RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4761/01**

MATHEMATICS (MEI)

Mechanics 1

THURSDAY 17 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 A cyclist starts from rest and takes 10 seconds to accelerate at a constant rate up to a speed of $15\,\mathrm{m\,s^{-1}}$. After travelling at this speed for 20 seconds, the cyclist then decelerates to rest at a constant rate over the next 5 seconds.

(i) Sketch a velocity-time graph for the motion. [3]

(ii) Calculate the distance travelled by the cyclist. [3]

2 The force acting on a particle of mass 1.5 kg is given by the vector $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$ N.

(i) Give the acceleration of the particle as a vector. [2]

(ii) Calculate the angle that the acceleration makes with the direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. [2]

(iii) At a certain point of its motion, the particle has a velocity of $\begin{pmatrix} -2 \\ 3 \end{pmatrix}\,\mathrm{m\,s^{-1}}$. Calculate the displacement of the particle over the next two seconds. [3]

3

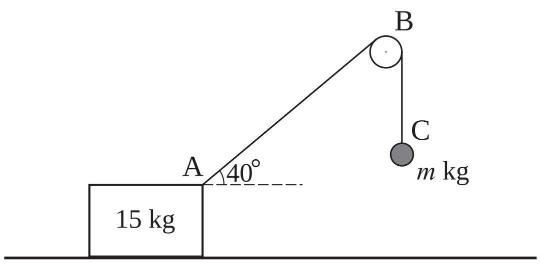

Fig. 3

Fig. 3 shows a block of mass 15 kg on a rough, horizontal plane. A light string is fixed to the block at A, passes over a smooth, fixed pulley B and is attached at C to a sphere. The section of the string between the block and the pulley is inclined at 40° to the horizontal and the section between the pulley and the sphere is vertical.

The system is in equilibrium and the tension in the string is 58.8 N.

(i) The sphere has a mass of m kg. Calculate the value of m. [2]

(ii) Calculate the frictional force acting on the block. [3]

(iii) Calculate the normal reaction of the plane on the block. [3]

4 Force $\mathbf{F}$ is $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ N and force $\mathbf{G}$ is $\begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$ N.

(i) Find the resultant of $\mathbf{F}$ and $\mathbf{G}$ and calculate its magnitude. [4]

(ii) Forces $\mathbf{F}$, $2\mathbf{G}$ and $\mathbf{H}$ act on a particle which is in equilibrium. Find $\mathbf{H}$. [3]

5

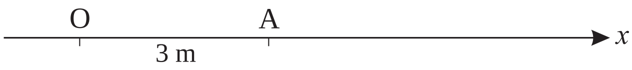

Fig. 5

A toy car is moving along the straight line Ox, where O is the origin. The time t is in seconds. At time $t = 0$ the car is at A, 3 m from O as shown in Fig. 5. The velocity of the car, $v\,\mathrm{m\,s^{-1}}$, is given by

$$v = 2 + 12t - 3t^2.$$

Calculate the distance of the car from O when its acceleration is zero. [8]

Section B (36 marks)

6 A helicopter rescue activity at sea is modelled as follows. The helicopter is stationary and a man is suspended from it by means of a vertical, light, inextensible wire that may be raised or lowered, as shown in Fig. 6.1.

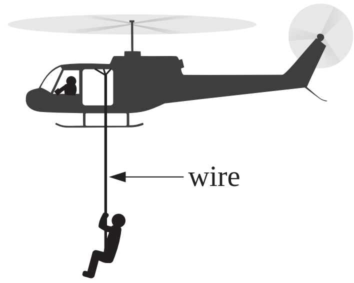

Fig. 6.1

(i) When the man is descending with an acceleration $1.5\,\text{m}\,\text{s}^{-2}$ downwards, how much time does it take for his speed to increase from $0.5\,\text{m}\,\text{s}^{-1}$ downwards to $3.5\,\text{m}\,\text{s}^{-1}$ downwards?

How far does he descend in this time? [4]

The man has a mass of 80 kg. All resistances to motion may be neglected.

(ii) Calculate the tension in the wire when the man is being lowered

(A) with an acceleration of $1.5\,\text{m}\,\text{s}^{-2}$ downwards,

(B) with an acceleration of $1.5\,\text{m}\,\text{s}^{-2}$ upwards. [5]

Subsequently, the man is raised and this situation is modelled with a constant resistance of 116 N to his upward motion.

(iii) For safety reasons, the tension in the wire should not exceed 2500 N. What is the maximum acceleration allowed when the man is being raised? [4]

At another stage of the rescue, the man has equipment of mass 10 kg at the bottom of a vertical rope which is hanging from his waist, as shown in Fig. 6.2. The man and his equipment are being raised; the rope is light and inextensible and the tension in it is 80 N.

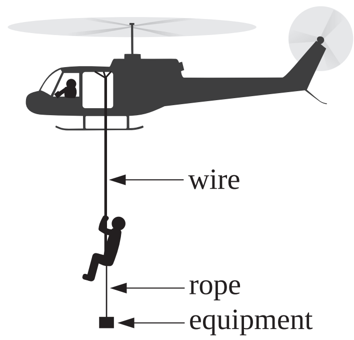

Fig. 6.2

(iv) Assuming that the resistance to the upward motion of the man is still 116 N and that there is negligible resistance to the motion of the equipment, calculate the tension in the wire. [4]

7 A small firework is fired from a point O at ground level over horizontal ground. The highest point reached by the firework is a horizontal distance of 60 m from O and a vertical distance of 40 m from O, as shown in Fig. 7. Air resistance is negligible.

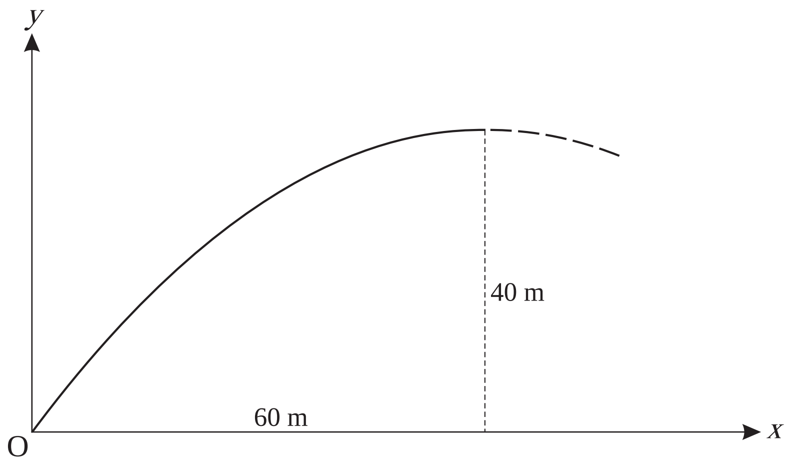

Fig. 7

The initial horizontal component of the velocity of the firework is $21\,\mathrm{m\,s^{-1}}$.

(i) Calculate the time for the firework to reach its highest point and show that the initial vertical component of its velocity is $28\,\mathrm{m\,s^{-1}}$. [4]

(ii) Show that the firework is $(28t - 4.9t^2)$ m above the ground t seconds after its projection. [1]

When the firework is at its highest point it explodes into several parts. Two of the parts initially continue to travel horizontally in the original direction, one with the original horizontal speed of $21\,\mathrm{m\,s^{-1}}$ and the other with a quarter of this speed.

(iii) State why the two parts are always at the same height as one another above the ground and hence find an expression in terms of t for the distance between the parts t seconds after the explosion. [3]

(iv) Find the distance between these parts of the firework

(A) when they reach the ground, [2]

(B) when they are 10 m above the ground. [5]

(v) Show that the cartesian equation of the trajectory of the firework before it explodes is $y = \frac{1}{90}(120x - x^2)$, referred to the coordinate axes shown in Fig. 7. [4]

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