiple angles RHS
	$\Rightarrow \quad \sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$	A1ft	Division by 32(j)
	$A = \frac{5}{8}$, $B = -\frac{5}{16}$, $C = \frac{1}{16}$	5	
(b)(i)	4^{th} roots of $-9j = 9e^{\frac{3}{2}\pi j}$ are $re^{j\theta}$ where		
	$r = \sqrt{3}$	B1	Accept $9^{\frac{1}{4}}$
	$\theta = \frac{3\pi}{8}$	B1	
	$\Rightarrow \quad \theta = \frac{3\pi}{8} + \frac{2k\pi}{4}$	M1	Implied by at least two correct (ft) further values
	$\Rightarrow \quad \theta = \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$	A1	Or stating k = (0), 1, 2, 3 Allow arguments in range $-\pi \le \theta \le \pi$
		M1 A1 6	Points at vertices of a square centre O or 3 correct points (ft) or 1 point in each quadrant
(ii)	Mid-point of SP has argument $\frac{\pi}{8}$	B1	
	and modulus of $\sqrt{\frac{3}{2}}$	B1	
	Argument of $w = 4 \times \frac{\pi}{8} = \frac{\pi}{2}$		
	and modulus $= \left(\sqrt{\frac{3}{2}}\right)^4 = \frac{9}{4}$	M1 A1 G1 5	Multiplying argument by 4 and modulus raised to power of 4 Both correct w plotted on imag. axis above level of P **16**

3 (a)(i)	$2\lambda^3 + \lambda^2 - 13\lambda + 6 = 0 \Rightarrow (\lambda - 2)(2\lambda^2 + 5\lambda - 3) = 0$ $\Rightarrow \quad \lambda = 2$ or $2\lambda^2 + 5\lambda - 3 = 0$ $\Rightarrow \quad (2\lambda - 1)(\lambda + 3) = 0$ $\Rightarrow \quad \lambda = \frac{1}{2}, \lambda = -3$	B1 M1 A1A1 **4**	Substituting $\lambda = 2$ or factorising Obtaining and solving a quadratic
(ii)	$\mathbf{M}\begin{pmatrix}3\\-3\\1\end{pmatrix} = 2\begin{pmatrix}3\\-3\\1\end{pmatrix} = \begin{pmatrix}6\\-6\\2\end{pmatrix}$ $\mathbf{M}^2\mathbf{v} = 2^2\mathbf{v} = 4\begin{pmatrix}1\\-1\\\frac{1}{3}\end{pmatrix} = \begin{pmatrix}4\\-4\\\frac{4}{3}\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}\frac{3}{2}\\-\frac{3}{2}\\\frac{1}{2}\end{pmatrix} = 2\begin{pmatrix}\frac{3}{2}\\-\frac{3}{2}\\\frac{1}{2}\end{pmatrix} = \begin{pmatrix}3\\-3\\1\end{pmatrix}$ $\Rightarrow \quad x = \frac{3}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$	B1 B2 M1 A1 **5**	 Give B1 for one component with the wrong sign Recognising that the solution is a multiple of the given RHS Correct multiple
(iii)	$2\lambda^3 + \lambda^2 - 13\lambda + 6 = 0$ $\Rightarrow \quad 2\mathbf{M}^3 + \mathbf{M}^2 - 13\mathbf{M} + 6\mathbf{I} = \mathbf{0}$ $\Rightarrow \quad \mathbf{M}^3 = -\frac{1}{2}\mathbf{M}^2 + \frac{13}{2}\mathbf{M} - 3\mathbf{I}$ $\Rightarrow \quad \mathbf{M}^4 = -\frac{1}{2}\mathbf{M}^3 + \frac{13}{2}\mathbf{M}^2 - 3\mathbf{M}$ $\Rightarrow \quad \mathbf{M}^4 = -\frac{1}{2}(-\frac{1}{2}\mathbf{M}^2 + \frac{13}{2}\mathbf{M} - 3\mathbf{I}) + \frac{13}{2}\mathbf{M}^2 - 3\mathbf{M}$ $\Rightarrow \quad \mathbf{M}^4 = \frac{27}{4}\mathbf{M}^2 - \frac{25}{4}\mathbf{M} + \frac{3}{2}\mathbf{I}$ $A = \frac{27}{4}, B = -\frac{25}{4}, C = \frac{3}{2}$	 M1 M1 M1 A1 **4**	 Using Cayley-Hamilton Theorem Multiplying by **M** Substituting for $\mathbf{M}^3$
(b)	$\mathbf{N} = \mathbf{PDP}^{-1}$ where $\mathbf{D} = \begin{pmatrix}-1 & 0\\0 & 2\end{pmatrix}$ and $\mathbf{P} = \begin{pmatrix}1 & -1\\2 & 1\end{pmatrix}$ $\Rightarrow \quad \mathbf{P}^{-1} = \frac{1}{3}\begin{pmatrix}1 & 1\\-2 & 1\end{pmatrix}$ $\Rightarrow \quad \mathbf{N} = \frac{1}{3}\begin{pmatrix}1 & -1\\2 & 1\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\-2 & 1\end{pmatrix}$ $= \frac{1}{3}\begin{pmatrix}-1 & -2\\-2 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\-2 & 1\end{pmatrix}$ $= \frac{1}{3}\begin{pmatrix}3 & -3\\-6 & 0\end{pmatrix} = \begin{pmatrix}1 & -1\\-2 & 0\end{pmatrix}$	B1 B1 B1 B1ft M1 A1	Order must be correct For B1B1, order must be consistent Ft their **P** Attempting matrix product
	OR Let $\mathbf{N} = \begin{pmatrix}a & c\\b & d\end{pmatrix}$ $\begin{pmatrix}a & c\\b & d\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} = -1\begin{pmatrix}1\\2\end{pmatrix}$ B1 $\begin{pmatrix}a & c\\b & d\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix} = 2\begin{pmatrix}-1\\1\end{pmatrix}$ B1 $\Rightarrow \quad a + 2c = -1, -a + c = -2$ B1 $b + 2d = -2, -b + d = 2$ B1 $\Rightarrow \quad a = 1, c = -1; b = -2, d = 0$ M1A1		 Or $\begin{pmatrix}a+1 & c\\b & d+1\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ Or $\begin{pmatrix}a-2 & c\\b & d-2\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ Solving both pairs of equations
		6	**19**

4 (i)	$2\sinh x\cosh x$		
	$= 2\times\frac{e^x+e^{-x}}{2}\times\frac{e^x-e^{-x}}{2}$		
	$=\frac{e^{2x}-e^{-2x}}{2}$	M1	Using exponential definitions and multiplying or factorising
	$=\sinh 2x$	A1 (ag)	
	Differentiating,		
	$2\cosh 2x = 2\cosh^2 x + 2\sinh^2 x$	B1	One side correct
	$\Rightarrow \quad \cosh 2x = \cosh^2 x + \sinh^2 x$	B1	Correct completion
		4	
(ii)		G1	Correct shape and through origin
	Volume $=\pi\int_0^2 (\cosh x-1)^2\,dx$	M1	$\int(\cosh x-1)^2\,dx$
	$=\pi\int_0^2 \cosh^2 x-2\cosh x+1\,dx$	A1	A correct expanded integral expression including limits 0, 2 (may be implied by later work)
	$=\pi\int_0^2 \tfrac{1}{2}\cosh 2x-2\cosh x+\tfrac{3}{2}\,dx$	M1	Attempting to obtain an integrable form Dep. on M1 above
	$=\pi\left[\tfrac{1}{4}\sinh 2x-2\sinh x+\tfrac{3}{2}x\right]_0^2$	A2	Give A1 for two terms correct
	$=\pi\left[\tfrac{1}{4}\sinh 4-2\sinh 2+3\right]$		
	$= 8.070$	A1	3 d.p. required. Condone 8.07
		7	
(iii)	$y = \cosh 2x + \sinh x$		
	$\Rightarrow \quad \frac{dy}{dx} = 2\sinh 2x + \cosh x$	B1	Any correct form
	At S.P. $2\sinh 2x + \cosh x = 0$		
	$\Rightarrow \quad 4\sinh x\cosh x + \cosh x = 0$	M1	Setting derivative equal to zero and using identity
	$\Rightarrow \quad \cosh x(4\sinh x + 1) = 0$	M1	Solving $\frac{dy}{dx}$ =0 to obtain value of $\sinh x$
	$\Rightarrow \quad \cosh x = 0$ (rejected)	A1	Repudiating $\cosh x = 0$
	$\Rightarrow \quad \sinh x = -\tfrac{1}{4}$	A1	
	$\Rightarrow \quad x = \ln\left(-\frac{1}{4}+\frac{\sqrt{17}}{4}\right)$	M1	Using log form of arsinh, or setting up and solving quadratic in e^x
		A1	A0 if extra "roots" quoted
		7	**18**

5(i)(A)	Circle	B1	
(B)		G1 G1	Sketch of circle, centre (0, 0) Sketch of “squarer” circle on same axes
(C)	Square	B1	
(D)	$-1 \le x \le 1$ $-1 \le y \le 1$	B1 B1 **6**	Give B1B0 for not all non-strict or unclear
(ii)(A)	Odd roots exist for all real numbers	B1	Any equivalent explanation
(B)	Line	B1	Sketch insufficient
(C)		G1	
	Asymptote: $x + y = 0$	B1	
(D)		G1 G1 **6**	Line $x + y = 0$ outside unit square Lines $y = 1$ and $x = 1$ on unit square
(iii)		G1	G0 if curve beyond (1, 0) or (0, 1)
	$0 \le x, y \le 1$	B1 **2**	Accept strict, or indication on graph
(iv)(A)		G2ft	Give G1 for a partial attempt. Ft from (iii) on shape
(B)	Limit is a “plus sign” where $x \to 0$ for $-1 \le y \le 1$ and vice versa	B1 B1 **4**	**18**

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553