OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4762/01**

MATHEMATICS (MEI)

Mechanics 2

MONDAY 16 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

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1 **(a)** Disc A of mass 6 kg and disc B of mass 0.5 kg are moving in the same straight line. The relative positions of the discs and the **i** direction are shown in Fig. 1.1.

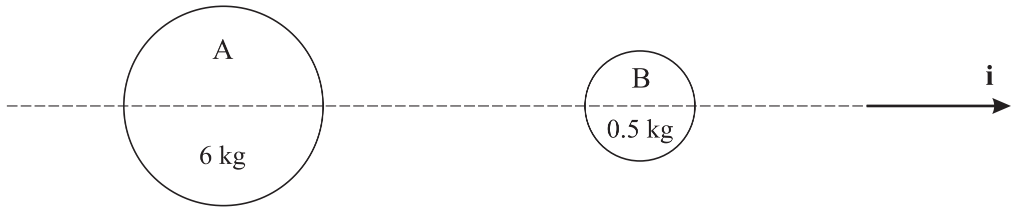

Fig. 1.1

The discs collide directly. The impulse on A in the collision is $-12\mathbf{i}$ N s and after the collision A has velocity $3\mathbf{i}\,\text{m s}^{-1}$ and B has velocity $11\mathbf{i}\,\text{m s}^{-1}$.

(i) Show that the velocity of A just before the collision is $5\mathbf{i}\,\text{m s}^{-1}$ and find the velocity of B at this time. [5]

(ii) Calculate the coefficient of restitution in the collision. [3]

(iii) After the collision, a force of $-2\mathbf{i}$ N acts on B for 7 seconds. Find the velocity of B after this time. [4]

(b) A ball bounces off a smooth plane. The angles its path makes with the plane before and after the impact are α and β, as shown in Fig. 1.2.

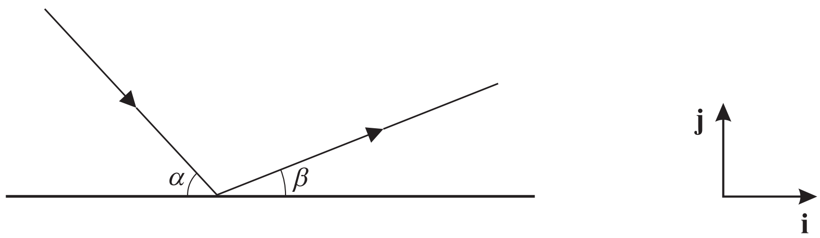

Fig. 1.2

The velocity of the ball before the impact is $u\mathbf{i} - v\mathbf{j}$ and the coefficient of restitution in the impact is e.

Write down an expression in terms of u, v, e, $\mathbf{i}$ and $\mathbf{j}$ for the velocity of the ball immediately after the impact. Hence show that $\tan\beta = e\tan\alpha$. [5]

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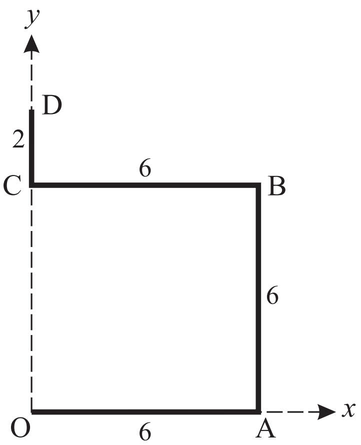

Fig. 2.1

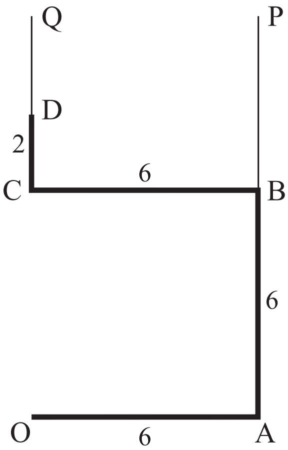

Fig. 2.2

A uniform wire is bent to form a bracket OABCD. The sections OA, AB and BC lie on three sides of a square and CD is parallel to AB. This is shown in Fig. 2.1 where the dimensions, in centimetres, are also given.

(i) Show that, referred to the axes shown in Fig. 2.1, the x-coordinate of the centre of mass of the bracket is 3.6. Find also the y-coordinate of its centre of mass. [6]

(ii) The bracket is now freely suspended from D and hangs in equilibrium.

Draw a diagram showing the position of the centre of mass and calculate the angle of CD to the vertical. [5]

(iii) The bracket is now hung by means of vertical, light strings BP and DQ attached to B and to D, as shown in Fig. 2.2. The bracket has weight 5 N and is in equilibrium with OA horizontal.

Calculate the tensions in the strings BP and DQ. [4]

The original bracket shown in Fig. 2.1 is now changed by adding the section OE, where AOE is a straight line. This section is made of the same type of wire and has length L cm, as shown in Fig. 2.3.

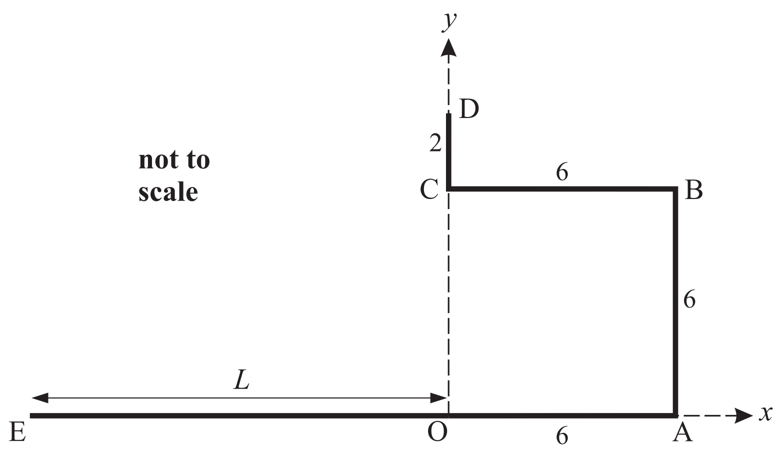

Fig. 2.3

The value of L is chosen so that the centre of mass is now on the y-axis.

(iv) Calculate L. [4]

3 **(a)** Fig. 3.1 shows a framework in a vertical plane constructed of light, rigid rods AB, BC, AD and BD. The rods are freely pin-jointed to each other at A, B and D and to a vertical wall at C and D. There are vertical loads of L N at A and $3L$ N at B. Angle DAB is 30°, angle DBC is 60° and ABC is a straight, horizontal line.

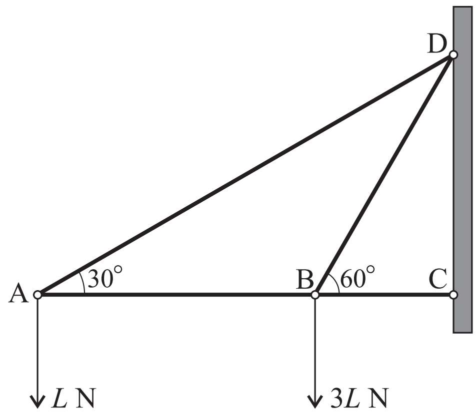

Fig. 3.1

(i) Draw a diagram showing the loads and the internal forces in the four rods. [2]

(ii) Find the internal forces in the rods in terms of L, stating whether each rod is in tension or in thrust (compression). [You may leave answers in surd form. Note that you are **not** required to find the external forces acting at C and at D.] [9]

(b) Fig. 3.2 shows uniform beams PQ and QR, each of length $2l$ m and of weight W N. The beams are freely hinged at Q and are in equilibrium on a rough horizontal surface when inclined at 60° to the horizontal. You are given that the total force acting at Q on QR due to the hinge is horizontal. This force, U N, is shown in Fig. 3.3.

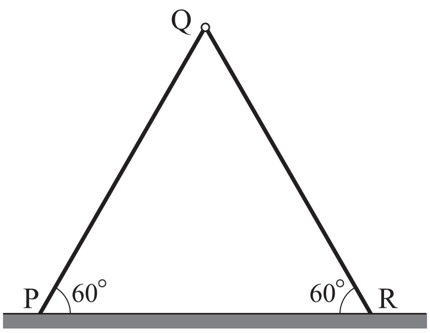

Fig. 3.2

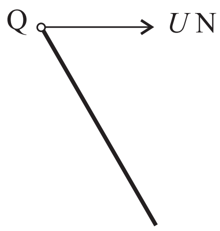

Fig. 3.3

Show that the frictional force between the floor and each beam is $\frac{\sqrt{3}}{6}W$ N. [7]

4 **(a)**

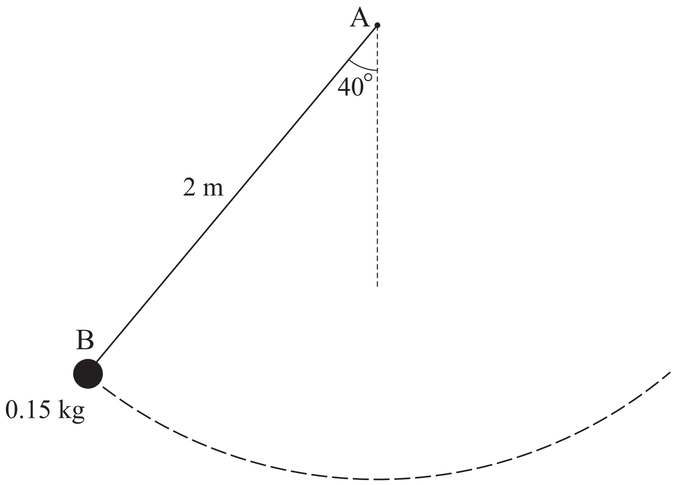

Fig. 4

A small sphere of mass 0.15 kg is attached to one end, B, of a light, inextensible piece of fishing line of length 2 m. The other end of the line, A, is fixed and the line can swing freely.

The sphere swings with the line taut from a point where the line is at an angle of 40° with the vertical, as shown in Fig. 4.

(i) Explain why no work is done on the sphere by the tension in the line. [1]

(ii) Show that the sphere has dropped a vertical distance of about 0.4679 m when it is at the lowest point of its swing and calculate the amount of gravitational potential energy lost when it is at this point. [4]

(iii) Assuming that there is no air resistance and that the sphere swings from rest from the position shown in Fig. 4, calculate the speed of the sphere at the lowest point of its swing. [2]

(iv) Now consider the case where

- there is a force opposing the motion that results in an energy loss of 0.6 J for every metre travelled by the sphere,
- the sphere is given an initial speed of $2.5\ \mathrm{m\,s^{-1}}$ (and it is descending) with AB at 40° to the vertical.

Calculate the speed of the sphere at the lowest point of its swing. [5]

(b) A block of mass 3 kg slides down a uniform, rough slope that is at an angle of 30° to the horizontal. The acceleration of the block is $\frac{1}{8}g$.

Show that the coefficient of friction between the block and the slope is $\frac{1}{4}\sqrt{3}$. [6]

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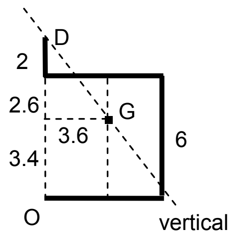
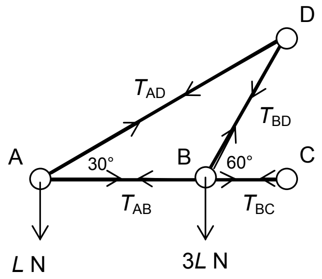
4762 Mechanics 2

Q 1		mark	comment	sub
(a) (i)	In **i** direction: $6u-12=18$ so u = 5 i.e. 5**i** m s^{-1}	M1 E1	Use of I-M Accept $6u-12=18$ as total working. Accept 5 instead of 5**i**.	
	either In **i** direction: $0.5v+12=0.5\times11$ $v=-13$ so -13 **i** m s^{-1} **or** 6 × 5 + 0.5 v = 6 × 3 + 0.5 × 11 v = – 13 so – 13**i** m s^{-1}	 M1 B1 A1 M1 A1 A1	 Use of I-M Use of + 12**i** or equivalent Accept direction indicated by any means PCLM Allow only sign errors Accept direction indicated by any means	5
(ii)	Using NEL: $\frac{11-3}{-13-5}=-e$ $e=4/9$ $(0.\dot{4})$	M1 F1 F1	Use of NEL. Condone sign errors but not reciprocal expression FT only **their** -13 (even if +ve) FT only **their** -13 and only if –ve (allow 1 s.f. accuracy)	3
(iii)	In **i** direction: $-2\times7=0.5v-0.5\times11$ v = -17 so -17 **i** m s^{-1} **or** – 2 **i** = 0.5 a so a = – 4 **i** m s^{-2} v = 11**i** – 4**i** × 7 v = -17 so -17 **i** m s^{-1}	M1 M1 A1 A1 M1 A1 M1 A1	Use of **I** = **F**t Use of **I** = m(**v** –**u**) For ±17 cao. Direction (indicated by any means) Use of **F** = m**a** For ±4 Use of **uvas***t* cao. Direction (indicated by any means)	4
(b)	u **i** + ev **j** $\tan\alpha=\frac{v}{u}$, $\tan\beta=\frac{ev}{u}$ $\tan\beta=e\left(\frac{v}{u}\right)=e\tan\alpha$	B1 B1 M1 B1 E1	For u For ev Use of tan. Accept reciprocal argument. Accept use of **their** components Both correct. Ignore signs. Shown. Accept signs not clearly dealt with.	5
		17		

Q 2		mark	comment	sub
(i)	$(2+3\times 6)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=6\begin{pmatrix}3\\0\end{pmatrix}+6\begin{pmatrix}6\\3\end{pmatrix}+6\begin{pmatrix}3\\6\end{pmatrix}+2\begin{pmatrix}0\\7\end{pmatrix}$	M1	Method for c.m.	
		B1	Total mass correct	
	$20\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}18+36+18\\18+36+14\end{pmatrix}=\begin{pmatrix}72\\68\end{pmatrix}$	B1	For any of the 1st 3 RHS terms	
		B1	For the 4th RHS term	
	$\bar{x}=3.6$	E1		
	$\bar{y}=3.4$	A1	cao [If separate cpts, award the 2nd B1 for 2 *x*- terms correct and 3rd B1 for 2×7 in *y* term]	
				6
(ii)		B1	Diagram showing G vertically below D	
		B1	3.6 and **their** 3.4 correctly placed (may be implied)	
	$\arctan\left(\frac{3.6}{2+(6-3.4)}\right)=\arctan\left(\frac{3.6}{4.6}\right)$	M1	Use of arctan on **their** lengths. Allow reciprocal of argument.	
			Some attempt to calculate correct lengths needed	
		B1	2 + (6 – **their** 3.4) seen	
	so 38.047… so 38.0° (3 s. f.)	A1	cao	
				5
(iii)	moments about D	M1	moments about D. No extra forces	
	$5\times 3.6=6\times T_{BP}$ so tension in BP is 3 N	F1	FT **their** values if calc 2nd	
	Resolve vert: $3+T_{DQ}=5$	M1	Resolve vertically or moments about B.	
	so tension in DQ is 2 N	F1	FT **their** values if calc 2nd	
				4
(iv)	We require *x*-cpt of c.m. to be zero **either** $(20+L)\bar{x}=20\times 3.6-\frac{1}{2}L^2$ **or** $2\times 6\times(0.5\times 6)+6\times 6-0.5\times L^2=0$	M1	A method to achieve this with all cpts	
		B1	For the $0.5\times L^2$	
		A1	All correct	
	L = 12	A1		
				4
		19		

Q 3		mark	comment	sub
(a) (i)		B1 B1	Internal forces all present and labelled All forces correct with labels and arrows (Allow the internal forces set as tensions, thrusts or a mixture)	2
(ii)	A ↑ T_{AD} sin 30 – L = 0 so T_{AD} = 2L so 2L N (T) A → T_{AB} + T_{AD} cos 30 = 0 so $T_{AB} = -\sqrt{3}L$ so $\sqrt{3}L$ N (C) B ↑ T_{BD} sin 60 – 3L = 0 so $T_{BD} = 2\sqrt{3}L$ so $2\sqrt{3}L$ N (T) B → T_{BC} + T_{BD} cos 60 – T_{AB} = 0 so $T_{BC} = -2\sqrt{3}L$ so $2\sqrt{3}L$ N (C)	M1 A1 M1 F1 M1 A1 M1 F1 E1	Equilibrium equation at a pin-joint attempted 1st ans. Accept + or –. Second equation attempted 2nd ans. FT any previous answer(s) used. Third equation attempted 3rd ans. FT any previous answer(s) used. Fourth equation attempted 4th ans. FT any previous answer(s) used. All T/C consistent [SC 1 all T/C correct WWW]	9
(b)	Leg QR with frictional force F ← moments c.w. about R $U \times 2l \sin 60 - Wl \cos 60 = 0$ Horiz equilibrium for QR $F = U$ Hence $\frac{1}{2}W = \sqrt{3}F$ and so $F = \frac{\sqrt{3}}{6}W$	M1 A1 A1 M1 E1 M1 E1	Accept only 1 leg considered (and without comment) Suitable moments equation. Allow 1 force omitted a.c. moments c.w. moments A second correct equation for horizontal or vertical equilibrium to eliminate a force (U or reaction at foot) [Award if correct moments equation containing only W and F] * This second equation explicitly derived Correct use of 2nd equation with the moments equation Shown. CWO but do not penalise * again.	7
		18		

Q 4		mark	comment	sub
(a) (i)	Tension is perp to the motion of the sphere (so WD, $Fd\cos\theta$ = 0)	E1		1
(ii)	Distance dropped is $2-2\cos 40$ = 0.467911.. GPE is *mgh* so $0.15\times 9.8\times 0.467911... = 0.687829...$ J	M1 E1 M1 B1	Attempt at distance with resolution used. Accept $\sin \leftrightarrow \cos$ Accept seeing $2-2\cos 40$ Any reasonable accuracy	4
(iii)	$0.5\times 0.15\times v^2 = 0.687829...$ so *v* = 3.02837… so 3.03 m s^{-1} (3 s. f.)	M1 F1	Using KE + GPE constant FT **their** GPE	2
(iv)	$\frac{1}{2}\times 0.15\left(v^2-2.5^2\right)$ $= 0.687829... - 0.6\times\frac{40}{360}\times 2\pi\times 2$ *v* = 2.06178… so 2.06 m s^{-1} (3 s. f.)	M1 B1 M1 A1 A1	Use of W-E equation (allow 1 KE term or GPE term omitted) KE terms correct WD against friction WD against friction correct (allow sign error) cao	5
(b)	N2L down slope: $3g\sin 30 - F = 3\times\frac{1}{8}g$ so $F=\frac{9g}{8}$ (= 11.025) $R = 3g\times\frac{\sqrt{3}}{2}$ (= 25.4611…) $\mu = \frac{F}{R} = \frac{\sqrt{3}}{4}$ (= 0.43301…)	M1 A1 A1 B1 M1 E1	Must have attempt at weight component Allow sign errors. Use of $F=\mu R$ Must be worked precisely	6
		18		