OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4763/01**

MATHEMATICS (MEI)

Mechanics 3

FRIDAY 23 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

1 **(a)** **(i)** Write down the dimensions of velocity, acceleration and force. [3]

A ball of mass m is thrown vertically upwards with initial velocity U. When the velocity of the ball is v, it experiences a force λv^2 due to air resistance where λ is a constant.

(ii) Find the dimensions of λ. [2]

A formula approximating the greatest height H reached by the ball is

$$H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2}$$

where g is the acceleration due to gravity.

(iii) Show that this formula is dimensionally consistent. [4]

A better approximation has the form $H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2} + \frac{1}{6}\lambda^2 U^\alpha m^\beta g^\gamma$.

(iv) Use dimensional analysis to find α, β and γ. [5]

(b) A girl of mass 50 kg is practising for a bungee jump. She is connected to a fixed point O by a light elastic rope with natural length 24 m and modulus of elasticity 2060 N. At one instant she is 30 m vertically below O and is moving vertically upwards with speed 12 m s^{-1}. She comes to rest instantaneously, with the rope slack, at the point A. Find the distance OA. [4]

2 A particle P of mass 0.3 kg is connected to a fixed point O by a light inextensible string of length 4.2 m.

Firstly, P is moving in a horizontal circle as a conical pendulum, with the string making a constant angle with the vertical. The tension in the string is 3.92 N.

(i) Find the angle which the string makes with the vertical. [2]

(ii) Find the speed of P. [4]

P now moves in part of a vertical circle with centre O and radius 4.2 m. When the string makes an angle θ with the downward vertical, the speed of P is v m s^{-1} (see Fig. 2). You are given that $v = 8.4$ when $\theta = 60°$.

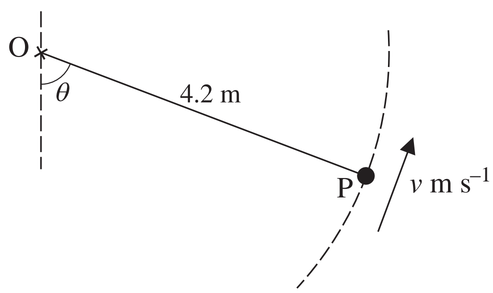

Fig. 2

(iii) Find the tension in the string when $\theta = 60°$. [3]

(iv) Show that $v^2 = 29.4 + 82.32 \cos\theta$. [4]

(v) Find θ at the instant when the string becomes slack. [5]

3 A small block B has mass 2.5 kg. A light elastic string connects B to a fixed point P, and a second light elastic string connects B to a fixed point Q, which is 6.5 m vertically below P.

The string PB has natural length 3.2 m and stiffness $35\,\mathrm{N\,m^{-1}}$; the string BQ has natural length 1.8 m and stiffness $5\,\mathrm{N\,m^{-1}}$.

The block B is released from rest in the position 4.4 m vertically below P. You are given that B performs simple harmonic motion along part of the line PQ, and that both strings remain taut throughout the motion. Air resistance may be neglected. At time t seconds after release, the length of the string PB is x metres (see Fig. 3).

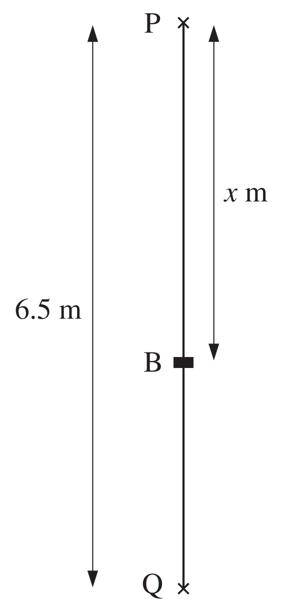

Fig. 3

(i) Find, in terms of x, the tension in the string PB and the tension in the string BQ. [3]

(ii) Show that $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = 64 - 16x$. [4]

(iii) Find the value of x when B is at the centre of oscillation. [2]

(iv) Find the period of oscillation. [2]

(v) Write down the amplitude of the motion and find the maximum speed of B. [3]

(vi) Find the time after release when B is first moving *downwards* with speed $0.9\,\mathrm{m\,s^{-1}}$. [4]

[Question 4 is printed overleaf.]

4 **(a)** A uniform solid of revolution is obtained by rotating through 2π radians about the y-axis the region bounded by the curve $y = 8 - 2x^2$ for $0 \leqslant x \leqslant 2$, the x-axis and the y-axis.

(i) Find the y-coordinate of the centre of mass of this solid. [7]

The solid is now placed on a rough plane inclined at an angle θ to the horizontal. It rests in equilibrium with its circular face in contact with the plane as shown in Fig. 4.

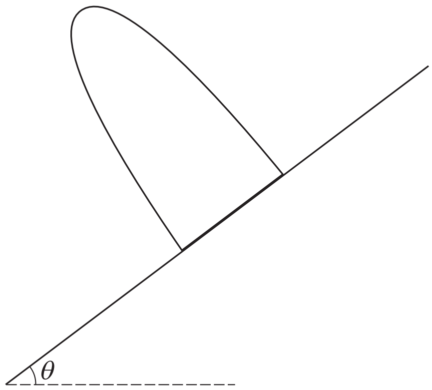

Fig. 4

(ii) Given that the solid is on the point of toppling, find θ. [4]

(b) Find the y-coordinate of the centre of mass of a uniform lamina in the shape of the region bounded by the curve $y = 8 - 2x^2$ for $-2 \leqslant x \leqslant 2$, and the x-axis. [7]

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1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*(Deduct 1 mark if kg, m, s are consistently used instead of M, L, T)*
(ii)	$[\lambda] = \dfrac{[\text{Force}]}{[v^2]} = \dfrac{\mathrm{M\,L\,T^{-2}}}{(\mathrm{L\,T^{-1}})^2}$ $= \mathrm{M\,L^{-1}}$	M1 A1 cao **2**	
(iii)	$\left[\dfrac{U^2}{2g}\right] = \dfrac{(\mathrm{L\,T^{-1}})^2}{\mathrm{L\,T^{-2}}} = \mathrm{L}$ $\left[\dfrac{\lambda U^4}{4mg^2}\right] = \dfrac{(\mathrm{M\,L^{-1}})(\mathrm{L\,T^{-1}})^4}{\mathrm{M}(\mathrm{L\,T^{-2}})^2}$ $= \dfrac{\mathrm{M\,L^3\,T^{-4}}}{\mathrm{M\,L^2\,T^{-4}}} = \mathrm{L}$ [*H*] = L ; all 3 terms have the same dimensions	B1 cao M1 A1 cao E1 **4**	*(Condone constants left in)* *Dependent on B1M1A1*
(iv)	$(\mathrm{M\,L^{-1}})^2 (\mathrm{L\,T^{-1}})^\alpha \mathrm{M}^\beta (\mathrm{L\,T^{-2}})^\gamma = \mathrm{L}$ $\beta = -2$ $-2 + \alpha + \gamma = 1$ $-\alpha - 2\gamma = 0$ $\alpha = 6$ $\gamma = -3$	B1 cao M1 A1 A1 cao A1 cao **5**	At least one equation in α, γ One equation correct

(b)	EE is $\frac{1}{2}\times\frac{2060}{24}\times 6^2 \quad (=1545)$ (PE gained) = (EE lost) + (KE lost) $50\times 9.8\times h = 1545+\frac{1}{2}\times 50\times 12^2$ $490h = 1545+3600$ $h = 10.5$ OA $= 30-h = 19.5$ m	B1 M1 F1 A1 **4**	Equation involving PE, EE and KE Can be awarded from start to point where string becomes slack *or* any complete method (e.g. SHM) for finding v^2 at natural length If B0, give A1 for $v^2 = 88.2$ correctly obtained *or* $0 = 88.2 - 2\times 9.8\times s \quad (s = 4.5)$ *Notes* $\frac{1}{2}\times\frac{2060}{24}\times 6$ *used as EE can earn B0M1F1A0* $\frac{2060}{24}\times 6$ *used as EE gets B0M0*

2 (i)	$T\cos\alpha = mg$ $3.92\cos\alpha = 0.3\times 9.8$ $\cos\alpha = 0.75$ Angle is $41.4°$ (0.723 rad)	M1 A1	**2**	Resolving vertically *(Condone sin / cos mix for M marks throughout this question)*
(ii)	$T\sin\alpha = m\dfrac{v^2}{r}$ $3.92\sin\alpha = 0.3\times\dfrac{v^2}{4.2\sin\alpha}$ Speed is $4.9\ \text{m s}^{-1}$	M1 B1 A1 A1	**4**	Force and acceleration towards centre (condone $v^2/4.2$ or $4.2\omega^2$) For radius is $4.2\sin\alpha$ $(=2.778)$ Not awarded for equation in ω unless $v=(4.2\sin\alpha)\omega$ also appears
(iii)	$T - mg\cos\theta = m\dfrac{v^2}{a}$ $T - 0.3\times 9.8\times\cos 60° = 0.3\times\dfrac{8.4^2}{4.2}$ Tension is 6.51 N	M1 A1 A1	**3**	Forces and acceleration towards O
(iv)	$\frac{1}{2}mv^2 - mg\times 4.2\cos\theta = \frac{1}{2}m\times 8.4^2 - mg\times 4.2\cos 60°$ $v^2 - 82.32\cos\theta = 70.56 - 41.16$ $v^2 = 29.4 + 82.32\cos\theta$	M1 M1 A1 E1	**4**	For $(-)mg\times 4.2\cos\theta$ in PE Equation involving $\frac{1}{2}mv^2$ and PE
(v)	$(T) - mg\cos\theta = m\dfrac{v^2}{a}$ $(T) - m\times 9.8\cos\theta = m\times\dfrac{29.4+82.32\cos\theta}{4.2}$ String becomes slack when $T=0$ $-9.8\cos\theta = 7 + 19.6\cos\theta$ $\cos\theta = -\dfrac{7}{29.4}$ $\theta = 104°$ (1.81 rad)	M1 M1 A1 M1 A1	**5**	Force and acceleration towards O Substituting for v^2 *Dependent on first M1* *No marks for* $v=0 \Rightarrow \theta = 111°$

3 (i)	$T_{PB} = 35(x-3.2) \quad [=35x-112]$ $T_{BQ} = 5(6.5-x-1.8)$ $= 5(4.7-x) \quad [=23.5-5x]$	B1 M1 A1	**3**	Finding extension of BQ
(ii)	$T_{BQ} + mg - T_{PB} = m\dfrac{d^2x}{dt^2}$ $5(4.7-x)+2.5\times9.8-35(x-3.2) = 2.5\dfrac{d^2x}{dt^2}$ $160-40x = 2.5\dfrac{d^2x}{dt^2}$ $\dfrac{d^2x}{dt^2} = 64-16x$	M1 A2 E1	**4**	Equation of motion (condone one missing force) Give A1 for three terms correct
(iii)	At the centre, $\dfrac{d^2x}{dt^2}=0$ $x=4$	M1 A1	**2**	
(iv)	$\omega^2 = 16$ Period is $\dfrac{2\pi}{\sqrt{16}} = \frac{1}{2}\pi = 1.57\text{ s}$	M1 A1	**2**	Seen or implied *(Allow M1 for $\omega = 16$)* Accept $\frac{1}{2}\pi$
(v)	Amplitude $A = 4.4-4 = 0.4\text{ m}$ Maximum speed is $A\omega$ $= 0.4\times4 = 1.6\text{ m s}^{-1}$	B1 ft M1 A1 cao	**3**	ft is $\lvert 4.4-\text{(iii)} \rvert$
(vi)	$x = 4+0.4\cos 4t$ $v = (-)\,1.6\sin 4t$ When $v=0.9$, $\sin 4t = -\dfrac{0.9}{1.6}$ $4t = \pi + 0.5974$ Time is 0.935 s	M1 A1 M1 A1 cao	**4**	For $v = C\sin\omega t$ or $C\cos\omega t$ *This M1A1 can be earned in (v)* Fully correct method for finding the required time e.g. $\frac{1}{4}\arcsin\dfrac{0.9}{1.6} + \frac{1}{2}\text{period}$
	OR $0.9^2 = 16(0.4^2 - y^2)$ $y = -0.3307$ — M1 $y = 0.4\cos 4t$ — A1 $\cos 4t = -\dfrac{0.3307}{0.4}$ $4t = \pi + 0.5974$ — M1 Time is 0.935 s — A1 cao			Using $v^2 = \omega^2(A^2-y^2)$ *and* $y = A\cos\omega t$ or $A\sin\omega t$ For $y = (\pm)\,0.331$ *and* $y = 0.4\cos 4t$

4 (a)(i)	$V = \int \pi x^2 \, dy = \int_0^8 \pi (4 - \frac{1}{2} y) \, dy$ $= \pi \left[4y - \frac{1}{4} y^2 \right]_0^8 = 16\pi$ $V \bar{y} = \int \pi y x^2 \, dy$ $= \int_0^8 \pi y (4 - \frac{1}{2} y) \, dy$ $= \pi \left[2y^2 - \frac{1}{6} y^3 \right]_0^8 = \frac{128}{3} \pi$ $\bar{y} = \frac{\frac{128}{3}\pi}{16\pi}$ $= \frac{8}{3} \quad (\approx 2.67)$	M1 A1 M1 A1 A1 M1 A1 **7**	*π may be omitted throughout* Limits not required for M marks throughout this question *Dependent on M1M1*
(ii)	CM is vertically above lower corner $\tan\theta = \frac{2}{\bar{y}} = \frac{2}{8/3} \quad (= \frac{3}{4})$ $\theta = 36.9° \quad (= 0.6435 \text{ rad})$	M1 M1 A1 A1 **4**	Trig in a triangle including θ *Dependent on previous M1* Correct expression for $\tan\theta$ or $\tan(90 - \theta)$ *Notes* $\tan\theta = \frac{2}{\text{cand's } \bar{y}}$ *implies M1M1A1* $\tan\theta = \frac{\text{cand's } \bar{y}}{2}$ *implies M1M1* $\tan\theta = \frac{1}{\text{cand's } \bar{y}}$ *without further evidence is M0M0*

(b)	$A=\int_{-2}^{2}(8-2x^2)\,dx$	M1	*May use* $0\le x\le 2$ *throughout* or $(2)\int_0^8 \sqrt{4-\frac{1}{2}y}\,dy$
	$=\left[\,8x-\frac{2}{3}x^3\,\right]_{-2}^{2}=\frac{64}{3}$	A1	
	$A\bar{y}=\int_{-2}^{2}\frac{1}{2}(8-2x^2)^2\,dx$	M1	or $(2)\int_0^8 y\sqrt{4-\frac{1}{2}y}\,dy$ *(M0 if ½ is omitted)*
	$=\left[\,32x-\frac{16}{3}x^3+\frac{2}{5}x^5\,\right]_{-2}^{2}$	M1	For $32x-\frac{16}{3}x^3+\frac{2}{5}x^5$ *Allow one error* or $-\frac{8}{3}y(4-\frac{1}{2}y)^{\frac{3}{2}}-\frac{32}{15}(4-\frac{1}{2}y)^{\frac{5}{2}}$ or $-\frac{64}{3}(4-\frac{1}{2}y)^{\frac{3}{2}}+\frac{16}{5}(4-\frac{1}{2}y)^{\frac{5}{2}}$
	$=\frac{1024}{15}$	A1	
	$\bar{y}=\frac{1024/15}{64/3}$ $=\frac{16}{5}=3.2$	M1	*Dependent on first two M1's*
		A1 **7**	