RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Reports on the Units

January 2010

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REPORTS ON THE UNITS

Chief Examiners' Report

Many teachers have said how helpful they find the Examiners' reports, which include accounts of mistakes and comparisons of the effectiveness of alternative approaches. Reports can seem to dwell on errors because the details take up more space than comments to the effect that 'candidates did very well and showed excellent understanding'. It is, therefore, appropriate to emphasise here that in every unit there are many well-prepared candidates whose performance shows evidence of expert instruction and their own diligence (including making a real effort to do their best on the day). In this context it is worth noting that the bad weather at the start of the examination series did not lead to many absences; this was a tribute to the energy and concern of centres and individual candidates.

For most of the MEI units, the numbers entering have risen compared with the 2009 January series. The distribution of the grades awarded this time shows no marked difference from that of last year.

Some centres have commented adversely on questions which differ in style from those set in recent sessions or found in the textbooks. While it is acknowledged that it can be comforting to candidates if they are answering questions that are similar to ones they have already seen, the examination assessment may legitimately require them to apply their understanding in ways that are not familiar to them. Of course, every effort is made to ensure that all the questions set are fair and that there is sufficient material on each paper accessible to weaker candidates so that they can show knowledge that may not be supported by deep understanding.

4751 Introduction to Advanced Mathematics (C1)

General Comments

Many candidates showed a good knowledge of basic skills and did well on many of the questions in section A. In particular, the remainder theorem and use of the discriminant to show that a line and curve did not intersect were done better than usual. Curve sketching remains a problem for a few and there were some quite poor attempts at both the quadratic and cubic curves.

Last January I commented that some candidates were thrown by anything that required them to think beyond questions that they had done in practice. This was certainly true for some parts of the questions in section B this year. In order to take account of this the 'a' threshold was set rather lower than in recent series.

The pre-printed answer booklet worked quite well in keeping responses in order so that they could be marked online. However, there were a few candidates who answered in the wrong place or allowed their answer to spill over to the next question. Centres who issued answer booklets instead of additional sheets to candidates who needed extra space created some unnecessary scanning and checking through of blank pages, especially when candidates had used only the first page of an 8 page booklet. Centres are also asked to note that candidates should not hand in their pages of rough work.

Comments on Individual Questions

Section A

1 This was generally well answered. The common error in the first step was to write $2c = \sqrt{a+b}$ and those that made it were usually able to take advantage of the follow through marks available.

2 A few 'equals signs' were seen and a few inequalities the wrong way round were in evidence, but in the main this was often well done using correct notation throughout. The most common error was getting $2x+5$ from multiplying $(x+5)$ by 2. There were quite a few who seemed unsatisfied with $\frac{13}{3}$ or $4\frac{1}{3}$ and felt they had to express the answer as a recurring decimal.

3 Part (i) was usually correct but some spoiled good work in obtaining $x = 4$ by giving (0,4), or found (0,10) or gave the intersections with both axes as though they were unsure which one was required.

In part (ii) there were many correct answers from the substitution method. The elimination approach was popular and these solutions were often wholly correct, but some were longwinded, getting there eventually but at a cost of time. A few sign errors were seen and there were a few who stopped after correctly finding the value for x. There were also quite a few who gave the y value as $(5-{}^{10}\!/_{3})$ or similar as if they could not perform the evaluation.

4 Many candidates omitted to describe the transformation as a translation (which was required for full marks), preferring terms such as 'movement' or 'shift', but most candidates showed they had the right idea. Responses ranged from a vector to quite verbose descriptions. Contradictory statements were not uncommon.

Many good, correct sketches were seen. Most candidates were able to produce a parabola but not always in the right place or right orientation – some were clearly sketching the curve from part (i). Failure to label all (or even any) of the intercepts often lost the second mark. A few did not attempt this question at all.

5 Many candidates were able to answer part (i) without a problem, but with varying amounts of working shown. Weaker candidates often knew that $144^{\frac{1}{2}} = \sqrt{144} = 12$ and gained a mark for this.

In part (ii), many correct expressions were seen, but the weaker candidates had problems here, often with wrong methods for adding fractions such as multiplying 4 by $5-\sqrt{7}$ instead of by $5+\sqrt{7}$. Denominators of 32 instead of 18 (from $25 + 7$ instead of $25 - 7$) were not uncommon and some final additions led to denominators of 36.

6 In part (i), most candidates drew a cubic curve and most had it the right way up. Most had a turning point at (–1, 0), with a few at (1, 0). The common error was not to identify the y -intercept, whilst some had the intercept on the positive x -axis at 5 instead of $\frac{5}{2}$.

Expanding the brackets was generally well done. Some did this in part (i), not realising that there was an easier way to find the *y*-intercept there. The most common error was to write $(x+1)^2$ as x^2+1. A small number of candidates tried to multiply all three brackets at once, and there were consequent errors.

7 Most candidates used the remainder theorem, and most were successful apart from minor slips. A small number used f(3). Long division was also quite a popular method; in this case, the final step turned out to be difficult unless matching coefficients was also used to obtain the constant as 8, and answers of 24 or 18 for k from this method were as common as 30.

8 This was much the worst answered question in section A. Some candidates tried to multiply out the brackets, but generally failed to find correct terms. Most used the binomial method and Pascal's triangle, not all being quite sure which row they were on. Candidates who wrote the coefficients as combinations often had errors. The biggest difficulty was the powers of 5/*x*, often with either numerator or denominator powered, but not both. This in turn led to terms which should have been kept separate being combined. It was quite common to see unsimplified terms such as $\frac{15x^2}{x}$ in the final answer. A few candidates, not liking algebraic fractions at all, insisted on multiplying through by x^3 at some stage of their working.

9 Showing that the line and the curve do not intersect was quite well done. The majority of candidates eliminated y and found the correct quadratic in x and most then knew what to do, pleasingly many confining themselves to the discriminant. There were occasional slips in the arithmetic, particularly writing ½√(64 - 68) as √(-2), but most candidates knew what was expected of them. A small number of candidates tried to draw, and then work with the proximity of, the two separate graphs.

Section B

10 **(i)** Proving that the lines are parallel was done very well, with nearly all candidates knowing the condition for the gradients of parallel lines and just a few candidates making errors such as 4/8 = 2.

(ii) Many candidates gained full marks here, although some of these wasted time by obtaining the lengths of all 4 sides. Some showed that the gradients of AD and BC were not equal and thought that this was sufficient, presumably thinking that they had shown the base angles were unequal. A few candidates successfully used other properties of an isosceles trapezium, such as showing that the line joining the midpoints of AB and DC is not perpendicular to those lines. A small proportion of candidates did not know what isosceles meant.

(iii) Some candidates had forgotten basic work on quadrilaterals and a significant proportion of candidates found the intersection of AD and BC instead of that of AC and BD. With the benefit of hindsight it would have helped if the question had read 'The diagonals AC and BD of the trapezium meet at M …' as the intention was to examine candidates' understanding of coordinate geometry and ability to apply that. Those who knew what diagonals were, and correctly worked with AC and BD, were usually successful in gaining the method marks in this part, although errors in arithmetic were fairly common.

(iv) This was often well done; however a very common error was to try to establish instead whether the diagonals were perpendicular, a result perhaps of familiarity with the term 'perpendicular bisector', although such candidates did not pay any attention to the 'bisecting' aspect.

11 **(i)** This part was done successfully by all but a handful of candidates. Very occasionally the signs for the coordinates were wrong and the radius was given as 25.

(ii) A large majority of candidates managed to pick up one or two marks in this question, but only about 30% of them succeeded in gaining all three. Very few candidates failed to show that the point with coordinates (6, -6) was on the circle. Most did it by substituting the coordinates into the equation of the circle, but a few did it by showing that the distance between the centre and that point was equal to the radius (5). Many candidates were not rigorous enough in showing that the point B, at the other end of the diameter, had the coordinates (0 , 2); very many of them considered that it was sufficient to simply show that B was on the circle or, alternatively, to show that the length of the line AB was twice the radius. Similarly a few candidates thought that it was sufficient to find the centre of the line AB but didn't relate this point to the centre of the circle in order to complete the argument. However, there were some fully correct arguments, with a few candidates choosing to do more than was really necessary in order to be certain, rather than following a specific line of reasoning. Of the few candidates who tried to use the equation of line AB, hardly any went on to show that (0, 2) was the intersection of the line and the circle. The method based on vectors was rarely used, though some candidates succeeded in showing this with the help of diagrams.

(iii) The majority of candidates understood what was required in terms of their knowledge about perpendicular gradients and of methods for determining the equation of a straight line. Although there were many correct solutions, answers were often spoiled by careless arithmetic or algebra. In finding the gradient of AB, some candidates made errors in working with the negative numbers, and the gradient of the tangent was sometimes written down as simply the reciprocal of that of AB. However the vast majority correctly used <u>their</u> gradient in conjunction with the coordinates (6, -6).

(iv) Only a few candidates made a success of this final part. There was little evidence of methodical thought in trying to determine the centre of this new circle. Those who drew diagrams tended to be more successful, and having found the coordinates of the centre nearly always wrote down correctly the equation of the required circle.

12 **(i)** This was quite well answered, with some candidates appreciating that with the factors of x and $(10 - x)$ they could just write down the answer. However, some did try to make it over-complicated via their algebra – multiplying out brackets, then using the quadratic formula, with errors often creeping in during the process.

(ii) Again, this was generally well answered although too many candidates gave a single answer of 5 only and thus just scored a single mark. A few candidates benefited from the follow through mark here.

(iii) A high proportion correctly identified their *d*. However, significantly fewer candidates went on to use the figure successfully to score either of the next two marks. A common error was to state *d* = 3, although a small number of candidates did go on and use their value and gain a method mark for doing so. The last A mark was earned in various successful ways - $\frac{91}{20}$ or $4\frac{11}{20}$ or 4.55 or $\frac{22.75}{5}$ were common. It was sad to see, however, just how many times this mark was lost through the candidates' inability to work with either fractions or decimals.

(iv) This part was omitted by 40% of the candidates, although some were able to make a fresh start here and gain several of the marks. The most common wrong approach by candidates who could not see how to begin otherwise was to consider the cross-sectional area of the lorry, whilst Pythagoras was attempted by some. A significantly high proportion of those who did know where to get started had insufficient algebraic skills to continue past obtaining the quadratic equation – a fair number scored the first two marks only. Some who obtained a correct form of the equation did not simplify the fractional coefficients before they attempted to apply the quadratic formula; they very often became confused when sorting out their end results and made spurious cancelling errors. The quadratic formula was usually well-quoted, which was pleasing, although a small number of candidates did benefit from the allowance of one error. A small number of candidates attempted to complete the square, often with success. Many candidates who obtained the correct pair of values for x failed to go on and work out the width of the lorry.
A few candidates opted for the alternative approach of expressing x in terms of the width and making the substitution – leading to an elegant solution.

4752 Concepts for Advanced Mathematics (C2)

General Comments

The paper was generally well received, with very few poor scripts. However, there were also fewer outstanding scripts than usual for a January series. Some high-scoring candidates lost marks through careless errors. A significant minority of candidates wasted time by using graph paper for an accurate plot when asked for a sketch. There were some very good scores in the second half of section A; many candidates scored full marks on questions 6, 7, 8 and 9.

Comments on Individual Questions

Section A

1 Many candidates lost marks on this question. Some candidates lost an easy mark by omitting "+ c". Weaker candidates failed to deal with $\frac{3}{x^2}$ correctly: sign errors were quite common, but some candidates gave the answer as $\frac{6}{x^3}$, in spite of obtaining the other term correctly.

2 This question defeated many candidates. Those candidates who did obtain the correct answer often presented no reasoning, in spite of a clear instruction to do so. A good number of candidates thought the period of the sequence was 5 or 10. Many candidates blindly applied the formula for arithmetic or geometric progressions in part (ii), or wrote 13.75 × 12 (which fortuitously gives the correct answer). Fully correct solutions were in the minority.

3 This question was generally well done. Some candidates obtained $\sqrt{11}$ or $\sqrt{5}$ for the third side of the triangle, or went straight for their calculators and presented all the numbers from their display, which didn't score.

4 This question was done very well indeed. Only a few candidates used $r\theta$ or $r\theta^2$; fewer still failed to manipulate the correct formula to obtain the correct answer. Even the majority of those who converted to degrees earned full marks by using their calculators effectively. A very small minority thought the angle was 0.4π radians.

5 There were many excellent responses to this question. In part (i) some candidates sketched 2f(x) or f(½x). The usual errors in part (ii) were to sketch 4f(x) or f(4x). Many candidates unnecessarily presented accurate plots on graph paper.

6 Part (i) was accessible to most. Nearly all candidates obtained the correct answer, with a small minority making arithmetical slips. Some weak candidates gave the answer as 51 + 4, or used the formula for the sum of the first 51 terms. A surprising number substituted n = 55 in otherwise correct working.
There were many excellent answers to part (ii). A few lost the accuracy mark by presenting the answer 8.3 or 8.33. Some weak candidates used r = 2.5, and did not score.

7 Most scored very well on this question. Some lost the accuracy mark in part (i) through premature rounding or truncating the final answer. A few used $\sin\theta = \frac{o}{h}$ and did not score. The majority of candidates scored full marks in part (ii), even when they used convoluted methods to get there. Some weaker candidates tried to use the Sine Rule, or used sin*θ* instead of cos*θ* in the Cosine Rule formula. Others failed to cope with the double negative.

8 The majority of candidates scored very well indeed on this question. Only a few candidates differentiated $x^{\frac{1}{6}}$, and a few slipped up by giving $\frac{\mathrm{d}y}{\mathrm{d}x}$ as $12x^{-\frac{1}{2}}$. Most found *y* = 24 correctly and obtained the following method mark. Some candidates used the gradient of the normal at this stage, thus losing the last two marks. A few candidates took *m* to be $3x^{-\frac{1}{2}}$.

9 Part (i) was very well done. Some candidates failed to score because they only sketched the curve in one quadrant, and a few lost a mark by failing to identify the *y*- intercept. In part (ii) most scored full marks. A few lost the final mark by presenting the answer to 3 s.f. $x = \frac{1}{2}(\frac{1}{\log 3} + 1)$ and $x = \frac{1}{2}\frac{1}{\log 3} - 1$ were occasionally seen.

Section B

10 **(i)** This was done very well indeed, with many candidates scoring full marks. A few candidates failed to justify their identification of the nature of the turning points, or used the second derivative and reached the wrong conclusion.

(ii) This was not done well. Some candidates went straight to the quadratic formula and consequently missed the value *x* = 0 and did not earn any marks. Most simply earned the first mark because they only presented decimal expansions of the two irrational roots instead of giving them in surd form.

(iii) Most candidates were able to score both marks, even if they had not been entirely successful in parts (i) and (ii).

11 **(i)** The majority of candidates scored full marks here. Most went straight to the composite form of the Trapezium Rule. A few candidates made arithmetic slips, thus losing the accuracy mark, and a small number lost all the marks by omitting the outside brackets or by substituting *h* = 12. A few candidates wasted time by calculating the area of each individual trapezium before summing.

(ii) Many candidates simply used the lower x value throughout, thus failing to score.

(iii) This was generally well done. A common slip was integrating 2.4 to obtain $24x$, and many candidates lost the final accuracy mark through poor calculator work. Some lost the last two marks because incorrect limits were used.

(iv) A number of candidates simply made a general comment here, which didn't score. Others substituted in their integrated function. Those who did obtain the correct value of 5.3… did not always appreciate the need to compare this with 5.2.

12 **(i)** Far too many candidates lost marks on this standard piece of bookwork, either through failing to produce a correct formula, or through presenting the correct formula after wrong working. Many candidates left $\log_{10} 10$ unsimplified, which was penalised.

(ii) The overwhelming majority of candidates scored full marks on this section. However, 2.36 was sometimes seen instead of 2.44, and lines were not always ruled, which cost a mark.

(iii) The expected approach of log*a* = intercept and *b* = gradient generally yielded the first three marks, although some failed to obtain a gradient within the required range. A few candidates were not sure whether "a" was c or 10^c. Only the best candidates then went on to present the equation in the required form.

(iv) *t* = 105 was often used appropriately, but *t* = 95, 2050, 105 and 110 were also seen. A good number lost an easy mark by presenting an ambiguous answer – it needed to be clear that the candidate was talking about thousands of millions of people, not thousands of people, to score. The final mark was rarely awarded. Many commented on natural disasters rather than commenting directly or indirectly on the problems associated with extrapolation well beyond the range of available data.

4753 Methods for Advanced Mathematics (Written Paper)

General Comments

This paper proved to be accessible to nearly all candidates. However, there were some questions, such as 3, 6 and 7(ii) which tested the abler candidates. Fewer than usual candidates scored full marks – the final part of question 7 proved the main stumbling block – but, equally, virtually all candidates scored above 20. There was no evidence of lack of time to complete the paper.

With reference to recent examiner's reports, it was pleasing to note that fewer candidates were using graph paper for their sketch in question 3. In general, some candidates seem to be insufficiently aware of the significance of words such as 'verify' (see question 8(ii)), and 'hence': most candidates missed the significance of this in questions 6 and 7(ii). Candidates also need to be clear what is meant by exact answers, or they will lose marks in this paper.

In general, the calculus topics continue to be well answered, albeit with some sloppy notation used in integration, with modulus, proof and inverse trigonometric functions being less securely understood. The standard of presentation varied from chaotic to exemplary.

Comments on Individual Questions

Section A

1 There were many good responses to this opening question, but there were a variety of incorrect approaches. One was to attempt to take logs before rearranging to $e^{2x} = 5e^x$. This led them to the error of $\ln(e^{2x} - 5e^x) = \ln e^{2x} - \ln 5e^x$. Other mistakes were in factoring out e^x, or inability to simplify $\ln 5e^x$ correctly.

2 This was extremely well done, with most candidates scoring full marks. A few candidates lost a mark for leaving *k* as –(1/5)ln(1/2). Over-accurate answers were not penalised in this instance. The laws of logarithms were generally correctly applied here.

3 This proved to be a straightforward test of the chain rule and implicit differentiation, with many candidates scoring full marks. Occasional index errors were made, and some candidates failed to simplify their answer to part (i), leaving it as $6x(1+3x^2)^{-2/3}/3$. The implicit differentiation in part (ii) was generally well done, and most candidates who had correctly done part (i) correctly showed the equivalence of their answers.

4 Part (i) was usually done correctly, with many candidates spotting the $f'(x)/f(x)$ form of the integrand and writing $\ln(x^2 + 1)$ directly rather than using integration by substitution. However, part (ii) proved considerably more searching. A large number of candidates tried integration by parts without success. Those who used the correct substitution either made substitution errors or failed to split the fraction before integrating. We also required some evidence for dx = du for the first M1 – this was usually correctly given.

5 Part (i) was more successfully answered than part (ii), with c = 1/3 the most common error. In part (ii), many tried to use a translation to account for the reflection of the sine curve, which did not fit in with the required form of the function.

6 Most candidates gave correct algebraic definitions of odd and even function, often accompanied by redundant geometric descriptions, though some candidates failed to put the brackets and negative signs in the right places. However, the second request was rarely convincing – most candidates just substituted particular functions to verify the result, and were unable to deal with the abstraction of the general argument.

7 This proof question was also generally not well done, despite the hint. Getting from θ = arccos x to x = cos θ appeared to be considerably harder than might be expected. Some candidates correctly deduced that x = cos θ and y = sin θ but failed to convince by appearing to argue that $x^2 + y^2 = 1 \Rightarrow \cos^2\theta + \sin^2\theta = 1$. Indeed, many candidates seem to fail to understand the concept of direction of argument in proof. Common errors seen were arcsin x = 1/sin x and cos x = sin y.

Section B

8 This question tested differentiation and integration techniques as applied to a trigonometric function. Some candidates worked in degrees which was condoned in part (i) but inappropriate thereafter. In general, many candidates scored well.

(i) This was perhaps the hardest part of the question for some candidates. Solutions in degrees were accepted, but quite a few made errors in dealing with the '3' in cos 3x = 0, and in finding the second intercept Q. Although condoned in this particular case, students should avoid making statements like P = π/6, and some wrote the coordinates the wrong way round.

(ii) The product rule was done well. In finding the gradient at P, some worked in degrees and gained M0. The given result on turning points was reasonably well done, though some wrote cos 3x / sin 3x = tan 3x.

(iii) Integration by parts was well known, with fewer candidates seeming to make errors with dv/dx = cos 3x $\Rightarrow$ v = 1/3 sin 3x. Limits from part (i), if expressed in radians, were followed through up to the final 'A' mark: errors in evaluating cos 0 = 1 were not uncommon.

9 This question relied on a good understanding of function language (domain, range, inverse etc.) and required careful use of notation to obtain full marks. The calculus in part (i) was well done, but that in part (iii) concerning the second derivative was less secure.

(i) The quotient rule was done well, though omitting necessary brackets forfeited the final 'E' mark. The deduction that f was increasing required justification that both the top and bottom of the derivative fraction were positive – many just verified it was positive for individual points.

(ii) The range required the student to evaluate f(2) to find the maximum: some assumed the domain was unbounded. We required correct inequalities (≤ not <) and y or f(x) for the range – some gave this as 2.4, which was disallowed.

(iii) This was less successfully done – setting the second derivative to zero to find the maximum of the derivative was less familiar.

(iv) We insisted on correct inequalities and consistent use of x, y or g(x) in the statement of domain and range, but followed through their answer to the range of f, provided this was bounded. For the sketch, most of these were recognisable as attempts to reflect in $y = x$, but needed three out of four of –1, $\sqrt{2}/2$, 1.4 and 2 to gain the A1. Some candidates misinterpreted 'on a copy of Fig. 9' as meaning that this copy should have been provided – we marked question papers which were included with scripts. On-screen marking will facilitate the provision of copies of figures such as this.

4754 Applications of Advanced Mathematics (C4)

General Comments

This paper was similar in standard to the January 2009 paper. It provided questions that were accessible to all candidates. As usual the entry for the January paper, although increasing, was relatively low. The standard of work was, once again, high. Good scores were obtained by many and some clear presentation was seen. The Comprehension proved to be well understood by all and scored highly. Section A of Paper A provided a wide range of accessible questions. The first part of question 7 in Section B provided the greatest challenge; here many solved the differential equation rather than verifying (as the question asked).

Candidates should be advised to read questions carefully. Many lost unnecessary marks by failing to 'verify' or 'show' fully when required. These words are part of the question and should not be overlooked. Similarly, 'explain' needs words and 'hence' should be seen as a big hint - even though alternative methods will sometimes gain credit. Questions requiring answers in degrees or radians are often answered in the alternative form or for the wrong range. Candidates could often avoid losing marks in these cases if they took more care.

Comments on Individual Questions

Paper A

Section A

1 The binomial expansion was well answered. There were few errors other than numerical slips such as using $2x$ instead of $-2x$. The most commonly lost marks were in the validity statement which was often omitted.

2 A few candidates failed to obtain the E mark in the first part by failing to state both $\cot 2\theta = 1/\tan 2\theta$ and the double angle formula for $\tan 2\theta$ even though one felt that they meant to write these down.
The second part was almost always successful. Most candidates found and solved the quadratic equation and found all four solutions.

3 Here dx/dt was sometimes seen with the coefficient ½ instead of 2. dy/dt was usually correct if approached as a product or quotient, but many missed this and gave $dy/dt = -2t/(1 + t)^2$ having only differentiated the denominator. The general method was usually correct but not all could accurately substitute $t = 0$.
In the second part, most approached from x rather than y. Generally $t = 1/2 \ln x$ was obtained, but substitution and simplification caused errors when finding y in terms of x.

4 Part (i) was almost always correct. Some gave the answer in co-ordinate form which was condoned on this occasion.
In part (ii) there were many completely correct solutions. However, a number failed to realise it was necessary to prove that the normal was perpendicular to two vectors in the plane in order to establish that it was a normal vector.
For the equation of the plane, common errors included using *i/j/k* instead of *x/y/z* or obtaining -5 or 7 instead of 5.

5 Whilst most candidates correctly found λ or μ or both, some failed to verify that these values worked for all the co-ordinates in both lines.
The angle between the lines was usually found correctly although a few candidates used the position vectors instead of the direction vectors.

Section B

6 **(i)** Angle BAC was usually found successfully but a few omitted this or made incorrect or unclear statements.

(ii) This was almost always correct. Most wrote down the correct formula and substituted for cos 60 and sin 60. A few lost the *b* when expanding the bracket.

(iii) For those that put h=0 and divided by cos θ this proved very successful. Some omitted this part. Occasionally incorrect statements such as sin $0 = \sqrt{3}b$, cos $\theta = 2a+b$ were seen. Others failed to substitute $h = 0$ but continued with the correct method on the RHS without changing the LHS. Full marks could be obtained by substituting tan $\theta = \sqrt{3}b/(2a+b)$ and working backwards to show that $h = 0$.

(iv) This was usually successful. Some candidates attempted only this part of question 6. Some lost a mark here for working in radians.

7 **(i)** This proved to be the most difficult part of the paper. The majority chose to solve the differential equation rather than to use verification as asked in the question. Those who chose to verify were usually successful and some efficient solutions were seen. The approach from the differential equation was prone to error. Whilst some good solutions were seen, common errors included omitting the constant of integration, integrating $1/(1-x)$ as $+\ln(1-x)$, using incorrect log rules (particularly with the constant) and when rearranging to make x the subject. The mark for checking the initial condition was usually obtained. Verification proved to be the easiest route and it is unclear why so few tried this.

(ii) This part was usually well done although some used values of x other than 0.75; 0.875 was the most common. Others failed to manipulate their equation correctly and found that they needed the logarithm of a negative number and others prematurely approximated their answer.

(iii) The partial fractions were well done by most but a surprising number equated the coefficients of x^2 as $0 = B + C$ instead of $0 = -B + C$ and so incorrectly obtained $B = -1$ .Others started with $1 = Ax(1-x) + Bx^2(1-x) + Cx^3$ and lost marks - in some cases for apparently correct values of the coefficients found from incorrect working.

(iv) As the answer to this part was given, there was some 'fiddling' of answers from those that failed to integrate $1/(1-x)$ as $-\ln(1-x)$. Some failed to show the separation of the variables and did not show t =, only working with the right hand side, and many also tried to integrate $1/x^2$ as a logarithm. The general method was usually known and most tried to find the constant of integration even following incorrect working.

(v) This was usually correct although some did substitute incorrect values of *x*.

Paper B
The Comprehension

1 This was usually correct although some wrote out the alphabet. Occasionally 5 or 11 were given instead of 15.

2 '*The mathematician*' was almost always given.

3 *M H X I Q* was usually correct.

4 Most candidates understood what was required here. A variety of acceptable reasons were given although some were not sufficiently precise or were unclear.

5 The majority of candidates realised that the values given were factors of 84 and 40. However some failed to make it clear that these were the **only** common factors. In some cases the words 'factor' and 'multiple' were confused.

6 Few fully understood what was required here but many obtained some credit for their answers. Many were unclear. Few realised that with a keyword of length 2 the strings would be longer and so the frequency analysis would make the letter frequency more obvious.

7 The correct answers were usually seen.

8 **(i)** There was usually evidence of the intermediate H although some encoded in the wrong order.

(ii) FACE was usually correct.

(iii) Some candidates miscalculated which row was needed. Others decoded by doing the ciphers in the wrong order. However, Row D followed by plaintext T and then plaintext R was usually correctly given.

4755 Further Concepts for Advanced Mathematics (FP1)

General comments

Most candidates showed a good mastery of the material and were well prepared for the examination; many were able to score highly.

The standard of algebra was generally high, but some candidates fell down on several questions through lack of algebraic fluency and it appeared that a small proportion of candidates had been entered for the examination before they were ready.

Comments on Individual Questions

1 Complex numbers

This was generally well done, though a few made careless sign errors. In part (ii) almost all could use the complex conjugate to carry out the division, but some incorrectly assumed the numerator of $\frac{\alpha}{\beta}$ would be -13 + 11j, the answer to part (i).

2 Matrices

(i) **AB** not possible and **CA** and **B** + **D** were mostly correctly found. Several errors seen were: **CA** and **AC** not possible; **AC** = **CA**; **CA** found but **AC** thought impossible. Surprisingly, these errors were often made even by the candidates who scored highly on the paper as a whole.

(ii) **DB** was usually chosen and calculated correctly, but a significant minority of candidates opted for **BD**.

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3 Relationships between the roots of a cubic

There were many very good answers, but there were several common errors:

- A surprising number of candidates thought that $3a = 3$ leads to $a = 3$;
- There were errors with signs in using the standard results for $\sum \alpha$, etc;
- Many forgot to take account of the coefficient of x^3 and so did not divide by 4 when using the standard results;
- Some did not give the roots of the equation, despite a specific request to do so.

Finding k was usually done using $\sum \alpha\beta$, but quite a few chose to substitute a root, usually $x = 1$, which is probably quicker and easier.

Some tried to expand the product $(x-a+d)(x-a)(x-a-d)$, but very few did so without error in lengthy algebraic expressions and those that did often did not take account of the coefficient of x^3.

4 Three by three matrix and simultaneous equations

Most candidates chose to multiply the given matrices and thus found $k = 5$ easily. Some attempted to find the determinant of **M** (finding the determinant of a 3×3 matrix is not on the FP1 specification and so is never needed in the examination), but most of those that were able to do so correctly then assumed that $k = \det$ **M**, failing to check the cofactors. A few candidates failed to use the inverse matrix to solve the equations, as required, thus limiting their marks.

5 Summation of a series using standard results

There were many fully correct answers to this question, but a common error was to assume $\sum 6 = 6$, rather than $6n$. The resulting lack of common factor *n* either led to fudged workings or to a reassessment of the expressions, which was not always accurately pursued.

6 Proof by induction

A good answer requires a clearly stated argument with all the logical steps shown explicitly. In many cases this was achieved, perhaps helped by the simplicity of the algebra needed to establish the result for $1\times2 + 2\times3 + \ldots + k\left(k+1\right)+\left(k+1\right)\left(k+2\right)$. This manipulation was often carefully expressed, with correctly placed brackets, but there was often careless treatment of the summation, including omitting it altogether, leading to clearly incorrect statements such as $k(k+1) = \dfrac{k(k+1)(k+2)}{3}$.

Some candidates do not appear to appreciate the conditional part of the argument, "<u>If true for $n = k$ then true for $n = k + 1$...</u>" and try to run together the $n = 1$, $n = k$ and $n = k + 1$ cases as all true. This is made even more illogical if there was no earlier statement about assuming the truth of the statement when $n = k$.

7 Curve sketching

There were many very good answers to this question.

(i) A fairly common incorrect answer was to give $y = \frac{5}{9}$ for the horizontal asymptote.

(ii) Most candidates showed their method, usually by substituting large positive and negative values of *x*. The results were not always used correctly in (iii).

(iii) The right-hand branch was often incorrect, with no maximum shown. Approaches to asymptotes were sometimes sketched carelessly and many candidates did not label the points where the curve crossed the axes. Several candidates' graphs had inverted sections, suggesting they did not check the behaviour of the function near to the vertical asymptotes.

(iv) This was often answered correctly, given a correctly sketched graph. Some candidates appeared to misread ≤ 0, and gave answers for ≥ 0. Some gave inclusive inequalities at the vertical asymptotes $x = 3/2$ and $x = -7/2$. Some became extremely muddled in an attempt to solve the inequality algebraically, rather than using the sketch of the graph.

8 Loci on the Argand diagram

This question was probably the least well answered of the paper.

(a) (i) Modulus lines were often omitted and "≤ 4" was inserted instead of "$= 4$", although an equation was clearly requested. There were a few correct Cartesian equations given, but also some incorrect ones.

(a) (ii) This produced some misunderstanding and curious notations, despite the instruction to write down two inequalities.

(b) (i) It was not uncommon to see -2 - j used as the apex of the region. Candidates' diagrams often showed lines from the point $2 + \mathrm{j}$ (or other) to the origin and lines parallel to the Real Axis. Many diagrams were spoilt by inadequate annotation (especially failing to show the angles of the half-lines), in the absence of a properly labelled grid.

(b) (ii) There were few correct answers. Most commonly candidates found $\arg(43+47\mathrm{j})$ instead of $\arg(41+46\mathrm{j})$. Occasionally $\tan^{-1}(41/46)$ or $\tan^{-1}(43/47)$ were used. Those candidates who recognised that $y = x - 1$ was relevant often did well, but some only had a vague idea of this and failed to produce a convincing explanation.

9 Series and proof by induction

This question was often done well, but there was evidence that a few candidates were pushed for time at the end of the paper.

(i) This was usually correctly done, but those who chose to use partial fractions (not required for FP1), rather than simply showing the result as requested, probably used up valuable time on two marks.

(ii) Part (ii) was not always clearly presented. With a given result to work towards, many candidates either fudged their algebra, or did not show sufficient working for a convincing argument. It is always good practice to show at least the first three and the last two terms of the series in full, so that cancelling can be properly justified.

(iii) This was usually answered correctly, but some candidates who considered the structure of the general term instead of the total were led to a give a limit of 0.

(iv) There were few correct answers. A large proportion of candidates calculated Sum(100 terms) – Sum(50 terms). A surprising number believed that "0.01" shows three significant figures. Some candidates successfully repeated the method of differences, rather than using the totals for 100 terms and 49 terms.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

It is pleasing to report an increase of about a quarter in the number of candidates for this paper, as compared with January 2009. There was no evidence that these extra candidates had been incorrectly entered: again, the number of candidates scoring under 20 marks was well below 10% of the total entry, and the mean mark was almost identical to January 2009, although the standard deviation was a little greater. There was a great deal of very good work, with nearly a quarter of candidates scoring 60 marks or more.

Even competent candidates sometimes succumbed to quite frightening errors in elementary algebra or calculus. These included: confusion of differentiation and integration (the derivative of 1 in Q4(i) often appeared as x); "elementary" differentiation errors (e.g. the derivative of e^t appeared as te^{t-1}); poor use of the laws of logarithms (e.g. $e^t + e^{-t} = 2.5$ in Q4 was followed by $t - t = \ln(2.5)$ or $t + \frac{1}{t} = \ln(2.5)$), ignorance of the laws of algebraic fractions (e.g. in Q2(c) $\frac{t}{t+\frac{1}{2}t^2}$ was very frequently followed by $\frac{t}{t} + \frac{t}{\frac{1}{2}t^2} = 1 + \frac{2}{t}$ or similar) and other wishful thinking (e.g. in Q1(a) $\int_0^1 \frac{1}{\sqrt{x}\,(x+1)}\,dx = \int_0^1 \frac{1}{\sqrt{x}}\,dx \times \int_0^1 \frac{1}{x+1}\,dx$ or $\frac{1}{\sqrt{x}} \int_0^1 \frac{1}{x+1}\,dx$). On the other hand, some processes were handled with remarkable efficiency: these included the inverse matrix in Q3(i) and the hyperbolic equation in Q4(ii). Questions 3 and 4 were slightly better done than Questions 1 and 2; Question 5 (Investigations of Curves) was attempted by only one candidate. There was little evidence of time trouble, although as usual some candidates used very inefficient methods to answer some parts of questions: this was particularly evident in Q1(b)(ii), Q2(b) and in some parts of Q4.

Presentation was generally good although once again there were candidates who split questions up and scattered them around the paper, and others who used up to three eight-page answer books. Candidates who used supplementary answer sheets often tagged them in the middle of their main answer book: it is much easier to mark the paper (and sometimes to turn the pages) if these are tagged at the end.

Comments on Individual Questions

1 Calculus of inverse trigonometric functions, polar curves

The mean mark for this question was just under 11.

(a) While there were correct and efficient solutions, very many candidates thought the derivative of $\arctan\sqrt{x}$ was $\frac{1}{1+x}$. This caused problems with the integral, with many candidates blundering ahead in the ways described above. The fact that the answer was given did not deter some candidates from producing $\frac{\pi}{4}$ or ln 2.

(b) The majority of candidates knew how to convert from Cartesian to polar co-ordinates, which was an improvement on previous series. Some of those candidates could not then handle the algebra required to obtain the printed answer, with $\frac{1}{2}$s becoming 2s and the like.

The responses to the second part were very varied. Many candidates did use properties of the sine function, but many of these thought that the minimum value of sin 2θ was 0, and others confused θ and 2θ. Worrying statements like "sin 2θ = 2", sometimes followed by or following "sin θ = 1", were seen fairly frequently. Many gave values of r^2 rather than of *r*. Other candidates differentiated, either implicitly or (worse) the square root of the given right hand side: this sometimes led to the correct answers, but more frequently to a mess which sometimes covered two pages. Others produced a table: if the correct increment for θ was chosen (by chance?) this sometimes led to an answer which could be credited.

The sketch was usually well done, although some of the ellipses acquired dents or sharp points, and other candidates were determined to draw "well-known" polar curves such as cardioids.

2 Complex numbers

The mean mark for this question was 11.

(a) This part was well answered by the majority of candidates, and it was pleasant to see the efficient methods many of them used. It is quite acceptable to replace cos θ by *c* and sin θ by *s*. One common starting point was cos 5θ = $(\cos\theta + \mathrm{j}\sin\theta)^5$: this was condoned, but it is not correct. Some candidates were determined to use the $z + \frac{1}{z}$ method, which was not appropriate. Errors included: minor mistakes with binomial coefficients or signs; ignoring the sine terms completely when equating real parts; including the sine terms as part of the constants *a*, *b* and *c*, and omitting j altogether from the expansion.

(b) Candidates generally did a lot more work than was required in this part. Most recognised what was required at the beginning, writing the series *C* + j*S* in exponential form and recognising a geometric series, and many obtained a correct expression for its sum, although finding a sum to infinity was very common and a substantial number experienced trouble in finding the common ratio, which often lost its j. Those who obtained any sort of answer then often spent several pages trying to "realise the denominator": this often included reconversion to trigonometrical form, the use of addition formulae and appeals to the periodicity of sine and cosine. All that was required was to show that the sum was 0, so it was sufficient to show that the numerator was 0, which followed straight from $e^{2\pi \mathrm{j}} = 1$: only a small minority of candidates appreciated this.

(c) Although the required Maclaurin series appears in the formula book (and was often correctly quoted) and the stem of the question was "Write down", many candidates spent time deriving it, sometimes incorrectly. Then most candidates could get as far as substituting the series into the given expression, obtaining $\frac{t}{t + \frac{1}{2}t^2}$ or $\frac{1}{1 + \frac{1}{2}t}$. A minority were able to proceed appropriately, either using the binomial theorem or an elegant argument involving multiplying by $1 - \frac{1}{2}t$ or equivalent. Unfortunately the majority, having reached this stage, started blundering about (see above) or just wrote down the given answer with no linking statements. Some candidates just substituted a few numbers, or omitted the part altogether. Quite a few converted the expression to one involving e^{-t}: this required great care in implementation, but was sometimes successful.

3 Matrices

The mean mark for this question was just under 12. (i) was done very well, and (ii) was very often done well, while (iii) proved much more of a challenge.

(i) Finding the inverse of a 3 × 3 matrix caused little problem to the vast majority of candidates: indeed, some were able to almost "write down" the answer, which was most impressive. There were the usual minor sign errors: for some reason the top element of the middle column was most often incorrect. A few candidates multiplied the cofactors by the elements of the original matrix, while one or two set $a = -1$ at the start, and derived the given inverse without involving a: this attracted one mark if done correctly.

(ii) Many candidates used the inverse in (i) and obtained the answer with the minimum of fuss, although some forgot to divide by 5 or made other slips. Those who tried to solve the equations from scratch were often successful as well, although as usual many eliminated x, then y, then z, obtaining three equations in two unknowns from which they could not make progress.

(iii) This part was much less well done. Some tried to use the inverse matrix, setting $a = 4$ and ignoring the inconvenient determinant. Others tried to use the solution to part (ii), ignoring the fact that the system of equations had changed: many of these appeared to believe that, because the second and third equations had not changed, neither would the values of y or z, so if x were eliminated, they could proceed to find b. Others still were determined to use their knowledge of eigenvalues and eigenvectors somewhere, and proceeded to try to find and solve the characteristic equation. Those who did try to eliminate one unknown in two different ways frequently made slips, often when subtracting negative quantities or in working with fractions, so the correct value of b was not seen very often.

The geometrical interpretation was rather badly done. "Line" or "sheaf" were seen (with "sheaf" sometimes becoming "sheath" or even "shaft") but more frequently candidates asserted that the planes intersected at a unique point, or formed a triangular prism, or made self-contradictory statements such as "the planes all intersect along a line and two of them are parallel". Many candidates omitted this part.

4 Hyperbolic functions

This question was, by a small margin, the best answered, with a mean mark of 12.

(i) The proof was done well, although weaker candidates did not always "prove from definitions involving exponentials" and used other (unproven) identities. The second part required candidates to differentiate the identity with respect to x and obtain $2\sinh 2x = 4 \sinh x \cosh x$ in the first instance. This rarely occurred. Many went back to the exponential definitions and differentiated or otherwise manipulated those: those who did attempt to differentiate the identity often lost the 2 on the left, and the right-hand side defeated many completely. Many candidates omitted this part altogether.

(ii) This equation was often solved very efficiently and was a very good source of marks for candidates. Most picked up the clue from (i), although a small minority converted everything to exponentials, obtaining a quartic which could not be factorised, although several almost managed to. Once values of arsinh x had been obtained, many candidates went back to first principles, obtaining and solving quadratics in e^x to find x; it would have been enough to quote and apply the logarithmic form of arsinh from the formula book.

(iii) This part gave candidates a value of cosh t and asked them to show "using exponential functions" that t was ±ln 2; it was not acceptable here to quote from the formula book (with or without remarks about cosh t being an even function) but several did. The vast majority proceeded in a more appropriate way, although the manner in which –ln 2 was obtained was sometimes rather opaque. A few candidates verified the given result, which was quite acceptable as long as it involved exponential functions. The integral was usually done well although the final answer was given as ±ln 2 rather frequently. One small point is that, although the integral has two forms in the formula book, it is not true that $\text{arcosh}\,\frac{x}{4} = \ln(x + \sqrt{x^2 - 16})$, which was commonly asserted.

5 Investigations of Curves

Only one candidate attempted this question this series.

4758 Differential Equations (Written paper)

General Comments

The standard of work was generally good, with the majority of candidates demonstrating a clear understanding of the techniques required. There were, however, a significant number of arithmetic and algebraic slips which led to loss of marks. Almost all candidates opted for Questions 1 and 4, with Question 3 being the least popular choice. Those who did attempt Question 3 usually gained very high marks for their solutions.

There was a slight improvement in the standard of the graph sketching, compared with previous series, but candidates need to realise that in this paper any known or previously calculated information should be indicated on the sketch. Any particular features, such as the behaviour in approaching an asymptote, should be clearly and carefully shown. No further calculations are required.

Comments on Individual Questions

1 Second order differential equation

(i) The method required here was well known, but errors in solving the linear simultaneous equations obtained in finding the particular integral were seen from many candidates.

(ii) Most candidates earned the method marks, but the accuracy of their differentiation and arithmetic was not of a good standard.

(iii) A description involving oscillatory motion with constant amplitude was expected here. It was not sufficient to find an expression for y for large t, with no further comment.

(iv) Most candidates substituted $t = 20\pi$ in their expressions for y and $\mathrm{d}y/\mathrm{d}x$ and gained the method mark.

(v) The majority of candidates gave their complementary function from part **(i)** as the general solution to this differential equation, but failed to realise that different arbitrary constants were required. The graphs were very variable in quality. Most candidates recognised that the solution curve approached zero for large t. It was rare, however, to see correct use of the values found in part **(iii)** as the initial values of the function and its gradient.

2 First order differential equation

(a) (i) There were two methods available here. The minority of candidates who opted for separation of variables almost always gained full marks for their solutions. Those who opted for use of the integrating factor fared less well. Many omitted the minus sign in front of $\tan x$ and could make little progress with the integral subsequently obtained on the right hand side of the differential equation. Others failed to recognise $\cos x \tan x$ as being equivalent to $\sin x$ and so were unable to integrate successfully.

(ii) There were very few correct graphs, often because the general solution obtained in part **(i)** was incorrect.

(b) (i) The general idea of the solution curves was universally known, but the drawing of them was often carelessly executed. Sketches should not cross the gradient lines in the given tangent field and need to be drawn with care to obtain full marks.

(ii) Euler's method was well-known and the calculations usually correct, but candidates need to ensure that they show sufficient working to justify an answer that is given in the question.

(iii) The limits for x given in part **(a)** of this question were often assumed to apply in part **(ii)**. This led to answers that noted that 1.6 is greater than $\frac{1}{2}\pi$, rendering the method invalid. Marks were only awarded when candidates realised the existence of the asymptote at $\frac{1}{2}\pi$.

3 First order differential equations

This was the least popular question, with candidates perhaps deterred by its algebraic nature. The question was, however, structured to lead candidates through the process and those who attempted it usually scored very high marks. The given answers enabled them to work more accurately, as they very sensibly checked back through their work when they did not immediately attain the given answer.

4 Simultaneous differential equations

The vast majority of candidates knew the methods involved and if they had been able to execute them accurately, would have obtained at least 23 of the 24 available marks.

(i) Solutions were almost always fully correct, exhibiting an accuracy of work that seemed to evaporate for the remaining parts of the question.

(ii) The complementary function was found successfully, but the particular integral less so. There were some erroneous assumptions about the form of the particular integral.

(iii) Solutions were marred by accuracy errors in differentiation of the general solution and/or in simplification and collection of like terms following substitution. This led to a disappointing loss of marks by candidates who clearly knew the methods but who were unable to carry them out carefully.

(iv) Again, the method was known, but accuracy errors in execution were made by many candidates.

(v) For full marks, a statement was expected to the effect that, for sufficiently large t, the exponential term in each of the particular solutions for x and y tends to zero.

4761 Mechanics 1

General Comments

There were many candidates who submitted good, well-presented scripts which showed their sound grasp of the principles and their ability to apply them accurately. Most of the candidates were able to make progress with all of the questions and many produced good answers to all of them; the mistakes mentioned in the detailed analysis below of the candidates' responses should be viewed in this context.

Two matters that were not done well by a surprisingly large number of the candidates were the interpretation and construction of kinematics diagrams in Q1 and the production of the force diagram in Q6 (v); there are more details about these matters below.

It was very pleasing to read the many excellent explanations given in answer to Q4 (iii) and to Q5 (iii) from candidates who clearly exactly understood the points they were trying to make.

Comments on Individual Questions

Section A (36 marks)

1 Construction and use of kinematics graphs

Many candidates failed to score full marks. The meaning of the language of kinematics is not completely understood by many candidates and quite a few do not understand at all well how to interpret a kinematics graph.

(i) This was done well by most but by no means all of the candidates. Some candidates did not realise they only had to find gradients; few of those who tried to use the *suvat* results instead made much progress. A common error was to give the speed instead of the velocity for $2 < t < 3.5$.

(ii) Most candidates did this part quite well but some did not use horizontal lines or plotted speeds instead of velocities for $2 < t < 3.5$, even when their answer in part (i) was correct.

(iii) Most candidates understood what was required but a few answered in terms of moving towards or away from O. Perhaps because of misreading the graph, some said that the ring was not moving when $t = 3.25$.

2 Kinematics and the use of Newton's second law in vector form

Quite a lot of the candidates did this question very well, working accurately throughout in vector forms. Quite a common error was to substitute the initial position where the initial velocity was required throughout the question or just in part (ii).

(i) Most candidates did this accurately but a few gave the magnitude as their final answer.

(ii) This part was not done so well. Most candidates worked throughout in vector form and gave a vector answer but many failed to take account of the initial position.

(iii) Most candidates did this part well.

3 **The magnitude and direction of a vector; solving a vector equation.**

(i) This was done well by most of the candidates with the common error being to find the wrong angle.

(ii) It was pleasing to see so many candidates working efficiently at this problem. Although many candidates knew exactly what to do, others failed to get $\mathbf{H} = 4\mathbf{F} + \mathbf{G}$ or, having found it, didn't know how to extract scalar equations from it. A surprisingly large number found *a* and *b* but did not then state $\mathbf{G}$ and $\mathbf{H}$ (or got them wrong).

4 **A box in equilibrium with two strings attached**

This static equilibrium problem received sensible analysis from many of the candidates. It seemed that fewer candidates than in recent series confused sine and cosine. As always, finding a normal reaction presented too much complexity to candidates who lacked a systematic approach.

(i) This part was done well by most of the candidates.

(ii) Although quite a few candidates unnecessarily 'started again', there were many correct answers. The most common error was to resolve the tension in the string at C (or the value found in part (i)) in the direction of AB instead of considering the horizontal components of the two strings.

(iii) Although there were many correct answers, many more were spoiled by one or more of the following: confusing sine and cosine; omission of a tension; omission of resolution of one or more of the tensions; sign errors.

(iv) Very many good answers were seen from candidates who had clearly understood that the size of the force in AB depended only on the horizontal component of the force acting at C and that this latter value had not changed. Common errors were simply to claim that it was because the system is still in equilibrium or instead (or as well) to list (some of) the modelling terms 'light' and 'smooth' used in the question.

5 **The kinematics of a particle with non-constant acceleration**

It was a pleasure to see so many completely correct answers to this question, including the explanation required in part (iii).

(i) Almost all of the candidates answered this part correctly, even those who used constant acceleration results in part (ii).

(ii) Some candidates tried to use the constant acceleration results but a large majority used the correct method of integration. A surprisingly common error was for the '14' to become '4' in a miscopy at some stage of the working. A quite common mistake was to find the values of the integral for t = 3 and t = 1 but not subtract them.

(iii) A very pleasing number of candidates clearly understood that the key point is that $v > 0$ at all times means that the particle does not change direction. A few candidates seemed to have the wrong idea but rather more did not express themselves clearly enough to receive full marks – often their difficulty seemed to come from not clearly distinguishing between distance and distance travelled.

Section B (36 marks)

6 **Situations on an inclined plane involving friction. After considering static equilibrium this is followed by application of Newton's second law to one body and then two connected bodies.**

There were many good answers to all parts of this question except (v). It was expected that the complete analysis of this situation would be challenging to all but the strongest candidates but it was surprising how few candidates could produce diagrams showing the forces acting on each barge. Many candidates did not apply the rule that forces should the have same label if and only if they are known to have the same magnitude. Many candidates missed out one force due to the coupling or did not get the weights or the normal reactions in the correct directions.

(i) Most candidates did this part correctly but some failed fully to show the given result.

(ii) Most candidates answered this well but some did not state the direction of the frictional force.

(iii) Many candidates knew what to do but quite a few of these made a mistake with signs. Common errors were to continue to involve the 6.4 N force or to start again and this time confuse sine and cosine.

(iv) Most of the candidates obtained full marks for this part. Most calculated the displacement directly but quite a few unnecessarily used a two-step method and found the time first.

(v) Most of the diagrams given were poor for the reasons given above and few candidates obtained both marks. Whether the equations of motion of each barge or the equation of motion of the pair of barges was attempted, the most common error was to omit either the weight components or, less commonly, the friction terms. A surprisingly large number of those candidates who considered the barges separately omitted the tension terms. Many candidates produced complete and accurate numerical solutions without full (or any) marks for the diagram.

7 Various questions about the motion of a projectile with the trajectory equation as the initial given information

No question about a projectile set in an earlier series has started with the equation of its trajectory and so this question was structured to lead the candidates through a strategy to establish the required results. After parts (i) and (ii) (which were managed by most candidates), the rest of the question, as usual, required increasing levels of understanding of projectile motion. Most candidates worked parts (iii) and (iv) in the order given and many of these made a lot of progress and scored full or nearly full marks; many of the candidates who tried a different order failed to make much progress. A few candidates adopted approaches to the solutions that involved their knowledge of a general trajectory equation and some of these were successful. A mistake seen more than once was differentiating to find $\frac{dy}{dx}$ and then treating it as if it were $\frac{dy}{dt}$.

(i) Almost all of the candidates scored this mark.

(ii) The most common method for finding the horizontal position of the highest point was to argue from symmetry but many candidates used calculus. Most candidates who used either argument clearly obtained the correct value. Almost all candidates found the required height by substituting the x value they had just found in the trajectory equation.

(iii) Many candidates knew how to do this but quite a few made sign errors in their substitution of $s = ut + \frac{1}{2}at^2$. Candidates were expected properly to show that their answer agreed to 0.565 to 3 significant figures and many did not do so.

(iv) Most candidates who attempted this could see how to obtain the given horizontal component of velocity. As in part (iii), candidates were expected properly to show that their answer agreed to the given one to 3 significant figures and many did not do so – they were not penalised for this omission in both parts (iii) and (iv).

Although there are many ways to find the time of flight from Q to R, a few candidates who attempted this part were unable to find any of them. This request also seems to have been overlooked by quite a few candidates.

(v) A pleasingly large proportion of the candidates knew what to do and did it accurately. By far the most common error was to give the vertical component of the velocity at R instead of the speed.

4762 Mechanics 2

General Comments

Many excellent responses to this paper were seen and the majority of candidates attempted at least some part of every question and gained credit for their efforts. The standard of presentation was variable and, for some candidates, poor notation and failure to state the principles or processes being employed led to avoidable errors and to loss of marks. This was seen particularly in those parts of questions where a candidate was required to *explain* or *show* a given answer; many candidates did not structure a logical argument or give enough detail in either case.

Question 4 appeared to pose fewest difficulties to the majority of candidates.

Comments on Individual Questions

1 This was a high scoring question with the vast majority of candidates able to produce work worthy of significant credit.

(a) **(i)** This part posed few problems for the majority of candidates but many lost marks for failing to indicate the direction of the velocity they had found.

(ii) Almost all of the candidates showed understanding of the principle of conservation of momentum and Newton's experimental law and many obtained full marks for this part. Errors when they occurred were usually with signs. Those candidates who drew and labelled a diagram were on the whole more successful than those who omitted to do so.

(iii) This part caused difficulties for a minority of candidates. These, on the whole, did not appreciate that R and S would move as projectiles.

(b) Many candidates tried to treat kinetic energy as a vector and stated, incorrectly, that no energy would be lost in the horizontal direction. Others wrongly applied Newton's experimental law to both components of the initial velocity.

2 Many candidates seemed to understand the principles required to answer this question and applied them effectively.

(i) The majority of candidates gained full marks for this part. For those that did not, the most common error was to omit the term for gravitational potential energy. It was pleasing to see that the majority of candidates used units consistently and knew to take the time involved as 120 seconds.

(ii) Most candidates gained most of the marks for this part but only a small minority gained full marks by explaining clearly why the resistance to motion was equal to the driving force.

(iii) Most candidates obeyed the instruction in the question to employ an energy method. Some candidates found it difficult to correctly identify the term in gravitational potential energy; others omitted either one of the kinetic energy terms or the work done.

(iv) This part of the question was poorly done by quite a few candidates. Many used inconsistent notation and/or failed to provide sufficient detail to show the given answer. Many arguments to explain why the acceleration in this case could not be constant were muddled and incomplete. Some candidates tried to argue from the structure of the expression by rearranging it and did not relate the argument to the physical situation under investigation.

3 Many excellent answers were seen to this question but a significant minority of candidates struggled to provide sufficient detail when establishing a given answer.

(i) Almost all of the candidates obtained full marks for this part.

(ii) Most of the candidates understood that they needed to take moments for this part but failed to give enough detail in the working to show the given answer properly.

(iii) This part of the question was well done on the whole with the majority of candidates able to obtain correct values for the friction and the reaction forces. However, many failed to appreciate that an inequality was required and stated without any qualification that $F = \mu R$ and went on to say, again with no reason given, that $\mu > \frac{5}{8}$. Others understood that an inequality was required but did not appreciate the significance of the cupboard *not* sliding.

4 This was the highest scoring question on the paper with many excellent and completely correct answers.

(i) The majority of the candidates gained all the marks for this part.

(ii) Those candidates that were successful in part (i) were usually as successful in this part as well.

(iii) Many fully correct solutions were seen. Diagrams in many cases were good; those that were not were usually too small to be helpful and did not clearly show the centre of mass below the point of suspension.

(iv) Many candidates successfully obtained full marks for this part of the question. A minority, however, attempted to reinvent the wheel instead of using the results already established. Of those who were unsuccessful, many included a top to the bin or failed to appreciate that there were two ends to it.

4763 Mechanics 3

General Comments

The standard of work on this paper was very high. Most candidates were able to demonstrate their competence in all the topics which were tested, and about three fifths of the candidates scored 60 marks or more (out of 72).

Comments on Individual Questions

1 Dimensional analysis and elastic energy

This question was answered well, with about one third of the candidates scoring full marks. The average mark was about 15 (out of 18).

(a) (i) Almost all candidates gave the dimensions of density and kinetic energy correctly, but about one third gave the wrong dimensions for power. Common misconceptions were that power may be calculated as (energy × time) or (force × distance).

(ii) The method for finding the dimensions of viscosity was well understood and usually applied accurately.

(iii) The method for finding the indices in the formula was very well known, although there were quite a few careless slips (usually sign errors) in this part.

(b) The great majority of candidates realised that they should apply conservation of energy, and used appropriate formulae for elastic and gravitational potential energy. Minor slips were sometimes made in the calculation of the terms, but by far the most common error was to neglect the elastic energy when the rock is at its highest point.

2 Centres of mass

This question was very well understood. About 40% of the candidates scored full marks, and the average mark was about 15.

(a) Almost all the candidates were able to derive the given result correctly.

(b) (i) Almost all candidates knew appropriate formulae for finding the centre of mass of a lamina, with just a very few confusing them with those for a solid of revolution. The integrations were usually done well, although common errors here were losing the factor ½ in the integral for the y-coordinate, and failing to multiply out $(2-\sqrt{x})^2$ correctly.

(ii) Most candidates knew that they should take moments to find the tensions in the strings. However, a very common error in this part was to proceed as if the x-coordinates of A and B were 0 instead of 1.

3 Circular motion

Circular motion has often been a cause of difficulty for many candidates in the past. However, this question was answered very well indeed, with nearly half the candidates scoring full marks.

(i) Most candidates obtained the given equation correctly. Just a few did not realise that conservation of energy was required, and tried to derive the result from the radial equation of motion.

(ii) Most candidates used the radial equation of motion to find the tension in the string correctly. The work was quite often spoilt by sign errors, or by omitting the component of the weight.

(iii) This was very well understood. Some candidates found the value of θ when the string becomes slack, but omitted to calculate the speed.

(iv) This problem, about a particle moving in a horizontal circle while attached to two strings, was answered much better than similar problems in the past. There were no common errors, and most candidates found both tensions correctly.

4 Simple harmonic motion

This was certainly found to be the most difficult question. Only about 15% of the candidates scored full marks, and the average mark was about 12.

(i) This was quite well done, with most candidates calculating the tensions in the given position and verifying that there is no resultant force parallel to the plane. Others considered a general position and formed an equation for the equilibrium position. Some tried to resolve horizontally or vertically, but all of these omitted the normal reaction from their calculations.

(ii) The tension in AP was quite often wrong, with the extension being taken as x or $(1.55+x)$ instead of $(0.05+x)$. However, the given expression for the tension in BP was almost always derived correctly.

(iii) Most candidates set up an equation of motion parallel to the plane, although there were very many sign errors here, and the component of the weight was sometimes omitted. A fair number of candidates did not state at what stage they had established that the motion is simple harmonic, but most knew how to find the period from the equation of motion.

(iv) Most candidates were able to obtain a time at which the speed is $0.2\,\mathrm{m\,s^{-1}}$, but finding the first time for which the particle has this speed when travelling down the plane proved to be quite challenging.

4766 Statistics 1 (G241 Z1)

General Comments

The level of difficulty of the paper appeared to be entirely appropriate for the candidates with a good range of marks obtained. High-scoring candidates scored heavily on all questions with the exception of question 6; low-scoring candidates gained the majority of their marks from questions 1, 2 and 7. Very few candidates seemed totally unprepared. There seemed to be no trouble in completing the paper within the time allowed, and although the last parts of Q7 and Q8 were sometimes not completed this appeared to be due to a lack of knowledge rather than a lack of time.

Most candidates supported their numerical answers with appropriate explanations and working although some rounding errors were noted, particularly in question 4. Arithmetic accuracy was generally good. Particularly amongst lower scoring candidates, there was evidence of the use of point probabilities in question 8. The Venn Diagram question was answered significantly better than in previous examinations with many candidates gaining full marks.

Comments on Individual Questions

1 **(i)** There were many fully correct answers, but a significant number of candidates did not include a key. Some used 50, 60, 70, 80 rather than 5, 6, 7, 8 for the stem, and others were not careful enough in aligning their leaves. Only a very small number of candidates did not know what a stem and leaf diagram was.

(ii) The median was almost always found correctly but many did not how to find the mid-range, often giving (81-52)/2 = 14.5 as the answer.

(iii) Most candidates identified the median as the preferred measure, often with a correct explanation, but "middle of the data" was a common wrong answer. Those realising that outliers may be involved were more successful in explaining the reason for their choice than those using skewness. Some candidates thought that the mid-range was a measure of spread which did not help in their comparison.

2 **(i)** (A)(B) The response to this question was rather variable. Many stated that there are $^5C_2$ = 10 combinations, then wrote 1/10 and 2/10 without explaining where the 1 and 2 came from, whereas others gave very clear explanations, which were often of the form 1/5 × ¼ × 2, 1/5 × ¼ × 4, 2/5 × ¼ × 2, etc. with no explanation of the 2 and 4 multipliers and benefit of doubt had to be given. Many others used a probability method, often giving creditable fractional/decimal multiplications to show the values necessary.

(ii) The vast majority of attempts at E(X) and Var(X) were correct. Only occasionally did a candidate have no idea of how to go about this. There were also fewer instances of dividing by a spurious number or square rooting the answer than in the past.

3 This question produced better answers overall than in previous series; with several candidates scoring full marks.

(i) This was very well answered although a fairly common error was to mark the regions on the diagram with probabilities 0.24, 0.13 and 0.57 instead of 0.18, 0.07 and 0.69. Another error was to replace the 0.69 with 0.63.

(ii) The lack of independence of the two events was often correctly shown. Those candidates with correct diagrams sometimes wrongly stated $0.18 \times 0.07 \neq 0.06$. A small number confused independence with mutual exclusivity. Those who attempted to show that $P(G|R) \neq P(G|R')$ or similar often made mistakes finding the conditional probabilities.

(iii) The conditional probability was often found correctly, with or without correct diagrams. However a considerable number of candidates tried to use the incorrect formula $P(R|G) = P(R \cap G) / P(R)$.

4 A large number of candidates scored full marks although a significant number of candidates failed to realise that this was a binomial question.

(i) This was nearly always answered correctly. Omitting the ${}^{30}C_{20}$ term was the only recurring mistake. A few very weak candidates just gave an answer of 0.6^{20}.

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(ii) The fact that the mean of a binomial distribution is *np* was well known. Rounding to a whole number was common, usually 12, but sometimes 11. Some even stated "…because you can't have 0.52 of a student." Most did this after they had written a more accurate answer and did not lose marks. However in future series, rounding to the nearest whole number after getting a correct decimal answer may be penalised.

5 **(i)** Answers of 1/9999 and $(1/9)^4$ were seen regularly as were attempts involving ${}^{1000}C_4$ or ${}^{1000}P_4$. Arithmetic errors such as 0.00001 or 1/1000 also occurred.

(ii) Many correct answers were seen. However many candidates realised that 4! or 24 had some relevance but failed to produce the correct probability. These candidates often gave a final answer of 24 or alternatively divided 24 by 10000.

6 This proved beyond most candidates. Few scored full marks and a significant number scored none.

(i) ${}^{20}C_3$ was a popular wrong answer; seen more often than the correct $20 \times 19 \times 18$.

(ii) Correct answers to this part were very rare, with a wide variety of wrong answers. Amongst the more popular of these were $20 \times 20 \times 19 = 7600$ and
$6840 + 20 \times 19 = 7220$.

7 **(i)** Very few wrong answers were seen.

(ii) Most candidates used the correct frequencies and found the mean as 9850/120, usually approximated to 82.08 or 82.1. However a significant number of attempts used frequencies of 1, 3.5, 5.5 and 2 (the frequency densities). Use of class boundaries or incorrect mid points was rare. Most candidates correctly stated that their answer was only an estimate because they were using the mid-points of the intervals.

(iii) The standard deviation was often found correctly although not always accurately due to using 82.1 or just 82 for the mean. Only a few candidates divided by n rather than $n - 1$, so finding the RMSD rather than standard deviation. A number of candidates misinterpreted Σfx^2, and instead used one of Σx^2, $\Sigma(fx)^2$, Σxf^2, $(\Sigma fx)^2$ or even $\Sigma f \Sigma x^2$. Attempts at $\Sigma(x - \bar{x})^2 f$ usually failed but some correct answers were achieved this way. As in part (ii) some candidates used frequency densities. The quickest way to find both mean and standard deviation was by use of calculator and a number of candidates used this method.

(iv) The formula for outliers of $\bar{x} \pm 2s$ was well known and most candidates scored at least the method marks by following through with their $\bar{x}$ and s, but there were some who insisted on using 1.5s. The conclusion as to whether there were outliers was often incorrect, many stating there were outliers rather than introducing the idea of doubt. Only a very few attempted to use quartiles and interquartile range to find outliers.

(v) Nearly all candidates stated that there was negative skewness, with only a few suggesting it was positive or in some cases describing it as unimodal.

(vi) Most candidates attempted a sensible cumulative frequency curve with the main and surprisingly frequent error being plotting at the mid-points rather than the upper class boundary of the intervals. The other common error was the omission of the point (60,0) or replacing it with (0,0). Labelling was better than in the past, at least most wrote something on both axes. It would be helpful to see all candidates give the cumulative frequency values in a table before they drew the graph. Very few who drew graphs failed to realise the shape of graph required. Some centres appeared not to provide graph paper, whilst some candidates obviously preferred not to use it.

8 **(i)** (A) Almost all candidates answered this correctly.

(i) (B) Answers to this fell into two roughly equal groups; those who realised that "medium or high" could be treated as one (i.e. "not low") and those who did not. The first group nearly always got the right answer. The second nearly always got the wrong answer. Attempts at exhaustive listings of LMH, LMM, MLH, MHL, …seldom included all 19 outcomes. The majority of correct answers were from candidates who simply calculated 1 – P(Low on no days).

(i) (C) Most candidates multiplied the three probabilities 0.5×0.35×0.15 but a lot left it at that or multiplied by 3 or cubed it. Another not infrequent wrong answer involved $({}^3C_1 \times 0.5^2 \times 0.5) \times ({}^3C_1 \times 0.65^2 \times 0.35) \times ({}^3C_1 \times 0.85^2 \times 0.15) = 0.0541$.

(ii) Here most did recognise that "low or medium" could be grouped together as "not high" and used the binomial, B(10, 0.15).

(A) There were very many fully correct answers usually from binomial expressions, but also occasionally from tables.

(B) There was more use of tables here but still the majority of candidates calculated the answer. Some failed to remember to include the binomial coefficient $^{10}C_1$.

(iii) The correct hypotheses and test value of 15 were given. Many candidates could not correctly find $P(X \geq 15)$. $P(X \geq 15) = 1 - P(X \leq 15)$ leading to 0.0059 was widespread; certainly more common than using the point probability, which was also often seen. Attempts at the critical region often showed similar problems with upper tail probabilities; many attempts resulting in {14,15,16,17,18,19,20}. Some candidates totally omitted a conclusion in context. The reason for H_1 being $p > 0.5$ was generally well explained.

4767 Statistics 2

General Comments

Once again a very good overall standard was seen. No single question stood out as being more difficult or more straightforward than the others, and there was no evidence that candidates were short of time in which to complete the examination. A variety of techniques for handling hypothesis tests was seen; in most cases, candidates demonstrated a decent understanding of the technique they were employing. The vast majority of candidates handled probability calculations efficiently and accurately, using appropriate probability distributions.

Comments on Individual Questions

1 Candidates were first required to draw a scatter diagram for some given data; full marks were awarded in most cases, with occasional marks lost for erroneous points or failing to label axes. Most correctly identified the independent variable and the dependent variable, but few obtained both of the marks for justifying their choice. Many recognised that *a* was in some way controlled by the pilot but few satisfactorily explained that *t* was subject to random variation - however, many gained credit for implying this without stating it explicitly. Most gained full marks for calculating the equation of the regression line *t* on *a*, using it to obtain estimates for *t* and commenting on their reliability. Odd marks were lost for writing equations in terms of *y* and *x* instead of *t* and *a*, working with too little accuracy or providing unsatisfactory comments such as 'the prediction for 800m is reliable as it lies on the line' or ' .... as it lies near the points in the scatter diagram'. Some candidates calculated the regression line *a* on *t* and were penalised quite severely although most of the remaining marks were available with appropriate working and comments. Most candidates realised that to calculate a residual they must first find an estimate for *t* then subtract it from 923, producing a negative value which indicated that the observed value was less than the predicted value; however, many showed a poor understanding of this area of the course.

2 This question was based around the binomial distribution and involved the use of suitable approximating distributions to calculate probabilities. Part (i) was well answered by those candidates realising that the binomial model was needed - frequently seen mistakes included using 0.2 for 2%, and in some cases 0.1 was used (presumably from the '1 of a batch of 10' mentioned in the question). In part (ii), most candidates scored both available marks for explaining when a Poisson distribution is appropriate as an approximation to the binomial distribution; those referring to 'the number' and 'the probability' instead of n and p were penalised as this was deemed imprecise. Part (iii) (*A*) was well answered; some candidates used 0.02 for their Poisson mean instead of 3. Part (iii) (*B*) was less well answered; $P(X > 3) = 1 - P(X < 2)$ was seen regularly. In part (iv), most candidates managed to correctly identify the 'exact distribution' as Bin(2000, 0.02) in (*A*), and go on to use a Normal approximation to calculate the required probability in (*B*). Some candidates did not seem to understand what was meant by exact distribution and left this part blank, or wrote down the mean and variance. In (*B*), many candidates used a Normal approximation to the Poisson distribution rather than the required Normal approximation to the binomial distribution. Other common mistakes involved the lack of a continuity correction or use of the wrong continuity correction. Generally, the resulting Normal calculation was handled well. With more candidates using graphical calculators, it was not surprising to see the use of a Poisson approximation to the binomial distribution; this could lead to full marks. Candidates should note that in questions of this type they should provide evidence of their method; stating the distribution being used and giving some indication that such a calculator has been used is recommended as a minimum.

3 This question involving the use of the Normal distribution was well answered; most lost marks occurred in parts (iii) and (iv). Most candidates scored full marks in part (i) with the occasional mark lost through failure to work to a sufficient level of accuracy - either by premature rounding or by neglecting to use the difference column in the Normal probability tables. Part (ii) was well answered with few mistakes seen. In part (iii), many scored full marks, but a large number failed to realise that $P(X = 111)$ meant finding $P(110.5 < X < 111.5)$. Variations on this were seen (e.g. $P(110 < X < 112)$) and were given some credit. Part (iv) was generally well answered with most candidates able to obtain appropriate equations and solve them simultaneously. Common errors included using probabilities instead of z-values (e.g. $22 = \mu + 0.8\sigma$) or use of 1 - 0.5244 instead of -0.5244 (to give $15 = \mu + 0.4756\sigma$).

4 The first part of this question involved a Chi-squared test for association. It was pleasing to see the majority of candidates using appropriate terminology and providing sufficient explanation in their answers. Part (i) was well handled with only a few candidates mixing the null and alternative hypotheses. With the answer given in Part (ii), the onus was on the candidates to justify it by working at a sufficient level of accuracy; in many cases the candidates' working would not lead to the given answer. Candidates should be encouraged not to round expected frequencies too severely as this can have a large effect on the resulting chi-squared test statistic. Part (iii) was well answered by most candidates; however, some were unsure in their use of the phrase 'not significant' and many referred to 'two-tailed' tests or used the 97.5% value rather than the 2.5% value. In part (iv) most candidates provided the correct hypotheses but the definition of μ as the population mean was not commonly seen. Most candidates managed to successfully complete the test and received all the remaining marks. A common error in this part of the question involved treating the observed value of 30.9 as a single observation rather than the mean of a random sample of 50; however, this was seen less often than this type of mistake used to be. Some candidates provided a critical value from the product moment correlation coefficient table. Others found difficulty if they tried to calculate $P(Z < -3.951)$ as the provided tables do not cover this z-value; even so, those providing a sensible argument were given full credit. Other methods (e.g. confidence interval approach) were seen and could achieve full marks if handled correctly. Once again, some candidates made inappropriate comparisons such as $-3.951 < 1.645$ therefore we reject the null hypothesis, etc. but it is pleasing to note that this was seen less often than in previous years.

4768 Statistics 3

General Comments

There were 280 candidates from 41 centres (January 2009: 291 from 40) for this sitting of the paper. Overall the general standard of many of the scripts seen compared favourably with those seen in recent series: there were many examples of good, thorough and well-organised work. However, at the same time, some candidates showed considerable carelessness, particularly in the quality of their comments, interpretations and explanations. For example, some candidates stated their hypotheses badly or neglected to state them at all, and the final conclusions were sometimes badly expressed. It should be noted that, when a test produces a significant result, it is not correct to say "there is no evidence to suggest that H_0 is true"; but rather "there is evidence to suggest that H_0 may be false." Furthermore, candidates need to be aware that, when asked to show a result that is given in the question, the working leading to that result needs to be absolutely explicit in order to convince the examiner that the candidate has genuinely carried out every step of the necessary work.

All four questions were attempted. Marks for Question 4 were found to be a little higher on average than Questions 1, 2 and 3, which were equally well answered. There was no evidence to suggest that candidates were unable to complete the paper through a shortage of time.

Comments on Individual Questions

1 **Chi-squared test of goodness of fit of a binomial model; numbers of eggs per nest that hatch.**

(i) Sometimes the hypotheses for this test were missing completely while on other occasions they were badly expressed: for example, "H_0: $p = ½$" etc was seen on many occasions.
In contrast, the calculation of the expected frequencies and the test statistic were usually done correctly. The last part, the critical value and the conclusion, was generally correct, although there were many, as in the past, whose choice of language was too assertive.

(ii) The mean was usually correct, but occasionally there were errors in finding the estimate of p.

(iii) The majority of candidates, but by no means all, realised that an adjustment to the number of degrees of freedom and hence to the critical value was appropriate, but, as in part (i), some candidates were not as careful about their conclusion as they should have been.

(iv) Marks were not awarded for simply repeating what had already been established in parts (i) and (iii); candidates were expected to consider the different outcomes in a little more depth, for example by attributing the improved fit of the model to the fact that p had been estimated from the data.

2 Continuous random variables: the cdf and the median; Wilcoxon single sample test: birth weights of lambs.

(a) (i) There were many failed attempts to set up the necessary integral correctly. Limits were either incorrect or omitted altogether. Sometimes candidates were seen attempting to backtrack, probably as a result of realising in part (iii) that something was wrong.

(ii) Many responses did not show the basic characteristics of a cumulative probability curve. For full marks the sketches were expected to show the horizontal portions for $x < 2$ and for $x > 8$.

(iii) When the answer to part (i) was correct then it was not difficult for candidates to derive the required cubic equation for m. However, for this and for the verification of the value of m, the working needed to be convincing.

(b) The Wilcoxon test was almost always carried out with little difficulty. There were just two minor common shortcomings that incurred a loss of marks. In the hypotheses it was necessary to indicate that it is the <u>population</u> median that is being tested. (Also, it is not a good idea to use μ for the median.) As in other tests, the conclusion must be expressed using non-assertive language.

3 Paired *t* test for the population mean reduction in cholesterol levels; confidence interval for the true population mean.

(i) Both the distributional assumption and the hypotheses were sometimes carelessly expressed. There were occasional errors in calculating the differences in the data, thus leading to an incorrect test statistic, but broadly speaking candidates knew what they were required to do. The test itself was usually correct apart from the conclusion, which was almost always deficient in that either it was too assertive and/or (more usually) it contained no recognition that <u>on average</u> cholesterol levels appeared to be reduced.

(ii) By working backwards from the confidence interval, the mean and standard deviation of the second sample were frequently worked out correctly and, in some cases, very efficiently indeed. However, there were also many instances of candidates who could set up the correct simultaneous equations but who could not then solve them correctly.
The final part of this question asked candidates to interpret this particular interval. The standard response of “95% of intervals …” was seen frequently but did not impress the examiners. Some candidates tried to link it to the mean of the sample in part (i). Many tried to interpret it in terms of individuals. Hardly any candidates discussed it in terms of the population mean, let alone concluding that, since 0 is in the interval, it probably meant no improvement on the whole.

4 Combinations of Normal distributions; confidence interval for the true population mean; packets of different varieties of tomatoes.

On the whole, this question was answered very well by very many candidates.

(i) Intended as an easy introduction to the question this part was almost always answered correctly. However there was a noticeable number of candidates who looked up $\Phi(0.99)$ in the Normal distribution tables instead of $\Phi(0.909)$.

(ii) The majority of candidates made good progress with this part, and many scored full marks. Difficulties arose in respect of the expression needed for the variance of the weight of tomatoes in the pack. This often resulted in some adjustment at a later stage. Sometimes the candidate was able to recover the situation. On other occasions it meant that spurious calculations were used to arrive at the given value. Once again, the working leading to a printed answer needed to be totally convincing.

(iii) As in part (ii) considerable progress was made in this part, with full marks often being awarded. Again the main problem was in the calculation of the variance, which was not always shown clearly enough to allow a judgement about the validity of the method used for it. This time a further error was sometimes seen when candidates neglected to include the weights of the different packagings.

(iv) This part was very well answered, and it was quite common for candidates to score full marks for the confidence interval.

4771 Decision Mathematics 1

General Comments

There were no significant problems with this paper. There was a wide range of outcomes. Out of 2456 entries, 20 scored full marks, and 85 scored 70 or above. At the other extreme 247 (10.4%) failed to achieve the grade E threshold of 33, and 66 scored 20 marks or fewer (... based on 2379 scripts on the system at the time of writing).

Comments on Individual Questions

1 CPA

This represented an easy starter and nearly all candidates did well on it. Those that did drop marks did so mostly in their forward pass (e.g. [2,3] for B's "j" event) or their backward pass (e.g. [3,4] for C's "i" event). A few got the logic wrong, with C and D sharing the same immediate predecessors. Only a very few attempted activity on node, which scored zero.

2 Algorithms

This was a very successful question. Some candidates were able to sail through it with full marks in very short order, while others struggled to make sense of it.

There is a philosophical issue here which emerged in one or two answers, and which again surfaces elsewhere: continuous v. discrete. It is perhaps unfortunate that "graph colouring" should be the terminology which has developed and stuck. It might have been better had it been called "graph numbering". The point is that most would regard colour as a continuous measure, corresponding to the wavelength of the light, whereas in graphical work "colour" is used as a substitute for (natural) number. Thus, in part (ii), some candidates were exercised about whether or not a computer could recognise colour, whereas a graph theorist naturally refers to colour 1, colour 2, etc.

One of the most popular answers to part (ii) was that Kempe does not provide for an initial colouring. This was allowed, although it is trivial to give an initial colouring: "Colour vertex *n* with colour *n*".

3 Graph Theory

Very few candidates were able to compose a phrase such as "no pair of vertices is directly connected by more than a single arc", even if one suspected that they understood the concept well enough. It was not uncommon for candidates to make a correct statement, and then immediately to contradict it, often within the same sentence.

Surprisingly few candidates were successful with part (iii). Zero marks were awarded if an arc was added which connected the two sets. The majority of candidates did just that.

4 LP

This was stunningly successfully done. Very many mostly correct answers were seen and candidates are collectively to be congratulated.

One of the more difficult marks to gain was for the use of the phrase "number of" in formulating the problem. Markers were required to be very strict with this: it is so important in understanding the underlying algebra.

5 Networks

This represented a fairly routine, almost context-free, trot through standard work. It caused few problems, except for part (iv). Here the correct answers were 3 miles for inclusion in the shortest route and 2 miles for inclusion in the minimum connector. The reduced distances of 9 miles and 10 miles respectively were allowed, if given instead of the reductions. However, many candidates reasoned that they needed to **force** AD into the shortest path or the minimum connector by making it shorter than its competitor arc(s). Answers of 4 and 3 for the reductions, or 8 and 9 for the reduced lengths, were extremely common, and were marked wrong.

Use of the phrase "amount of people" is to be discouraged. It suggests an underlying confusion between continuous and discrete, between measuring and counting.

6 Simulation

The majority of candidates managed this well enough, although not all were clear in showing what they were doing, particularly in part (iv). There it is important that the apple is picked at the start of the day, and not at the end, after falling has taken place. In most cases correct simulation could be induced from the outcomes, even when the explanation was missing, but if it could not then marks could not be awarded.

Some candidates did manage to contrive unusual apple-related simulation rules, one or two of which were OK. However, without explanation such unusual correct solutions run the risk of not being seen as correct.

Part (v) referred to "reliability", not to "efficiency". Only "repetition" was acceptable as an answer. Using two-digit random numbers impacts on efficiency. Considering other factors, such as the weather, relates to the simulation modelling, and not on the results of the given simulation.

4776 Numerical Methods (Written Paper)

General Comments

There was a lot of good work seen on this occasion, with few candidates appearing to be unready for the examination. Routine numerical calculations were generally carried out accurately, though it is yet again disappointing that so many candidates set their work out badly. This is an algorithmic subject and good work will reflect that. A poor layout is difficult to follow for the examiner – and difficult for the candidate to check.

Interpretation remains a weak area, with quite a number of candidates simply omitting such parts of questions. In particular, the requests for graphical explanations were often poorly done or not attempted at all.

Comments on individual questions

1 Bisection method to solve an equation

This proved a straightforward starter for most candidates. The best solutions set the method out in tabular form and kept careful track of the maximum possible error. Weaker solutions were more like a jumble of figures. The only errors of substance were failing to stop the process at the required point and counting the number of iterations incorrectly.

2 Numerical integration

This too proved straightforward for most candidates, with the equations linking the various integration rules being well understood. The accuracy of the final answer was sometimes misjudged, but most candidates got this right.

3 Error in f(x)

This appeared to be a less familiar area of the specification. The given result was generally established correctly, perhaps because it involves very simple algebra and calculus, but there were relatively few successful attempts to apply it. A few candidates answered part (ii) with no evidence that they had used the result in part (i). Given the explicit instruction "Hence …", it was not possible to credit these attempts.

4 Numerical error in computation

Again, the given result was obtained by almost all candidates. A smaller number, but still a majority, were able to show that the algebraic result and its numerical equivalent agreed for small k, but disagreed when k became large. The explanations of why this should be were often missing or poor. The two points being looked for were that computers and calculators do not carry out calculations exactly. More particularly the subtraction of nearly equal quantities, as in this case, is a common source of error.

5 Newton's forward difference formula

The difference table and the deduction from it proved straightforward. Most candidates were able to write down the appropriate cubic, but the simplification was often done poorly. Those who constructed a cubic in x had far more work to do than those who saw that they could take $x = 1.5$ from the outset.

6 Numerical differentiation

The graphical demonstration in part (i) that the central difference method is generally more accurate than the forward difference method was often poorly done. Very many candidates did *not* show the tangent to the curve; as a result they had nothing to compare the central and forward chords with.

Parts (ii) and (iii), however, were usually very well done. The numerical results were obtained accurately and the correct inference was drawn from the ratios of differences of estimates.

7 Numerical solution by fixed point iteration

The sketches in part (i) varied greatly in quality. The simplest approach was to sketch $y = x$ and $y = 3 \sin x$. Candidates were divided as to whether there were any other non-zero roots.

The numerical work in part (ii) was done well, but the cobweb diagrams were frequently poor. Even those that were essentially correct mostly omitted vital information about the direction to be taken around the spiral.

In part (iii) the algebraic rearrangement defeated a surprising number. The numerical work, once again, was done well.

Coursework

Administration

Centres will be aware of the new computerised process for submitting marks and the generation of a sample request. This worked well for the majority of centres. However, it failed completely in cases where OCR has not been provided with a valid working email address. In these circumstances the moderator was advised of the sample chosen and requested but no work arrived because the centre had not received the request. It is crucial that OCR is provided with a valid and working email address.

Most centres adhered to the deadline set by OCR very well and if the first despatch was only the MS1 then they responded rapidly to the sample request. A small minority, however, caused problems with the process by being late with the coursework despatch. A number of centres submitted their marks so close to Christmas that a sample request could not be dealt with before the break. We would ask that all centres heed the deadlines published by the Board and organise their own processes of assessment, internal moderation and administration to enable these deadlines to be met.

The marks of most centres were appropriate and acknowledgement is made of the amount of work that is involved to mark and internally moderate. The unit specific comments are offered for the sake of centres which have had their marks adjusted for some reason.

Assessors are asked to ensure that the final mark on the cover sheet agrees with the submitted mark on the MS1 and is the sum of criteria marks.

Additionally, some assessors only give domain marks. This might be fine if the candidate deserves full marks (or zero!) for a domain, but it makes it very difficult for external moderators to understand the marking if a mark has been withheld – in this case we do not know which of the criteria have, in the opinion of the assessor, not been met adequately.

Teachers should note that all the comments offered have been made before. These reports should provide a valuable aid to the marking process and we would urge all Heads of Departments to ensure that these reports are read by all those involved in the assessment of coursework.

Core 3 – 4753/02

The marking scheme for this component is very prescriptive. However, there are a significant number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

<u>Change of Sign</u>
This method (and the other three as well!) should be illustrated graphically. Many candidates merely draw a graph of the function and this is accepted by assessors as an illustration. We expect to see either an annotation of the function or a number of "zoom ins" to demonstrate clearly how the iterates match the graph.

Illustrations of failure often cause problems.

- The equation chosen is such that the table of values includes the value 0. In this case the root has been found and so the method has not failed.
- The equation chosen is declared to touch the axis whilst closer inspection reveals that it does not. In this case there is no root to be found.

Newton Raphson

The intention of the task in this method is that it should demonstrate the ability to find all the roots. An equation that has only one root should not be used. If a candidate does use such an equation the second mark cannot be given.

Some illustrations are not clear enough to inform the reader and therefore should not earn the mark. In particular, if the candidate is reliant on "Autograph" to illustrate the convergence by the drawing of tangents, then it is the responsibility of the candidate to use an appropriate scale to ensure that the tangents can clearly be seen – we expect to see 2 such tangents.

Failure of the method to converge to an expected root should be demonstrated "despite a starting value close to it". By this we expect an integer value either side of the root and not a value that is artificially too far away or a contrived value that happens to be a turning value. Some candidates believe that if x_2 is further away from the root than x_1 then the process is not converging. This is not, of course, the case, and so credit in these situations should not be given.

Rearrangement

It is not expected that a candidate will simply derive $g'(x)$ and then calculate its value at x_0 or closer and claim convergence because this value fits the criterion $-1 < g'(x) < 1$. The discussion should also be based on the intersection of the curve $y = g(x)$ and the line $y = x$. Indeed, it was not one of the original intentions of this section that any differentiation should be done (and some candidates use rearrangements where $g'(x)$ cannot be calculated by them).

Comparison

For a meaningful comparison of speed of convergence to take place, the same equation should be used for all three methods, as required, the same starting point should be used and the same accuracy used. Some of these requirements were missing, yet full credit was given.

Notation

It still surprises and disappoints us (especially since it has been mentioned in reports every series so far!) that full credit is given where there is poor or incorrect notation. We see on numerous occasions an assessor writing "all correct" on the cover sheet and on the first page a statement by the candidate "I am going to solve the equation $y = \ldots$.."

Oral

The specification asks for a written report.

Differential Equations - 4758/02

There is a very small entry in the winter for this unit so any generalisations may be a little misleading.

By far the most popular task is "Aeroplane Landing". There are three recurrent difficulties in the assessment of this task for which a scaling may be made.

The essential function of the coursework element of this module is to test the candidate's ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. Most candidates can easily set up the model by making a suggestion regarding the air resistance and the reason for the rather sudden change in the pattern of values for the velocity at around $t = 9$. However, in many cases the model for the whole motion is not described by differential equations and even if it is they are not solved. In many instances the candidate will only solve the first differential equation for the first 9 seconds. The wording of the criteria clearly states that differential equations should be established and solved to model the situation which, in this case, is the whole 26 seconds. Candidates who deal only with the first 9 seconds before revising the model have not completed the task and should therefore not earn full credit.

Additionally, the development of the model assuming that air resistance is constant is not appropriate since it produces a differential equation that is too easy for this specification. The revision of the assumption that resistance is proportional to the velocity, to proportional to the square of the velocity must be justified. If it is not then there is a real danger that all the candidate will say is "the model is better because the curves are closer" and this is not following the modelling cycle but engaging in a curve fitting exercise.

Marks often seem to be automatically allocated for Domain 3 (Collection of data) when there is little discussion of the source or potential accuracy of the data.

If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Numerical Methods – 4776/02

There continue to be several cases where incorrect work had been ticked. Assessors are requested not to tick work unless it has been checked thoroughly.

The most popular task is to find the value of an integral numerically. In spite of the fact that these comments have been made before, assessors are giving credit where it is not appropriate and the particular criterion has not been met.

In domain 1 the basic requirement of a formal statement of the problem is often not met.

In domain 2 candidates describe what method they are to use but fail to say why.

In domain 3, a "substantial" application is to find values of M_n, T_n or S_n up to at least $n = 64$.

In domain 4 it is not enough to state what software is being used, nor is it enough to give a general printout of the formulae being used. A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout.

In domain 5 it is not appropriate to compare values obtained with "the real value". This might be π. Additionally, it is accepted that candidates will use a function that they are unable to integrate (because of where they are in the course) but which is integrable. However, it is not then appropriate to state a value found by direct integration.

Many candidates, as a result of their insubstantial application, will state the value to which the ratio of differences is converging without justification from their values. This can of course lead to inaccuracy, and the failure to provide an “improved solution”. Indeed, some candidates use the “theoretical” value regardless of the values they are getting (or not if they do not work the ratio of differences) far too early giving inaccurate solutions. These are often credited, leading to some very generous marking.

In domain 6 most of the marks are dependent on satisfactory work in the error analysis domain and so often a rather generous assessment of that domain led also to a rather generous assessment here as well. Teachers should note that comments justifying the accuracy of the solution are appropriate here, but comments on the limitations of Excel are not usually creditworthy. Assessors should also note that the criteria for this task demands a solution to “at least 6 figure accuracy”. Candidates should be finding a solution to an accuracy which they can justify and that should be least 6 significant figures. For example, if their working can justify 9 significant figures then they should give that level of accuracy with justification.

Grade Thresholds

Advanced GCE Mathematics 3895 7895
January 2010 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	52	46	40	34	28	0
4752	Raw	72	59	52	45	38	32	0
4753/01	Raw	72	57	50	43	36	29	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	74	65	56	48	40	0
4755	Raw	72	55	47	39	31	24	0
4756	Raw	72	54	46	39	32	25	0
4758/01	Raw	72	61	53	45	37	29	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	58	49	41	33	25	0
4762	Raw	72	62	54	46	38	31	0
4763	Raw	72	64	56	48	41	34	0
4766/G241	Raw	72	58	50	42	35	28	0
4767	Raw	72	62	54	46	39	32	0
4768	Raw	72	55	48	41	34	27	0
4771	Raw	72	60	53	46	39	33	0
4776/01	Raw	72	60	53	46	40	33	0
4776/02	Raw	18	14	12	10	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	27.9	61.3	84.3	95.7	98.7	100	395
7896	54.3	62.9	88.6	100	100	100	35
7897							0
7898							0
3895	27.1	54.1	74.2	88.2	97.3	100	947
3896	41.3	67.5	86.3	95	100	100	80
3897	100	100	100	100	100	100	1
3898	50	50	100	100	100	100	2

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums/index.html

Statistics are correct at the time of publication.

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