RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4752/01**

MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

Section A (36 marks)

1 Differentiate $10x^4 + 12$. [2]

2 A sequence begins

1 2 3 4 5 1 2 3 4 5 1 ...

and continues in this pattern.

(i) Find the 48th term of this sequence. [1]

(ii) Find the sum of the first 48 terms of this sequence. [2]

3 You are given that $\tan\theta = \frac{1}{2}$ and the angle θ is acute. Show, without using a calculator, that $\cos^2\theta = \frac{4}{5}$. [3]

4

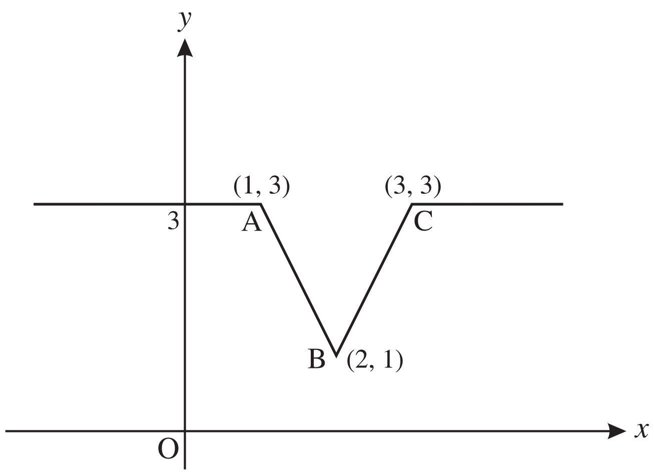

Fig. 4

Fig. 4 shows a sketch of the graph of $y = \mathrm{f}(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to A, B and C.

(i) $y = 2\mathrm{f}(x)$ [2]

(ii) $y = \mathrm{f}(x + 3)$ [2]

5 Find $\int (12x^5 + \sqrt[3]{x} + 7)\,\mathrm{d}x$. [5]

6 **(i)** Sketch the graph of $y = \sin\theta$ for $0 \leqslant \theta \leqslant 2\pi$. [2]

(ii) Solve the equation $2\sin\theta = -1$ for $0 \leqslant \theta \leqslant 2\pi$. Give your answers in the form $k\pi$. [3]

7 **(i)** Find $\sum_{k=2}^{5} 2^k$. [2]

(ii) Find the value of n for which $2^n = \frac{1}{64}$. [1]

(iii) Sketch the curve with equation $y = 2^x$. [2]

8 The second term of a geometric progression is 18 and the fourth term is 2. The common ratio is positive. Find the sum to infinity of this progression. [5]

9 You are given that $\log_{10} y = 3x + 2$.

(i) Find the value of x when $y = 500$, giving your answer correct to 2 decimal places. [1]

(ii) Find the value of y when $x = -1$. [1]

(iii) Express $\log_{10}(y^4)$ in terms of x. [1]

(iv) Find an expression for y in terms of x. [1]

Section B (36 marks)

10

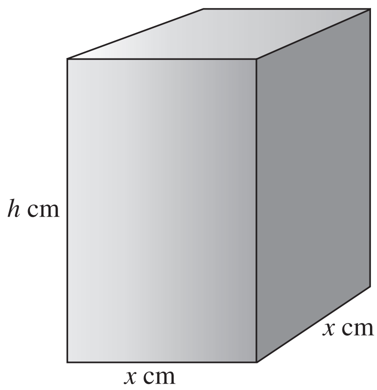

Fig. 10

Fig. 10 shows a solid cuboid with square base of side x cm and height h cm. Its volume is $120\,\text{cm}^3$.

(i) Find h in terms of x. Hence show that the surface area, $A\,\text{cm}^2$, of the cuboid is given by $A = 2x^2 + \dfrac{480}{x}$. [3]

(ii) Find $\dfrac{dA}{dx}$ and $\dfrac{d^2A}{dx^2}$. [4]

(iii) Hence find the value of x which gives the minimum surface area. Find also the value of the surface area in this case. [5]

11 **(i)** The course for a yacht race is a triangle, as shown in Fig. 11.1. The yachts start at A, then travel to B, then to C and finally back to A.

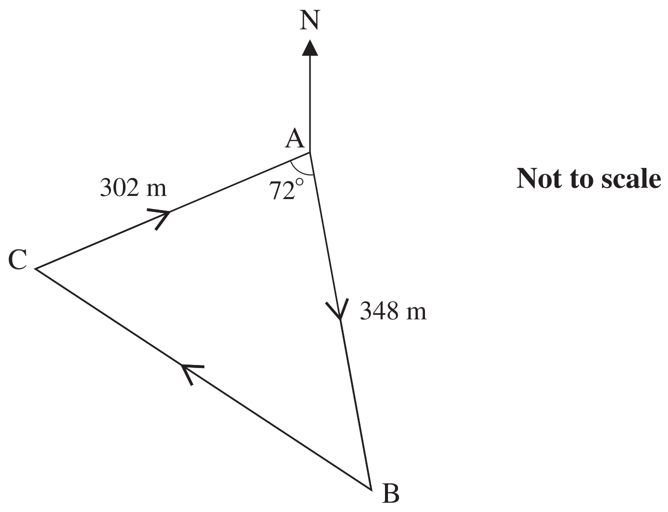

Fig. 11.1

(*A*) Calculate the total length of the course for this race. [4]

(*B*) Given that the bearing of the first stage, AB, is 175°, calculate the bearing of the second stage, BC. [4]

(ii) Fig. 11.2 shows the course of another yacht race. The course follows the arc of a circle from P to Q, then a straight line back to P. The circle has radius 120 m and centre O; angle POQ = 136°.

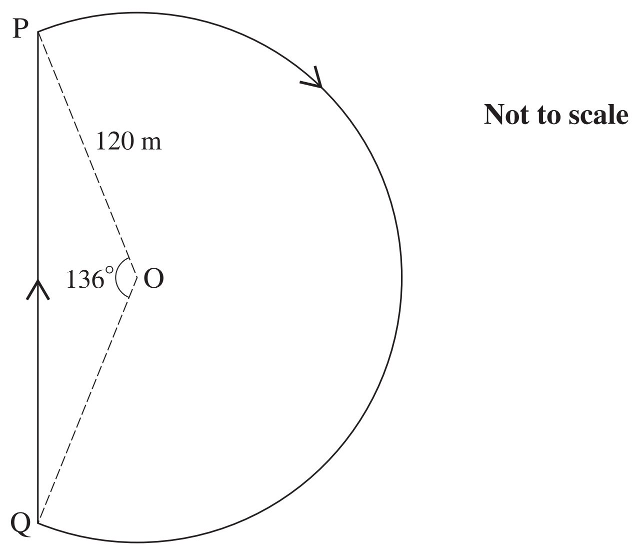

Fig. 11.2

Calculate the total length of the course for this race. [4]

12 **(i)**

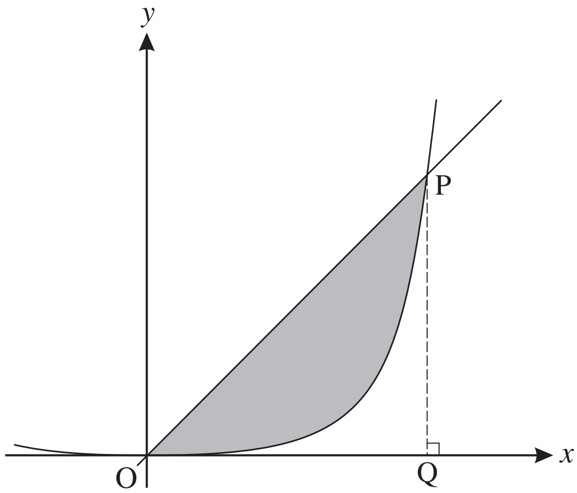

Fig. 12

Fig. 12 shows part of the curve $y = x^4$ and the line $y = 8x$, which intersect at the origin and the point P.

(A) Find the coordinates of P, and show that the area of triangle OPQ is 16 square units. [3]

(B) Find the area of the region bounded by the line and the curve. [3]

(ii) You are given that $\mathrm{f}(x) = x^4$.

(A) Complete this identity for $\mathrm{f}(x+h)$.

$$\mathrm{f}(x+h) = (x+h)^4 = x^4 + 4x^3h + \ \ldots$$ [2]

(B) Simplify $\dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$. [2]

(C) Find $\lim\limits_{h\to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$. [1]

(D) State what this limit represents. [1]

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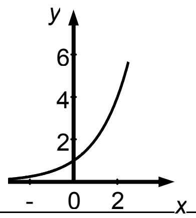
4752 (C2) Concepts for Advanced Mathematics

Section A

1	$40x^3$	2	-1 if extra term	2
2	(i) 3 (ii) 141	1 2	 M1 for $9 \times (1 + 2 + 3 + 4 + 5) + 1 + 2 + 3$	 3
3	right angled triangle with 1 and 2 on correct sides Pythagoras used to obtain hyp = $\sqrt{5}$ $\cos\theta = \frac{a}{h} = \frac{2}{\sqrt{5}}$	M1 M1 A1	or M1 for $\sin\theta = \frac{1}{2}\cos\theta$ and M1 for substituting in $\sin^2\theta + \cos^2\theta = 1$ E1 for sufficient working	 3
4	(i)line along $y = 6$ with V (1, 6), (2, 2), (3, 6) (ii) line along $y = 3$ with V (−2,3), (−1,1), (0,3)	2 2	1 for two points correct 1 for two points correct	 4
5	$2x^6 + \frac{3}{4}x^{\frac{4}{3}} + 7x + c$	5	1 for $2x^6$; 2 for $\frac{3}{4}x^{\frac{4}{3}}$ or 1 for other $kx^{\frac{4}{3}}$; 1 for $7x$; 1 for $+c$	5
6	(i) correct sine shape through O amplitude of 1 and period 2π shown (ii) $7\pi/6$ and $11\pi/6$	1 1 3	 B2 for one of these; 1 for $-\pi/6$ found	 5
7	(i) 60 (ii) −6 (iii) 	2 1 1 1	M1 for $2^2 + 2^3 + 2^4 + 2^5$ o.e. Correct in both quadrants Through (0, 1) shown dep.	 5
8	$r = 1/3$ s.o.i. $a = 54$ or ft $18 \div$ their r S = $\frac{a}{1-r}$ used with $-1 < r < 1$ S = 81 c.a.o.	2 M1 M1 A1	1 mark for $ar = 18$ *and* $ar^3 = 2$ s.o.i.	 5
9	(i) 0.23 c.a.o. (ii) 0.1 or 1/10 (iii) $4(3x + 2)$ or $12x + 8$ (iv) [y =] 10^{3x+2} o.e.	1 1 1 1	 10^{-1} not sufficient	 4

Section B

10	**i**	$h = 120/x^2$ $A = 2x^2 + 4xh$ o.e. completion to given answer	B1 M1 A1	 at least one interim step shown	 3
	ii	$A' = 4x - 480/x^2$ o.e. $A'' = 4 + 960 / x^3$	2 2	1 for kx^{-2} o.e. included ft their A' only if kx^{-2} seen ; 1 if one error	 4
	iii	use of $A' = 0$ $x = \sqrt[3]{120}$ or 4.9(3..) Test using A' or A'' to confirm minimum Substitution of their x in A A = 145.9 to 146	M1 A1 T1 M1 A1	 Dependent on previous M1	 5
11	**iA**	$BC^2 = 348^2 + 302^2 - 2 \times 348 \times 302 \times \cos 72°$ BC = 383.86… 1033.86…[m] or ft 650 + their BC	M2 A1 1	M1 for recognisable attempt at Cosine Rule to 3 sf or more accept to 3 sf or more	 4
	iB	$\frac{\sin B}{302} = \frac{\sin 72}{\text{their } BC}$ B = 48.4.. 355 − their B o.e. answer in range 306 to307	M1 A1 M1 A1	Cosine Rule acceptable or Sine Rule to find C or 247 + their C	 4
	ii	Arc length PQ = $\frac{224}{360} \times 2\pi \times 120$ o.e. or 469.1... to 3 sf or more QP = 222.5…to 3 sf or more answer in range 690 to 692 [m]	M2 B1 A1	M1 for $\frac{136}{360} \times 2\pi \times 120$	 4
12	**iA**	$x^4 = 8x$ (2, 16) c.a.o. PQ = 16 and completion to show ½ × 2 × 16 = 16	M1 A1 A1	 NB answer 16 given	 3
	iB	$x^5/5$ evaluating their integral at their co-ord of P and zero [or 32/5 o.e.] 9.6 o.e.	M1 M1 A1	 ft only if integral attempted, not for x^4 or differentiation c.a.o.	 3
	iiA	$6x^2h^2 + 4xh^3 + h^4$	2	B1 for two terms correct.	2
	iiB	$4x^3 + 6x^2h + 4xh^2 + h^3$	2	B1 for three terms correct	2
	iiC	$4x^3$	1		1
	iiD	gradient of [tangent to] curve	1		1