RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4776/01**

MATHEMATICS (MEI)

Numerical Methods

MONDAY 16 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks for each question is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

  [Turn over

Section A (36 marks)

1 The equation $f(x) = 0$ is known to have a single root in the interval (3, 3.5). Given that $f(3) = 0.5$ and $f(3.5) = -0.8$, estimate the root using linear interpolation.

State the maximum possible error in this estimate. [6]

2 The function $f(x)$ has the values shown in the table. The value of k is to be determined.

x	1	3	5	7	9
$f(x)$	2	1	5	k	2

Use a difference table to obtain the value of k, assuming that $f(x)$ is a cubic. [6]

3 The function $f(x) = \sqrt{1 + 3^x}$ is to be differentiated numerically.

Use the central difference method with $h = 0.2$ to estimate the derivative at $x = 2$. Obtain further estimates with $h = 0.1$ and $h = 0.05$.

By considering the differences between successive estimates, find the value of the derivative to an accuracy of 3 decimal places. [8]

4 Show that a Newton-Raphson iteration to find the cube root of 25 is

$$x_{r+1} = x_r - \frac{x_r^3 - 25}{3x_r^2}.$$

Perform three steps of this iteration, beginning with $x_0 = 4$. Show, by considering the differences between successive iterates, that the convergence is faster than first order. [8]

5 **(i)** Find sin 86° – sin 85° to the accuracy given by your calculator. [1]

(ii) A simple spreadsheet works to an accuracy of 6 significant figures. All intermediate answers used in calculations are rounded to 6 significant figures.

Write down the values of sin 86° and sin 85° as given by this spreadsheet. Hence find the value the spreadsheet gives for sin 86° – sin 85°. [3]

(iii) You are now *given* that sin 86° – sin 85° = 2 cos 85.5° sin 0.5°. Find the value the spreadsheet gives for this expression. [2]

(iv) Use your working from parts **(ii)** and **(iii)** to explain how two expressions that are mathematically identical can nevertheless evaluate differently. [2]

Section B (36 marks)

6 The integral $\int_1^3 \sqrt{1 + \sin x}\, dx$, where x is in radians, is to be evaluated numerically.

(i) Copy and complete the following table. [7]

h	Mid-point rule estimate	Trapezium rule estimate
2	$M_1 = 2.763\,547$	$T_1 =$
1	$M_2 =$	$T_2 =$
0.5	$M_4 =$	$T_4 =$

(ii) Show that the differences between successive mid-point rule estimates reduce by a factor of about 4.

State a result about the differences between successive trapezium rule estimates. [4]

(iii) Now let $S_1 = \frac{1}{3}(2M_1 + T_1)$, with S_2 and S_4 defined similarly.

Calculate S_1, S_2, S_4 and the differences $S_2 - S_1$, $S_4 - S_2$. By considering these differences, give the value of the integral to the accuracy that appears justified. [7]

7 The equation $x^2 = 4 + \frac{1}{x}$ has three roots.

(i) Show graphically that the equation has exactly one root for $x > 0$. Find the integer a such that this positive root lies in the interval $(a, a + 1)$.

Use the fixed-point iteration

$$x_{r+1} = \sqrt{\left(4 + \frac{1}{x_r}\right)} \qquad (*)$$

to determine the positive root correct to 4 decimal places. [7]

(ii) The equation also has two negative roots. Without doing any calculations, explain why the iteration (*) cannot be used to find these negative roots.

Use the fixed-point iteration

$$x_{r+1} = -\sqrt{\left(4 + \frac{1}{x_r}\right)} \qquad (**)$$

to find a negative root near to $x = -2$ correct to 4 decimal places. [5]

(iii) The third root of the equation lies in the interval $(-1, 0)$. Show that the iteration (**) used in part **(ii)** will not converge to this third root. Use another fixed point iteration to find the third root correct to 4 decimal places. [6]

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1

x	3	3.5	root = (3 x (-0.8) - 3.5 x 0.5) / (-0.8 - 0.5)	**[M1A1A1]**
f(x)	0.5	-0.8	= .192308 (3.192, 3.19)	**[A1]**

(-) mpe is 3.5 - 3.192308 = 0.307602 (0.308, 0.31) **[M1A1]**

[TOTAL 6]

2

1	2				
3	1	-1			
5	5	4	5		
7	k	k-5	k-9	k-14	
9	2	2-k	7-2k	16-3k	**[M1A1A1A1]**

16-3k = k-14 hence k = 7.5 **[M1A1]**

[TOTAL 6]

3

h	f(2+h)	f(2-h)	f '(2)		**derivatives**
0.2	.494507	.867869	.566594		**[M1A1A1A1]**
0.1	.323418	.010586	.564163	-0.00243	**differences**
0.05	.241636	.085281	.563555	-0.00061	**[M1A1]**

differences reducing by a factor 4 so next estimate about 1.56340. **[M1]**

1.563 secure to 3 dp. **[B1]**

[TOTAL 8]

4

$f(x) = x^3-25$ $f'(x) = 3x^2$ **[M1A1A1]**

$x_{r+1} = x_r - (x_r^3-25)/3x_r^2$ (a.g.)

r	0	1	2	3	
x_r	4	3.1875	.945197	2.92417	**[M1A1]**
diffs		-0.8125	-0.2423	-0.02103	**[B1]**
ratios			.298219	.086783	**[B1]**

differences reducing at an increasing rate *(hence faster than first order)* **[E1]**

[TOTAL 8]

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5 (i) 0.001 369 352 (accept 0.001 369 4) **[B1]**

(ii) $\sin 86^o = 0.997$
564 $\sin 85^o = 0.996$
195 **[B1B1]**

$\sin 86^o - \sin 86^o = 0.001\ 369$ **[A1]**

(iii) 2 x 0.0784591 x 0.008 726 54 **[M1]**

= 0.00136935 **[A1]**

(iv) Rounding has different effects in the two expressions *(may be implied)* **[E1]**

First method involves subtraction of nearly equal numbers and so loses accuracy **[E1]**

[TOTAL 8]

6 (i)

h	M	T		
2	2.763547	**2.425240**		
1	**2.677635**	**2.594393**	mid-point:	**[M1A1A1]**
0.5	**2.656743**	**2.636014**	trapezium:	**[M1A1A1A1]**

[subtotal 7]

(ii)

M: 2.763547	diffs	
2.677635	-0.08591	
2.656743	-0.02089	reducing by a factor 4 *(may be implied)*

[M1A1E1]

Differences in T reduce by a factor 4, too **[B1]**

[subtotal 4]

(iii)

M	T	S		
2.763547	2.425240	2.650778		
2.677635	2.594393	2.649888	-0.00089033	S values:
2.656743	2.636014	2.649833	-0.000054333	diffs

[M1] **[A1A1]** **[A1]**

Differences in S reducing fast e.g by a factor of (about) 16 **[E1]**
How this leads to an answer, e.g:
Next difference about -0.0000034 and/or next answer about 2.649830 **[E1]**
Accept 2.6498 or 2.64983 **[A1]**

[subtotal 7]

[TOTAL 18]

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7 (i) Eg: graph of x^2 and $4 + 1/x$ for $x > 0$ showing single intersection **[G2]**
Change of sign to find interval (2,3) - i.e. a = 2 **[B1]**

r	0	1	2	3	4	5
x_r	2.5	2.097618	2.115829	2.114859	2.11491	2.114907

[M1A1A1]

2.1149 secure to 4 dp **[A1]**

[subtotal 7]

(ii) The iteration gives positive values only. **[E1]**

r	0	1	2	3	4	5
x_r	-2	-1.87083	-1.86158	-1.86087	-1.86081	-1.86081

[M1A1A1]

-1.8608 secure to 4 dp **[A1]**

[subtotal 5]

(iii) Eg

r	0	1	2	3	4
x_r	-0.5	-1.41421	-1.81463	-1.85713	-1.86052

not converging to required root (converging to previous root) **[M1A1]**

Eg $x_{r+1} = 1 / (x_r^2 - 4)$ **[M1]**

r	0	1	2	3	4	5
x_r	-0.5	-0.26667	-0.25452	-0.25412	-0.2541	-0.2541

[M1A1]

-0.2541 secure to 4 dp **[A1]**

[subtotal 6]

[TOTAL 18]