OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4776/01

Numerical Methods

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Show that the equation

$$2^x + \left(\frac{1}{2}\right)^x = 3$$

has a root between $x = 1.3$ and $x = 1.5$. Use the bisection method to find an estimate of this root with a maximum possible error less than 0.02.

Determine how many further iterations would be required to reduce the maximum possible error to less than 0.001. **[8]**

2 An integral, $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$, is being evaluated numerically. Some mid-point rule and trapezium rule estimates are shown in the table.

h	Mid-point rule	Trapezium rule
1	2.579 768	2.447 490
0.5	2.547 350	

Find the trapezium rule estimate for $h = 0.5$.

Find two Simpson's rule estimates and hence state, with a reason, the value of the integral to the accuracy that appears justified. **[7]**

3 **(i)** Given that $\mathrm{f}(x) = x^3 - x^2 + 1$, find f(0.5).

Use the formula $\mathrm{f}(x + h) \approx \mathrm{f}(x) + h\,\mathrm{f}'(x)$ to show that

$$\mathrm{f}(0.5 + h) \approx 0.875 - 0.25h.$$ **[3]**

(ii) Hence determine the approximate range of values of x for which $\mathrm{f}(x) = 0.875$ correct to 3 decimal places. **[4]**

4 **(i)** Show algebraically that

$$(k + 1)^2 + (k - 1)^2 - 2k^2 = 2 \qquad (*)$$

for all values of k. **[2]**

(ii) Use your calculator to evaluate the left hand side of (*) for increasingly large values of k (e.g. 10^3, 10^6, 10^9, …). State briefly two important results in numerical methods that are illustrated by your working. **[4]**

5 A function $f(x)$ has the following values correct to 3 decimal places.

x	0	1	2	3	4
$f(x)$	1.883	2.342	2.874	3.491	4.206

(i) Show, by means of a difference table, that a cubic polynomial fits these data points closely but not exactly. **[4]**

(ii) Use Newton's forward difference formula to estimate the value of f(1.5). **[4]**

Section B (36 marks)

6 **(i)** The derivative of a function is to be estimated numerically. Show, with the aid of a sketch, that the central difference method will generally be more accurate than the forward difference method. **[4]**

(ii) The table shows two values of $\tan x^\circ$ correct to 7 significant figures.

x	60	62
$\tan x^\circ$	1.732 051	1.880 726

Use these two values to estimate the derivative of $\tan x^\circ$ at $x = 60$.

Use your calculator to find two further estimates of this derivative, using the forward difference method and taking $h = 1$ and $h = 0.5$. **[4]**

(iii) Use the central difference method with $h = 2$, $h = 1$ and $h = 0.5$ to obtain three estimates of the derivative of $\tan x^\circ$ at $x = 60$. **[4]**

(iv) Show that the differences between the estimates in part **(ii)** reduce by a factor of about 0.5 as h is halved.

By considering the differences between the estimates in part **(iii)** show that the central difference method seems to converge more rapidly than the forward difference method. **[6]**

[Question 7 is printed overleaf.]

7 **(i)** Show, by means of a sketch or otherwise, that the equation

$$x = 3 \sin x, \qquad \textbf{(*)}$$

where x is in radians, has a root, α, in the interval $(\frac{1}{2}\pi, \pi)$. Determine how many other non-zero roots, if any, the equation has. **[3]**

(ii) Determine whether or not the iteration

$$x_{r+1} = 3 \sin x_r,$$

starting with $x_0 = 2$, converges to α. Illustrate your answer with a staircase or cobweb diagram as appropriate. **[7]**

(iii) Show that equation (*) may be rearranged into the form

$$x = \sin x + \tfrac{2}{3}x.$$

Show that the corresponding iteration, starting with $x_0 = 2$, converges rapidly. State to 5 decimal places the value to which the iteration converges. Verify that this value for α is correct to 5 decimal places. **[8]**