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RECOGNISING ACHIEVEMENT

ADVANCED GCE **4757/01**

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

FRIDAY 6 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

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Option 1: Vectors

1 A tetrahedron ABCD has vertices A $(-3, 5, 2)$, B $(3, 13, 7)$, C $(7, 0, 3)$ and D $(5, 4, 8)$.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$, and hence find the equation of the plane ABC. [4]

(ii) Find the shortest distance from D to the plane ABC. [3]

(iii) Find the shortest distance between the lines AB and CD. [4]

(iv) Find the volume of the tetrahedron ABCD. [4]

The plane P with equation $3x - 2z + 5 = 0$ contains the point B, and meets the lines AC and AD at E and F respectively.

(v) Find λ and μ such that $\overrightarrow{AE} = \lambda\overrightarrow{AC}$ and $\overrightarrow{AF} = \mu\overrightarrow{AD}$. Deduce that E is between A and C, and that F is between A and D. [5]

(vi) Hence, or otherwise, show that P divides the tetrahedron ABCD into two parts having volumes in the ratio 4 to 17. [4]

Option 2: Multi-variable calculus

2 You are given $\mathrm{g}(x, y, z) = 6xz - (x + 2y + 3z)^2$.

(i) Find $\dfrac{\partial \mathrm{g}}{\partial x}$, $\dfrac{\partial \mathrm{g}}{\partial y}$ and $\dfrac{\partial \mathrm{g}}{\partial z}$. [4]

A surface S has equation $\mathrm{g}(x, y, z) = 125$.

(ii) Find the equation of the normal line to S at the point P $(7, -7.5, 3)$. [3]

(iii) The point Q is on this normal line and is close to P. At Q, $\mathrm{g}(x, y, z) = 125 + h$, where h is small. Find the vector $\mathbf{n}$ such that $\overrightarrow{PQ} = h\mathbf{n}$ approximately. [5]

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. [4]

(v) Find the two points on S at which the tangent plane is parallel to $x + 5y = 0$. [8]

Option 3: Differential geometry

3 The curve C has parametric equations $x = 8t^3$, $y = 9t^2 - 2t^4$, for $t \geqslant 0$.

(i) Show that $\dot{x}^2 + \dot{y}^2 = (18t + 8t^3)^2$. Find the length of the arc of C for which $0 \leqslant t \leqslant 2$. [6]

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant t \leqslant 2$ is rotated through 2π radians about the x-axis. [6]

(iii) Show that the curvature at a general point on C is $\dfrac{-6}{t(4t^2 + 9)^2}$. [5]

(iv) Find the coordinates of the centre of curvature corresponding to the point on C where $t = 1$. [7]

Option 4: Groups

4 A binary operation $*$ is defined on real numbers x and y by

$$x * y = 2xy + x + y.$$

You may assume that the operation $*$ is commutative and associative.

(i) Explain briefly the meanings of the terms 'commutative' and 'associative'. [3]

(ii) Show that $x * y = 2\left(x + \frac{1}{2}\right)\left(y + \frac{1}{2}\right) - \frac{1}{2}$. [1]

The set S consists of all real numbers greater than $-\frac{1}{2}$.

(iii) (A) Use the result in part **(ii)** to show that S is closed under the operation $*$.

(B) Show that S, with the operation $*$, is a group. [9]

(iv) Show that S contains no element of order 2. [3]

The group $G = \{0, 1, 2, 4, 5, 6\}$ has binary operation $\circ$ defined by

$x \circ y$ is the remainder when $x * y$ is divided by 7.

(v) Show that $4 \circ 6 = 2$. [2]

The composition table for G is as follows.

$\circ$	0	1	2	4	5	6
0	0	1	2	4	5	6
1	1	4	0	6	2	5
2	2	0	5	1	6	4
4	4	6	1	5	0	2
5	5	2	6	0	4	1
6	6	5	4	2	1	0

(vi) Find the order of each element of G. [3]

(vii) List all the subgroups of G. [3]

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Every day, a security firm transports a large sum of money from one bank to another. There are three possible routes A, B and C. The route to be used is decided just before the journey begins, by a computer programmed as follows.

On the first day, each of the three routes is equally likely to be used.

If route A was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.4, 0.5 respectively.

If route B was used on the previous day, route A, B or C will be used, with probabilities 0.7, 0.2, 0.1 respectively.

If route C was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.6, 0.3 respectively.

The situation is modelled as a Markov chain with three states.

(i) Write down the transition matrix **P**. [2]

(ii) Find the probability that route B is used on the 7th day. [4]

(iii) Find the probability that the same route is used on the 7th and 8th days. [3]

(iv) Find the probability that the route used on the 10th day is the same as that used on the 7th day. [4]

(v) Given that $\mathbf{P}^n \to \mathbf{Q}$ as $n \to \infty$, find the matrix **Q** (give the elements to 4 decimal places). Interpret the probabilities which occur in the matrix **Q**. [4]

The computer program is now to be changed, so that the long-run probabilities for routes A, B and C will become 0.4, 0.2 and 0.4 respectively. The transition probabilities after routes A and B remain the same as before.

(vi) Find the new transition probabilities after route C. [4]

(vii) A long time after the change of program, a day is chosen at random. Find the probability that the route used on that day is the same as on the previous day. [3]

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1 (i)	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}6\\8\\5\end{pmatrix}\times\begin{pmatrix}10\\-5\\1\end{pmatrix}=\begin{pmatrix}33\\44\\-110\end{pmatrix}$ ABC is $3x+4y-10z=-9+20-20$ $3x+4y-10z+9=0$	B2 M1 A1 **4**	*Ignore subsequent working* Give B1 for one element correct SC1 for minus the correct vector For $3x+4y-10z$ Accept $33x+44y-110z=-99$ etc
(ii)	Distance is $\dfrac{3\times5+4\times4-10\times8+9}{\sqrt{3^2+4^2+10^2}}$ $=(-)\ \dfrac{40}{\sqrt{125}}\quad(=\dfrac{8}{\sqrt{5}})$	M1 A1 ft A1 **3**	Using distance formula (or other complete method) *Condone negative answer* *Accept a.r.t. 3.58*
(iii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}6\\8\\5\end{pmatrix}\times\begin{pmatrix}-2\\4\\5\end{pmatrix}=\begin{pmatrix}20\\-40\\40\end{pmatrix}\quad[\ =20\begin{pmatrix}1\\-2\\2\end{pmatrix}\ ]$ Distance is $\overrightarrow{AC}\,.\,\hat{\mathbf{n}}=\dfrac{\begin{pmatrix}10\\-5\\1\end{pmatrix}.\begin{pmatrix}1\\-2\\2\end{pmatrix}}{\sqrt{1^2+2^2+2^2}}$ $=\dfrac{22}{3}$	M1 A1 M1 A1 **4**	Evaluating $\overrightarrow{AB}\times\overrightarrow{CD}$ *or* method for finding end-points of common perp PQ *or* P $(3/2, 11, 23/4)$ & Q $(71/18, 55/9, 383/36)$ *or* $\overrightarrow{PQ}=(22/9, -44/9, 44/9)$
(iv)	Volume is $\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC})\,.\,\overrightarrow{AD}$ $=\dfrac{1}{6}\begin{pmatrix}33\\44\\-110\end{pmatrix}.\begin{pmatrix}8\\-1\\6\end{pmatrix}$ $=(-)\ \dfrac{220}{3}$	M1 A1 M1 A1 **4**	Scalar triple product *Accept a.r.t. 73.3*
(v)	E is $(-3+10\lambda,\ 5-5\lambda,\ 2+\lambda)$ $3(-3+10\lambda)-2(2+\lambda)+5=0$ $\lambda=\dfrac{2}{7}$ F is $(-3+8\mu,\ 5-\mu,\ 2+6\mu)$ $3(-3+8\mu)-2(2+6\mu)+5=0$ $\mu=\dfrac{2}{3}$ Since $0<\lambda<1$, E is between A and C Since $0<\mu<1$, F is between A and D	M1 A1 M1 A1 B1 **5**	

(vi)	$V_{\text{ABEF}} = \frac{1}{6}(\overrightarrow{\text{AB}}\times\overrightarrow{\text{AE}})\,.\,\overrightarrow{\text{AF}}$ $= \frac{1}{6}\lambda\mu(\overrightarrow{\text{AB}}\times\overrightarrow{\text{AC}})\,.\,\overrightarrow{\text{AD}}$ $= \lambda\mu V_{\text{ABCD}}$ $= \frac{4}{21}V_{\text{ABCD}}$ Ratio of volumes is $\frac{4}{21} : \frac{17}{21}$ $= 4 : 17$	M1 A1 M1 A1 ag	4	($13\frac{61}{63}$) *ft if numerical* Finding ratio of volumes of two parts SC1 for 4 : 17 deduced from $\frac{4}{21}$ without working
2 (i)	$\frac{\partial g}{\partial x} = 6z - 2(x+2y+3z) = -2x-4y$ $\frac{\partial g}{\partial y} = -4(x+2y+3z)$ $\frac{\partial g}{\partial z} = 6x - 6(x+2y+3z) = -12y-18z$	M1 A1 A1 A1	4	Partial differentiation *Any correct form, ISW*
(ii)	At P, $\frac{\partial g}{\partial x} = 16,\ \frac{\partial g}{\partial y} = -4,\ \frac{\partial g}{\partial z} = 36$ Normal line is $\mathbf{r} = \begin{pmatrix} 7 \\ -7.5 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$	M1 A1 A1 ft	3	Evaluating partial derivatives at P All correct *Condone omission of* '**r =**'
(iii)	$\delta g \approx 16\,\delta x - 4\,\delta y + 36\,\delta z$ If $\overrightarrow{\text{PQ}} = \lambda\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$, $\delta g \approx 16(4\lambda) - 4(-\lambda) + 36(9\lambda) \quad (= 392\lambda)$ $h = \delta g$, so $h \approx 392\lambda$ $\overrightarrow{\text{PQ}} \approx \frac{h}{392}\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$, so $\mathbf{n} = \frac{1}{392}\begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$	M1 M1 A1 ft M1 A1	5	*Alternative:* *M3 for substituting* $x = 7 + 4\lambda$ *, …* *into* $g = 125 + h$ *and neglecting* λ^2 *A1 ft for linear equation in* λ *and* h *A1 for* **n** *correct*
(iv)	Require $\frac{\partial g}{\partial x} = \frac{\partial g}{\partial y} = 0$ $-2x-4y = 0$ and $x+2y+3z = 0$ $x + 2y = 0$ and $z = 0$ $g(x, y, z) = 0 - 0^2 = 0 \ne 125$ Hence there is no such point on *S*	M1 M1 M1 A1	4	Useful manipulation using both eqns Showing there is no such point on *S* Fully correct proof
(v)	Require $\frac{\partial g}{\partial z} = 0$ and $\frac{\partial g}{\partial y} = 5\frac{\partial g}{\partial x}$ $-4x-8y-12z = 5(-2x-4y)$	M1 M1 M1		Implied by $\frac{\partial g}{\partial x} = \lambda,\ \frac{\partial g}{\partial y} = 5\lambda$ *This M1 can be awarded for* $-2x-4y=1$ and $-4x-8y-12z=5$

	$y=-\frac{3}{2}z$ and $x=5z$	A1	*or* $z=-\frac{2}{3}y$ and $x=-\frac{10}{3}y$ *or* $y=-\frac{3}{10}x$ and $z=\frac{1}{5}x$ *or* $x=-\frac{5}{4}\lambda,\ y=\frac{3}{8}\lambda,\ z=-\frac{1}{4}\lambda$ *or* $x:y:z=10:-3:2$
	$6(5z)z-(5z)^2=125$ $z=\pm 5$ Points are $(25,\ -7.5,\ 5)$ and $(-25,\ 7.5,\ -5)$	M1 M1 A1 A1 ft **8**	Substituting into $g(x,y,z)=125$ Obtaining one value of *x, y, z or* λ *Dependent on previous M1* *ft is minus the other point, provided all M marks have been earned*
3 (i)	$\dot{x}^2+\dot{y}^2=(24t^2)^2+(18t-8t^3)^2$ $=576t^4+324t^2-288t^4+64t^6$ $=324t^2+288t^4+64t^6$ $=(18t+8t^3)^2$ Arc length is $\int_0^2 (18t+8t^3)\,dt$ $=\left[\ 9t^2+2t^4\ \right]_0^2$ $=68$	B1 M1 A1 ag M1 A1 A1 **6**	*Note* $\int_0^2 (18+8t^3)\,dt\ =\left[\ 18t+2t^4\ \right]_0^2=68$ *earns M1A0A0*
(ii)	Curved surface area is $\int 2\pi y\,ds$ $=\int_0^2 2\pi(9t^2-2t^4)(18t+8t^3)\,dt$ $=\int_0^2 \pi(324t^3+72t^5-32t^7)\,dt$ $=\pi\left[\ 81t^4+12t^6-4t^8\ \right]_0^2$ $=1040\pi\quad(\approx 3267)$	M1 M1 A1 M1 M1 A1 **6**	Using $ds=(18t+8t^3)\,dt$ Correct integral expression including limits *(may be implied by later work)*
(iii)	$\kappa=\dfrac{\dot{x}\ddot{y}-\ddot{x}\dot{y}}{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}=\dfrac{(24t^2)(18-24t^2)-(48t)(18t-8t^3)}{(18t+8t^3)^3}$ $=\dfrac{48t^2(9-12t^2-18+8t^2)}{8t^3(9+4t^2)^3}=\dfrac{-48t^2(9+4t^2)}{8t^3(9+4t^2)^3}$ $=\dfrac{-6}{t(4t^2+9)^2}$	M1 A1A1 M1 A1 ag **5**	Using formula for κ (or ρ) For numerator and denominator Simplifying the numerator

(iv)	When $t=1,\ x=8,\ y=7,\ \kappa=-\frac{6}{169}$ $\rho=(-)\dfrac{169}{6}$ $\dfrac{dy}{dx}=\dfrac{\dot{y}}{\dot{x}}=\dfrac{18t-8t^3}{24t^2}=\dfrac{10}{24}$ $\hat{\mathbf{n}}=\begin{pmatrix}5/13\\-12/13\end{pmatrix}$ $\mathbf{c}=\begin{pmatrix}8\\7\end{pmatrix}+\frac{169}{6}\begin{pmatrix}5/13\\-12/13\end{pmatrix}$ Centre of curvature is $(18\frac{5}{6},\ -19)$	B1 M1 M1 A1 M1 A1A1 **7**	Finding gradient (or tangent vector) Finding direction of the normal Correct unit normal (either direction)
4 (i)	*Commutative:* $x*y=y*x$ (for all *x, y*) *Associative:* $(x*y)*z=x*(y*z)$ (for all *x, y, z*)	B1 B2 **3**	Accept e.g. 'Order does not matter' Give B1 for a partial explanation, e.g. 'Position of brackets does not matter'
(ii)	$2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=2xy+x+y+\frac{1}{2}-\frac{1}{2}$ $=2xy+x+y=x*y$	B1 ag **1**	*Intermediate step required*
(iii)(*A*)	If $x,y\in S$ then $x>-\frac{1}{2}$ and $y>-\frac{1}{2}$ $x+\frac{1}{2}>0$ and $y+\frac{1}{2}>0$, so $2(x+\frac{1}{2})(y+\frac{1}{2})>0$ $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}>-\frac{1}{2}$, so $x*y\in S$	M1 A1 A1 **3**	
(*B*)	0 is the identity since $0*x=0+x+0=x$ If $x\in S$ and $x*y=0$ then $2xy+x+y=0$ $y=\dfrac{-x}{2x+1}$ $y+\frac{1}{2}=\dfrac{1}{2(2x+1)}>0$ (since $x>-\frac{1}{2}$) so $y\in S$ *S* is closed and associative; there is an identity; and every element of *S* has an inverse in *S*	B1 B1 M1 A1 M1 A1 **6**	or $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=0$ or $y+\frac{1}{2}=\dfrac{1}{4(x+\frac{1}{2})}$ *Dependent on M1A1M1*
(iv)	If $x*x=0,\ \ 2x^2+x+x=0$ $x=0$ or -1 0 is the identity (and has order 1) -1 is not in *S*	M1 A1 A1 **3**	

(v)	$4*6=48+4+6=58$ $=56+2=7\times8+2$ So $4\circ6=2$	B1 B1 ag **2**	
(vi)	Element: 0, 1, 2, 4, 5, 6 Order: 1, 6, 6, 3, 3, 2	B3 **3**	Give B2 for 4 correct B1 for 2 correct
(vii)	$\{0\}$, G $\{0,\ 6\}$ $\{0,\ 4,\ 5\}$	B1 B1 B1 **3**	*Condone omission of G* If more than 2 non-trivial subgroups are given, deduct 1 mark (from final B1B1) for each non-trivial subgroup in excess of 2

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.1 & 0.7 & 0.1\\ 0.4 & 0.2 & 0.6\\ 0.5 & 0.1 & 0.3\end{pmatrix}$	B2 **2**	Give B1 for two columns correct
(ii)	$\mathbf{P}^6\begin{pmatrix}1/3\\ 1/3\\ 1/3\end{pmatrix}=\begin{pmatrix}0.328864\\ 0.381536\\ 0.2896\end{pmatrix}$ P(*B* used on 7th day) $=0.3815$	M1 M1 M1 A1 **4**	Using $\mathbf{P}^6$ (or $\mathbf{P}^7$) For matrix of initial probabilities For evaluating matrix product *Accept 0.381 to 0.382*
(iii)	$0.328864\times0.1+0.381536\times0.2+0.2896\times0.3$ $=0.1961$	M1 M1 A1 **3**	Using diagonal elements from **P** Correct method *Accept a.r.t. 0.196*
(iv)	$\mathbf{P}^3=\begin{pmatrix}0.352 & 0.328 & 0.304\\ 0.364 & 0.404 & 0.372\\ 0.284 & 0.268 & 0.324\end{pmatrix}$ $0.328864\times0.352+0.381536\times0.404+0.2896\times0.324$ $=0.3637$	M1 M1 M1 A1 **4**	For evaluating $\mathbf{P}^3$ Using diagonal elements from $\mathbf{P}^3$ Correct method *Accept a.r.t. 0.364*
(v)	$\mathbf{Q}=\begin{pmatrix}0.3289 & 0.3289 & 0.3289\\ 0.3816 & 0.3816 & 0.3816\\ 0.2895 & 0.2895 & 0.2895\end{pmatrix}$ 0.3289, 0.3816, 0.2895 are the long-run probabilities for the routes *A, B, C*	B1 B1 B1 B1 **4**	*Deduct 1 if not given as a (3×3) matrix* *Deduct 1 if not 4 dp* *Accept 'equilibrium probabilities'*
(vi)	$\begin{pmatrix}0.1 & 0.7 & a\\ 0.4 & 0.2 & b\\ 0.5 & 0.1 & c\end{pmatrix}\begin{pmatrix}0.4\\ 0.2\\ 0.4\end{pmatrix}=\begin{pmatrix}0.4\\ 0.2\\ 0.4\end{pmatrix}$ $0.04+0.14+0.4a=0.4$, so $a=0.55$ $0.16+0.04+0.4b=0.2$, so $b=0$ $0.2+0.02+0.4c=0.4$, so $c=0.45$ After *C*, routes *A, B, C* will be used with probabilities 0.55, 0, 0.45	M1 M1 A2 **4**	 Obtaining a value for *a, b* or *c* Give A1 for one correct
(vii)	$0.4\times0.1+0.2\times0.2+0.4\times0.45$ $=0.26$	M1 M1 A1 **3**	Using long-run probs 0.4, 0.2, 0.4 Using diag elements from new matrix

Post-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.1 & 0.4 & 0.5\\0.7 & 0.2 & 0.1\\0.1 & 0.6 & 0.3\end{pmatrix}$	B2 **2**	Give B1 for two rows correct
(ii)	$\left(\frac{1}{3}\ \ \frac{1}{3}\ \ \frac{1}{3}\right)\mathbf{P}^6=\left(0.328864\ \ 0.381536\ \ 0.2896\right)$ P(*B* used on 7th day) $=0.3815$	M1 M1 M1 A1 **4**	Using $\mathbf{P}^6$ (or $\mathbf{P}^7$) For matrix of initial probabilities For evaluating matrix product *Accept 0.381 to 0.382*
(iii)	$0.328864\times0.1+0.381536\times0.2+0.2896\times0.3$ $=0.1961$	M1 M1 A1 **3**	Using diagonal elements from **P** Correct method *Accept a.r.t. 0.196*
(iv)	$\mathbf{P}^3=\begin{pmatrix}0.352 & 0.364 & 0.284\\0.328 & 0.404 & 0.268\\0.304 & 0.372 & 0.324\end{pmatrix}$ $0.328864\times0.352+0.381536\times0.404+0.2896\times0.324$ $=0.3637$	M1 M1 M1 A1 **4**	For evaluating $\mathbf{P}^3$ Using diagonal elements from $\mathbf{P}^3$ Correct method *Accept a.r.t. 0.364*
(v)	$\mathbf{Q}=\begin{pmatrix}0.3289 & 0.3816 & 0.2895\\0.3289 & 0.3816 & 0.2895\\0.3289 & 0.3816 & 0.2895\end{pmatrix}$ 0.3289, 0.3816, 0.2895 are the long-run probabilities for the routes *A, B, C*	B1B1B1 B1 **4**	*Deduct 1 if not given as a (3×3) matrix* *Deduct 1 if not 4 dp* *Accept 'equilibrium probabilities'*
(vi)	$\left(0.4\ \ 0.2\ \ 0.4\right)\begin{pmatrix}0.1 & 0.4 & 0.5\\0.7 & 0.2 & 0.1\\a & b & c\end{pmatrix}=\left(0.4\ \ 0.2\ \ 0.4\right)$ $0.04+0.14+0.4a=0.4$, so $a=0.55$ $0.16+0.04+0.4b=0.2$, so $b=0$ $0.2+0.02+0.4c=0.4$, so $c=0.45$ After *C*, routes *A, B, C* will be used with probabilities 0.55, 0, 0.45	M1 M1 A2 **4**	 Obtaining a value for *a, b* or *c* Give A1 for one correct
(vii)	$0.4\times0.1+0.2\times0.2+0.4\times0.45$ $=0.26$	M1 M1 A1 **3**	Using long-run probs 0.4, 0.2, 0.4 Using diag elements from new matrix