OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Two complex numbers are given by $\alpha = -3 + \mathrm{j}$ and $\beta = 5 - 2\mathrm{j}$.

Find $\alpha\beta$ and $\dfrac{\alpha}{\beta}$, giving your answers in the form $a + b\mathrm{j}$, showing your working. **[5]**

2 You are given that $\mathbf{A} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 5 & 1 & 8 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}$.

(i) Calculate, where they exist, **AB**, **CA**, **B** + **D** and **AC** and indicate any that do not exist. **[5]**

(ii) Matrices **B** and **D** represent transformations B and D respectively. Find the single matrix that represents transformation B followed by transformation D. **[2]**

3 The roots of the cubic equation $4x^3 - 12x^2 + kx - 3 = 0$ may be written $a - d$, a and $a + d$. Find the roots and the value of k. **[6]**

4 You are given that if $\mathbf{M} = \begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}$ then $\mathbf{M}^{-1} = \dfrac{1}{k}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$.

Find the value of k. Hence solve the following simultaneous equations. **[6]**

$$\begin{aligned} 4x \quad\quad + \quad z &= 9 \\ -6x + \quad y + \quad z &= 32 \\ 5x + 2y + 5z &= 81 \end{aligned}$$

5 Use standard series formulae to show that $\displaystyle\sum_{r=1}^{n}(r+2)(r-3) = \tfrac{1}{3}n(n^2 - 19)$. **[6]**

6 Prove by induction that $1 \times 2 + 2 \times 3 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$ for all positive integers n. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{5x - 9}{(2x - 3)(2x + 7)}$.

(i) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(ii) Describe the behaviour of the curve for large positive and large negative values of x, justifying your answers. **[3]**

(iii) Sketch the curve. **[3]**

(iv) Solve the inequality $\dfrac{5x - 9}{(2x - 3)(2x + 7)} \leqslant 0$. **[3]**

8 **(a)** Fig. 8 shows an Argand diagram.

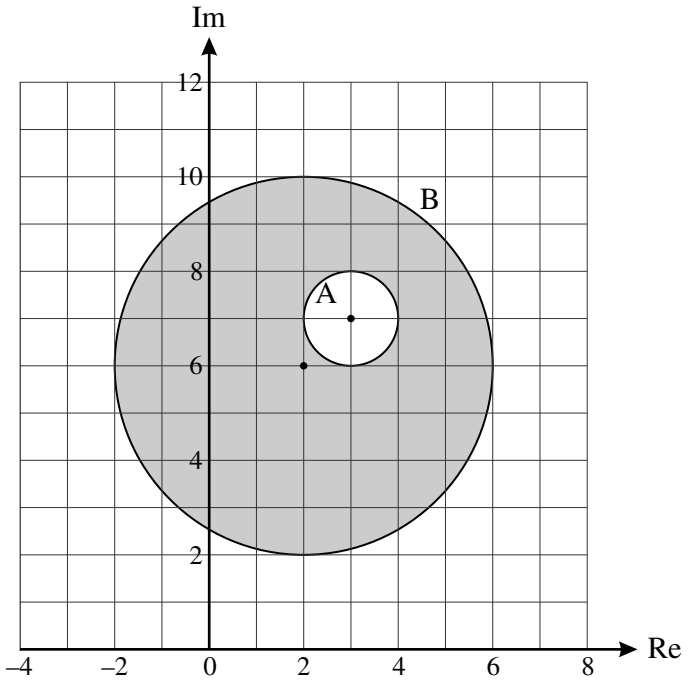

Fig. 8

(i) Write down the equation of the locus represented by the circumference of circle B. **[3]**

(ii) Write down the two inequalities that define the shaded region between, but not including, circles A and B. **[3]**

(b) **(i)** Draw an Argand diagram to show the region where

$$\frac{\pi}{4} < \arg\big(z - (2 + \mathrm{j})\big) < \frac{3\pi}{4}.$$ **[3]**

(ii) Determine whether the point $43 + 47\mathrm{j}$ lies within this region. **[3]**

9 **(i)** Verify that $\dfrac{4+r}{r(r+1)(r+2)} = \dfrac{2}{r} - \dfrac{3}{r+1} + \dfrac{1}{r+2}$. **[2]**

(ii) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{4+r}{r(r+1)(r+2)} = \frac{3}{2} - \frac{2}{n+1} + \frac{1}{n+2}.$$ **[6]**

(iii) Write down the limit to which $\displaystyle\sum_{r=1}^{n} \frac{4+r}{r(r+1)(r+2)}$ converges as n tends to infinity. **[1]**

(iv) Find $\displaystyle\sum_{r=50}^{100} \frac{4+r}{r(r+1)(r+2)}$, giving your answer to 3 significant figures. **[3]**