RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4776/01**

MATHEMATICS (MEI)

Numerical Methods

THURSDAY 24 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

CUP/T38505

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks for each question is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 The equation $f(x) = 0$ is known to have a single root. Given that $f(2) = 0.24$ and $f(3) = 0.03$, use the secant method to obtain an estimate of the root. Show, by means of a sketch, that this estimate could be very inaccurate. [5]

2 For the integral $I = \int_0^1 \frac{2 - \sqrt{x}}{2 + \sqrt{x}}\,dx$, find the values given by

(A) the trapezium rule with $h = 1$,

(B) the mid-point rule with $h = 1$.

Use these two values to obtain a further trapezium rule estimate and a Simpson's rule estimate of the integral. [8]

3 The function $f(x)$ has the values shown in the table.

x	0	1	3
$f(x)$	2.00	2.57	3.85

Use Lagrange's method to find the estimate of $f(2)$ given by fitting a quadratic function to the data. [7]

4 Show that the equation $x^3(2 - x) = 1$ has a root in the interval (1.5, 2). Use the bisection method to find the root with maximum possible error 0.0625. [6]

Determine how many further iterations of the bisection process would be required to reduce the maximum possible error to less than 0.005. [2]

5 A numerical derivative is being found using the forward difference approximation. Show, by means of a sketch, that a large value of h may lead to a large error. [3]

The function $g(x)$ has the values shown in the table correct to 3 decimal places.

x	2	2.001	2.01	2.1
$g(x)$	3.610	3.612	3.633	3.849

Obtain three estimates of the derivative of the function at $x = 2$. Use your answers to show that, in numerical differentiation, a smaller value of h may not always lead to greater accuracy. [5]

Section B (36 marks)

6 The function f(x) has the values shown in the table.

x	3	4	5	6
f(x)	1	3	–1	–10

(i) Use Newton's forward difference interpolation formula to fit a quadratic to the points at $x = 3, 4, 5$. Use this quadratic to estimate

(A) the value of x at which f(x) takes its maximum value,

(B) the value of x in the interval (4, 5) for which f(x) = 0.

Show that the quadratic does not pass through the fourth data point. [12]

(ii) Use Newton's forward difference interpolation formula to estimate f(4.5) using a cubic. (Note that you are not required to find the cubic in terms of x.)

Hence obtain a Simpson's rule estimate of $\int_3^6 \mathrm{f}(x)\,\mathrm{d}x$. [6]

[Question 7 is printed overleaf.]

7 **(i)** The number 2.506 628 is known to be correct to 6 decimal places. Write down the maximum possible error and calculate the maximum possible relative error. [3]

(ii) A computer adds up 1000 numbers each of which has been *rounded* to 6 decimal places. Calculate the maximum possible error in the sum. Explain why an error of this magnitude is unlikely to arise in practice. [3]

(iii) A computer adds up 1000 numbers each of which has been *chopped* to 6 decimal places. Calculate the maximum possible error in the sum. What is the most likely error in practice? Explain your answer. [5]

(iv) A computer program in which numbers are rounded to 7 significant figures is used to sum the following numbers. All intermediate answers used in calculations are rounded to 7 significant figures.

1, 0.000 000 1, 0.000 000 2, 0.000 000 3, 0.000 000 4.

Find the answers the program will give if the numbers are summed

(A) from left to right,

(B) from right to left.

Explain the difference in the two answers. [3]

(v) A simple computer program is written to find the following sum.

$$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \dots + \frac{1}{1000^3}.$$

The answer obtained is 1.202 051. When the terms are summed in reverse order the answer is 1.202 056. State, with an explanation, which of these is likely to be more accurate.

When the same two calculations are performed on a spreadsheet the two answers that are displayed are identical. What *two* features of a spreadsheet explain why this happens? [4]

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1

x	2	3
f(x)	0.24	0.03

root = (2 x 0.03 - 3 x 0.24) / (0.03 - 0.24) **[M1A1]**
= 3.142857 **[A1]**

Eg: graph showing turning point at x = 3 with root some way to the left or the right. **[G2]**

[TOTAL 5]

2

x	f(x)				
0	1				
1	0.333333	T1 =	0.666667		**[M1A1]**
0.5	0.477592	M =	0.477592		**[M1A1]**
		hence	T2 = (T1 + M)/2 =	0.572129	**[M1A1]**
		and	S = (T1 + 2*M)/3 =	0.540617	**[M1A1]**

[TOTAL 8]

3

x	0	1	3
f(x)	2	2.57	3.85

3 terms: **[M1]**
form: **[M1]**
use x=2: **[M1]**

f(2) = 2(2-1)(2-3)/(0-1)(0-3) + 2.57(2-0)(2-3)/(1-0)(1-3) + 3.85(2-0)(2-1)/(3-0)(3-1) **[A1A1A1]**
= 3.186667 (3.19) **[A1]**

[TOTAL 7]

4

x	1.5	2
x^3(2-x)-1	0.6875	-1

change of sign, so root *(may be implied)* **[M1A1]**

a	**b**	**x**	**x^3(2-x)-1**	**mpe**	
1.5	2	1.75	0.339844	0.25	**[M1A1]**
1.75	2	1.875	-0.17603	0.125	**[A1]**
1.75	1.875	**1.8125**		0.0625	**[A1]**

4 further iterations reqd: mpe 0.0325, 0.015625, 0.0078125, 0.00390625 **[M1A1]**

[TOTAL 8]

5

Sketch showing curve, tangent, chord, h. Makes clear that tangent and chord have substantially different gradients. **[G3]**

h	0	0.1	0.01	0.001
g(2 + *h*)	3.61	3.849	3.633	3.612
est g '(2)		2.39	2.3	2

[M1A1A1A1]

Clear loss of significant figures as h is reduced **[E1]**

[TOTAL 8]

6 (i)

x	f(x)	Δf	$\Delta^2 f$	$\Delta^3 f$	
3	1				
4	3	2			
5	-1	-4	-6		
6	-10	-9	-5	1	**[M1A1A1]**

quadratic $= 1 + 2(x-3) - 6(x-3)(x-4)/2$ **[M1A1]**
$= 1 + 2x-6 - 3x^2+21x-36$ **[A1]**
$= -3x^2 +23x -41$ **[A1]**

$q'(x) = -6x + 23 = 0$ at $x = 23/6$ (= 3.833...) **[M1A1]**

$q(x) = 0$ at $x = 4.847(127)$; *also at 2.81954 - not reqd.* **[M1A1]**

$q(6) = -11$ (or point out that the second differences not constant) **[A1]**
[subtotal 12]

(ii) cubic est $= 1 + 2(4.5-3) - 6(4.5-3)(4.5-4)/2 + 1(4.5-3)(4.5-4)(4.5-5)/6$ **[M1A1A1]**
$= 1.6875$ **[A1]**

$S = 1.5/3\ (1 + 4\times1.6875 -10) = -1.125$ **[M1A1]**
[subtotal 6]

[TOTAL 18]

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7 (i) mpe 0.000 000 5 **[B1]**
mpre 0.000 000 5 / 2.506 628
= 1.99×10^{-7} **[M1A1]**
[subtotal 3]

(ii) mpe 1000 x 0.000 000 5 = 0.000 5 **[M1A1]**
In practice the positive and negative errors will tend to cancel out **[E1]**
[subtotal 3]

(iii) mpe 1000 x 0.000 001 = 0.001 **[M1A1]**
In practice 1000 x 0.000 000 5 = 0.000 5 **[M1A1]**
because average error in chopping will be 0.000 000 5 **[E1]**
[subtotal 5]

(iv) L to R: 1 (or 1.000 000) **[B1]**
R to L: 1.000 001 **[B1]**
L to R requires 8 sf, (R to L doesn't) **[E1]**
[subtotal 3]

(v) Reverse order more accurate **[E1]**
as that way allows the very small terms at the end of the series
to contribute to the sum. **[E1]**

The spreadsheet is likely to work to greater accuracy **[E1]**
The spreadsheet works to more sf than are displayed **[E1]**
[subtotal 4]

[TOTAL 18]