RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4764**

Mechanics 4

Mark Scheme for June 2010

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1(i)	$(m-\lvert\delta m\rvert)(v+\delta v)+\lvert\delta m\rvert(v-u)-mv=-mg\delta t$	M1	Impulse = change in momentum	
		A1	Accept sign errors in δm	
	$m\delta v-u\lvert\delta m\rvert-\lvert\delta m\rvert\delta v=-mg\delta t$			
	$m\frac{\delta v}{\delta t}+u\frac{\delta m}{\delta t}+\delta m\frac{\delta v}{\delta t}=-mg$	M1	Form DE	
	$\Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t}+u\frac{\mathrm{d}m}{\mathrm{d}t}=-mg$	E1	Complete argument (including signs)	
				4
(ii)	$\frac{\mathrm{d}m}{\mathrm{d}t}=-k\Rightarrow m=m_0-kt$	M1		
	So $(m_0-kt)\frac{\mathrm{d}v}{\mathrm{d}t}-uk=-(m_0-kt)g$	A1		
	$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{uk}{m_0-kt}-g$			
	$v=\int\left(\frac{uk}{m_0-kt}-g\right)dt$	M1	Integrate	
	$=-u\ln(m_0-kt)-gt+c$	A1		
	$t=0, v=0\Rightarrow 0=-u\ln m_0+c$	M1	Use condition	
	$v=-u\ln\left(1-\frac{k}{m_0}t\right)-gt$	A1		
	Fuel burnt when $m_0-kt=0.25m_0$	M1		
	$v=-u\ln 0.25-\frac{0.75m_0g}{k}$	A1		
				8

2(i)	$m\frac{\mathrm{d}v}{\mathrm{d}t} = -mkv^{\frac{3}{2}}$	M1	N2L	
		A1		
	$\int -v^{-\frac{3}{2}}\mathrm{d}v = \int k\,\mathrm{d}t$	M1	Separate and integrate	
	$2v^{-\frac{1}{2}} = kt + c$	A1		
	$t = 0, v = 25 \Rightarrow c = \frac{2}{5}$	M1	Use condition	
	$2v^{-\frac{1}{2}} = kt + \frac{2}{5}$	M1	Rearrange	
	$v = 4\left(kt + \frac{2}{5}\right)^{-2}$	E1		
				7
(ii)	$x = \int 4\left(kt + \frac{2}{5}\right)^{-2}\mathrm{d}t$			
	$= -\frac{4}{k}\left(kt + \frac{2}{5}\right)^{-1} + A$	M1	Integrate	
	$t = 0, x = 0 \Rightarrow A = \frac{10}{k}$	M1	Use condition	
	$x = \frac{1}{k}\left(10 - \frac{4}{kt + \frac{2}{5}}\right)$	A1		
				3
(iii)	The speed decreases, tending to zero	B1		
	The displacement tends to $\frac{10}{k}$	B1	Cv $(10/k)$	
				2

3(i)	$V = -mga\sin\theta + \frac{\lambda}{2(2a)}(3a\sin\theta)^2$	M1	GPE term	
		M1	EPE term	
		A1		
	$\frac{dV}{d\theta} = -mga\cos\theta + \frac{\lambda}{4a}\cdot 9a^2 \cdot 2\sin\theta\cdot\cos\theta$	M1	Differentiate	
		A1		
	$= a\cos\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right)$	E1		
				6
(ii)	$\frac{dV}{d\theta} = 0 \Leftrightarrow \cos\theta = 0$ or $\sin\theta = \frac{2mg}{9\lambda}$	M1	Solve $\frac{dV}{d\theta} = 0$	
(A)	$\lambda > \frac{2}{9}mg$			
	$\theta = \frac{\pi}{2}$	A1		
	and $\theta = \sin^{-1}\frac{2mg}{9\lambda}$	A1		
	$\frac{d^2V}{d\theta^2} = -a\sin\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right) + a\cos\theta\left(\frac{9}{2}\lambda\cos\theta\right)$	M1	Second derivative (or other valid method)	
		A1	Any correct form	
	$= a\left(\frac{9}{2}\lambda(1-2\sin^2\theta) + mg\sin\theta\right)$			
	$V''\left(\frac{\pi}{2}\right) = a\left(-\frac{9}{2}\lambda + mg\right) < 0$	M1	Substitute $\theta = \frac{\pi}{2}$	
	$\Rightarrow$ unstable	A1	Deduce unstable	
	$V''\left(\sin^{-1}\left(\frac{2mg}{9\lambda}\right)\right) = a\left(\frac{9}{2}\lambda\left(1 - 2\left(\frac{2mg}{9\lambda}\right)^2\right) + \frac{2(mg)^2}{9\lambda}\right)$	M1	Substitute other value	

	$= \frac{9}{2}\lambda a\left(1 - \left(\frac{2mg}{9\lambda}\right)^2\right)$			
	$\lambda > \frac{2}{9}mg \Rightarrow \left(\frac{2mg}{9\lambda}\right)^2 < 1 \Rightarrow V'' > 0$	M1	Consider second derivative	
	$\Rightarrow$ stable	A1	Complete argument	
				10
(B)	$\lambda < \frac{2}{9}mg \Rightarrow$	M1	Consider solutions	
	$\theta = \frac{\pi}{2}$ only	A1		
	$V''\left(\frac{\pi}{2}\right) = a\left(-\frac{9}{2}\lambda + mg\right) > 0$	M1	Consider second derivative	
	$\Rightarrow$ stable	A1	Complete argument	
				4
(C)	$\lambda = \frac{2}{9}mg$ gives $\theta = \frac{1}{2}\pi$ only (from both factors)	M1	Consider solutions	
		A1		
	$V''\left(\frac{\pi}{2}\right) = 0$			
	$V'\left(\frac{\pi}{2} - \epsilon\right) = (+)(-) = (-)$			
	$V'\left(\frac{\pi}{2} + \epsilon\right) = (-)(-) = (+)$	M1	Valid method	
	Hence stable	A1	Complete argument	
				4

4(i)	Mass of slice $\approx \rho\pi y^2\delta x$	M1		
	So $I_{\text{slice}} \approx \frac{1}{2}(\rho\pi y^2\delta x)y^2$	M1		
	$= \frac{1}{32}\rho\pi x^4\delta x$	A1		
	So $I_{\text{cone}} \approx \int_0^{2a} \frac{1}{32}\rho\pi x^4 \mathbf{d}x$	M1		
	$= \left[\frac{1}{160}\rho\pi x^5\right]_0^{2a}$	A1	ft	
	$= \frac{1}{5}\pi\rho a^5$	A1		
	$\rho = \dfrac{M}{\frac{2}{3}\pi a^3}$	M1		
	$\Rightarrow I_{\text{cone}} = \frac{3}{10}Ma^2$	E1		
				8
(ii)	Mass of small cone $= \left(\frac{1}{2}\right)^3 M = \frac{1}{8}M$			
	Mass of frustum $= \frac{7}{8}M$	B1		
	$I_{\text{large cone}} = I_{\text{small cone}} + I$	M1		
	$\frac{3}{10}Ma^2 = \frac{3}{10}\left(\frac{1}{8}M\right)\left(\frac{1}{2}a\right)^2 + I$	M1	Moment of inertia of small cone	
	$\Rightarrow I = \frac{93}{320}Ma^2$			
	$\frac{7}{8}M = 2.8, a = 0.1 \Rightarrow I = 0.0093$	E1		
				4

(iii)	$C = I\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{0.05}{0.0093}$	M1		
		A1		
	$t = \frac{10}{\ddot{\theta}} = 1.86$	M1		
		A1		
				4
(iv)	Centre of mass:			
	$\frac{7}{8}M\bar{x} + \frac{1}{8}M \cdot \frac{3a}{4} = M \cdot \frac{3a}{2}$	M1		
		A1		
	OG $= \bar{x} = \frac{45a}{28} = \frac{4.5}{28} \approx 0.1607$	A1	Any distance which locates G	
	i.e. G is $\frac{1.7}{28} \approx 0.0607$ m from the small circular face			
				3
(v)	$0.1J = I(10 - 5)$	M1	Moment of impulse = ang. momentum	
	$J = 0.465$	A1		
	Radius at G is $\frac{1}{2}\bar{x}$	B1		
	$\left(\frac{4.5}{56}\right)J = I(5 - \omega)$	M1	Moment of impulse = ang. momentum	
	$\Rightarrow \omega = \frac{55}{56} \approx 0.98$	A1		
				5

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