RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4762/01**

MATHEMATICS (MEI)

Mechanics 2

WEDNESDAY 20 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

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1 **(a)** Disc A of mass 6 kg is on a smooth horizontal plane. The disc is at rest and then a constant force of magnitude 9 N acts on it for 2 seconds.

(i) Find the magnitude of the impulse of the force on the disc. Hence, or otherwise, find the speed of the disc after the two seconds. [2]

Without losing speed, disc A now collides directly with disc B of mass 2 kg which is also on the plane. Just before the collision, disc B is travelling at 1 m s^{-1} in the opposite direction to the motion of A, as shown in Fig. 1.1. On impact the two discs stick together to form the combined object AB.

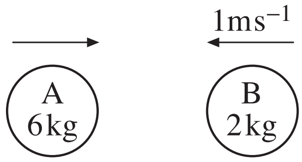

Fig. 1.1

(ii) Show that AB moves off with a speed of 2 m s^{-1} in the original direction of motion of disc A. [3]

(iii) Find the impulse that acts on disc B in the collision. [2]

The combined object AB now collides directly with disc C of mass 10 kg, which is moving on the plane at 1.8 m s^{-1} in the same direction as AB. After this collision the speed of AB is $v \text{ m s}^{-1}$ in the same direction as its speed before the impact, and disc C moves off with speed 1.9 m s^{-1}.

(iv) (*A*) Draw a diagram indicating the velocities before and after the collision. [1]

(*B*) Calculate the value of v. [3]

(*C*) Calculate the coefficient of restitution in the collision. [3]

(b) A small ball is thrown horizontally with a speed of 8 m s^{-1}. The point of projection is 10 m above a smooth horizontal plane, as shown in Fig. 1.2.

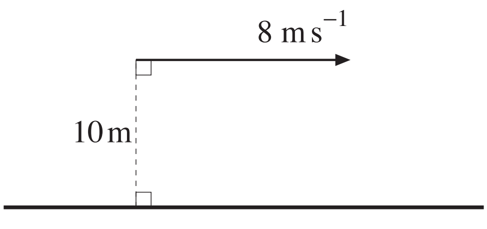

Fig. 1.2

The coefficient of restitution in the impact between the ball and the plane is $\frac{4}{7}$.

Calculate the vertical component of the velocity of the ball immediately after its first impact with the plane and also the angle at which the ball rebounds from the plane. [5]

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2 The position of the centre of mass, G, of a uniform wire bent into the shape of an arc of a circle of radius r and centre C is shown in Fig. 2.1.

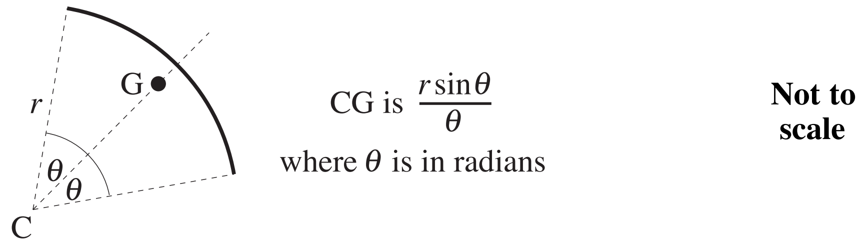

Fig. 2.1

(i) Use this information to show that the centre of mass, G, of the uniform wire bent into the shape of a semi-circular arc of radius 8 shown in Fig. 2.2 has coordinates $\left(-\frac{16}{\pi}, 8\right)$. [3]

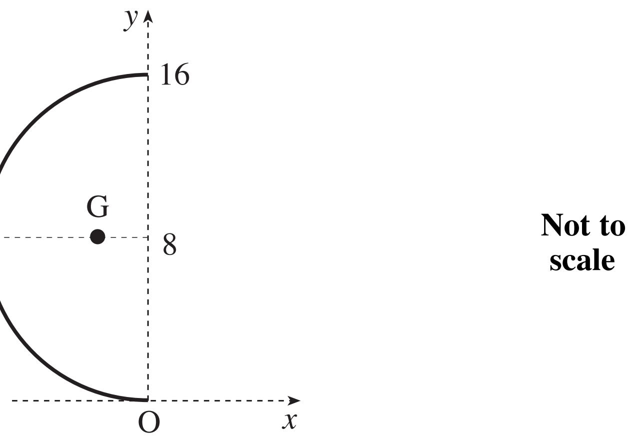

Fig. 2.2

A walking-stick is modelled as a uniform rigid wire. The walking-stick and coordinate axes are shown in Fig. 2.3. The section from O to A is a semi-circular arc and the section OB lies along the x-axis. The lengths are in centimetres.

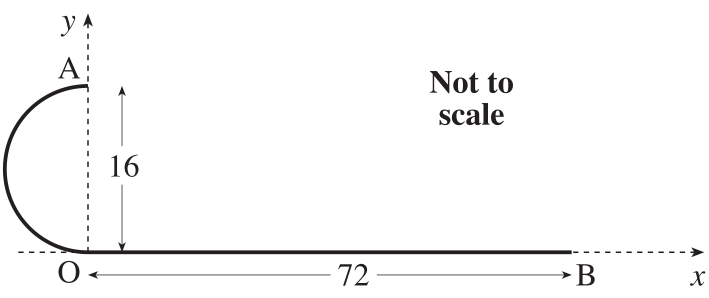

Fig. 2.3

(ii) Show that the coordinates of the centre of mass of the walking-stick are $(25.37, 2.07)$, correct to two decimal places. [6]

The walking-stick is now hung from a shelf as shown in Fig. 2.4. The only contact between the walking-stick and the shelf is at A.

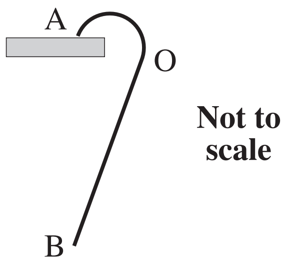

Fig. 2.4

(iii) When the walking-stick is in equilibrium, OB is at an angle α to the vertical.

Draw a diagram showing the position of the centre of mass of the walking-stick in relation to A.

Calculate α. [5]

(iv) The walking-stick is now held in equilibrium, with OB vertical and A still resting on the shelf, by means of a vertical force, F N, at B. The weight of the walking-stick is 12 N. Calculate F. [3]

3 A uniform plank is 2.8 m long and has weight 200 N. The centre of mass is G.

(i) Fig. 3.1 shows the plank horizontal and in equilibrium, resting on supports at A and B.

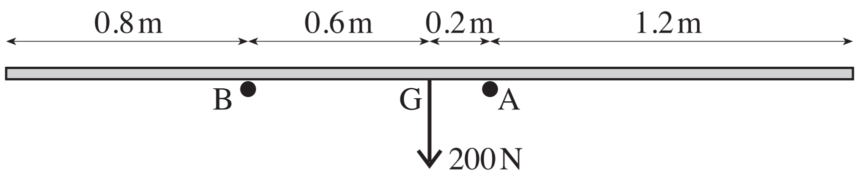

Fig. 3.1

Calculate the reactions of the supports on the plank at A and at B. [4]

(ii) Fig. 3.2 shows the plank horizontal and in equilibrium between a support at C and a peg at D.

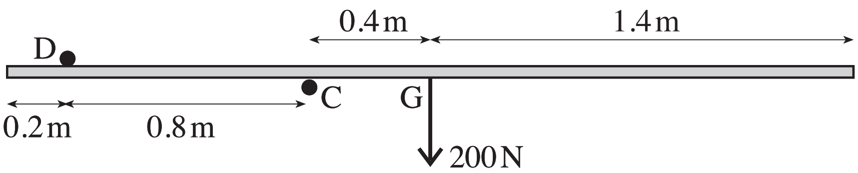

Fig. 3.2

Calculate the reactions of the support and the peg on the plank at C and at D, showing the directions of these forces on a diagram. [5]

Fig. 3.3 shows the plank in equilibrium between a support at P and a peg at Q. The plank is inclined at α to the horizontal, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$.

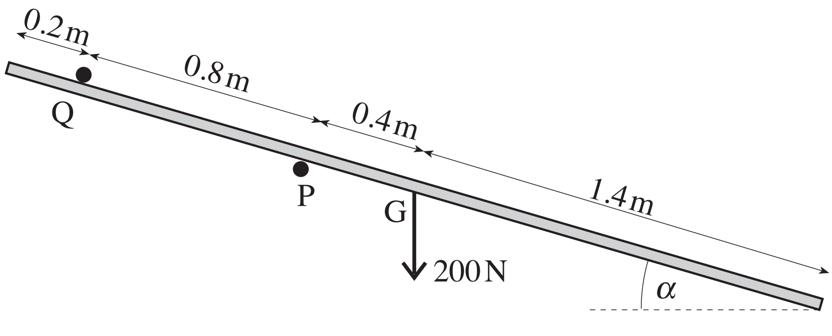

Fig. 3.3

(iii) Calculate the normal reactions at P and at Q. [6]

(iv) Just one of the contacts is rough. Determine which one it is if the value of the coefficient of friction is as small as possible. Find this value of the coefficient of friction. [4]

4 Jack and Jill are raising a pail of water vertically using a light inextensible rope. The pail and water have total mass 20 kg.

In parts **(i)** and **(ii)**, all non-gravitational resistances to motion may be neglected.

(i) How much work is done to raise the pail from rest so that it is travelling upwards at $0.5\,\text{m s}^{-1}$ when at a distance of 4 m above its starting position? [4]

(ii) What power is required to raise the pail at a steady speed of $0.5\,\text{m s}^{-1}$? [3]

Jack falls over and hurts himself. He then slides down a hill.

His mass is 35 kg and his speed increases from $1\,\text{m s}^{-1}$ to $3\,\text{m s}^{-1}$ while descending through a vertical height of 3 m.

(iii) How much work is done against friction? [5]

In Jack's further motion, he slides down a slope at an angle α to the horizontal where $\sin\alpha = 0.1$. The frictional force on him is now constant at 150 N. For this part of the motion, Jack's initial speed is $3\,\text{m s}^{-1}$.

(iv) How much further does he slide before coming to rest? [5]

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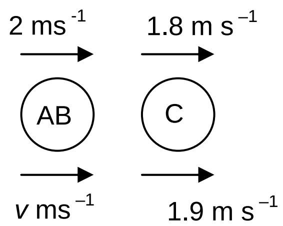
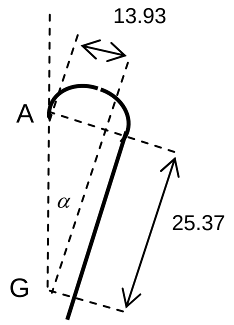
Mark Scheme 4762
June 2007

Q 1				
(a) (i)	Impulse has magnitude $2 \times 9 = 18$ N s speed is $\frac{18}{6} = 3 \text{ m s}^{-1}$.	B1 B1		2
(ii)	PCLM $\rightarrow$ $3 \times 6 - 1 \times 2 = 8v$ $v = 2$ so 2 m s^{-1} in orig direction of A	M1 A1 E1	Use of PCLM + combined mass RHS All correct Must justify direction (diag etc)	3
(iii)	$\rightarrow \quad 2 \times 2 - 2 \times -1 = 6$ N s	M1 A1	Attempted use of m**v** - m**u** for 6 N s dir specified (accept diag)	2
(iv) (A)		B1	Accept masses not shown	1
(B)	PCLM $\rightarrow$ $2 \times 8 + 10 \times 1.8 = 8v + 10 \times 1.9$ $v = 1.875$	M1 A1 A1	PCLM. All terms present Allow sign errors only	3
(C)	NEL $\frac{1.9 - 1.875}{1.8 - 2} = -e$ so $e = 0.125$	M1 A1 F1	Use of NEL with **their** v Any form. FT **their** v FT **their** v (only for $0 < e \le 1$)	3
(b)	Using $v^2 = u^2 + 2as$ $v = \sqrt{2 \times 10 \times 9.8} = 14$ rebounds at $14 \times \frac{4}{7}$ = 8 m s^{-1} No change to the horizontal component Since both horiz and vert components are 8 m s^{-1} the angle is 45°	B1 M1 F1 B1 A1	Allow ± 14 Using **their** *vertical* component FT from **their** 14. Allow $\pm$ Need not be explicitly stated cao	5
		19		

Q 2				
(i)	$\theta = \frac{\pi}{2}$	B1		
	gives CG $= \dfrac{8\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \dfrac{16}{\pi}$	E1		
	$\left(-\frac{16}{\pi}, 8\right)$ justified	E1		3
(ii)	$(8\pi + 72)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 8\pi\begin{pmatrix}-\frac{16}{\pi}\\ 8\end{pmatrix} + 72\begin{pmatrix}36\\ 0\end{pmatrix}$	M1	Method for c.m.	
		B1 A1 A1	Correct mass of 8 or equivalent 1st RHS term correct 2nd RHS term correct	
	$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}25.3673...\\ 2.06997...\end{pmatrix} = \begin{pmatrix}25.37\\ 2.07\end{pmatrix}$ (4 s. f.)	E1 E1		
			[If separate cpts award the A1s for *x*- and *y*- cpts correct on RHS]	6
(iii)		B1	General position and angle (lengths need not be shown)	
	$\tan\alpha = \dfrac{13.93}{25.37}$	M1	Angle or complement attempted. arctan or equivalent.	
		M1 A1	Attempt to get 16 – 2.0699… Obtaining 13.93… cao Accept use of 2.0699… but not 16.	
	α = 28.7700… so 28.8° (3 s. f.)	A1	cao	5
(iv)			[FT use of 2.0699…]	
	c. w. moments about A $12 \times 13.93 - 16F = 0$	M1 A1	Moments about any point, all forces present	
	so F = 10.4475…	A1	(1.5525... if 2.0699… used)	3
		17		

Q 3				
(i)	Moments c.w. about B $200 \times 0.6 - 0.8R_A = 0$ $R_A = 150$ so 150 N Resolve or moments $R_B = 50$ so 50 N	 M1 A1 M1 F1	 Accept about any point. Allow sign errors.	4
(ii)	Moments c.w. about D $-0.8R_C + 1.2 \times 200 = 0$ $R_C = 300$ ↑ Resolve or moments $R_D = 100$ ↓	 M1 A1 M1 A1 E1	 Or equiv. Accept about any point. All terms present. No extra terms. Allow sign errors. Neglect direction Or equiv. All terms present. No extra terms. Allow sign errors. Neglect direction Both directions clearly shown (on diag)	5
(iii)	Moments c.w. about P $0.4 \times 200\cos\alpha - 0.8R_Q = 0$ $R_Q = 96$ so 96 N resolve perp to plank $R_P = 200\cos\alpha + R_Q$ $R_P = 288$ so 288 N	 M1 A1 A1 M1 A1 A1	 Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working] Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working]	6
(iv)	Need one with greatest normal reaction So at P Resolve parallel to the plank $F = 200\sin\alpha$ so $F = 56$ $\mu = \frac{F}{R}$ $= \frac{56}{288} = \frac{7}{36}$ (= 0.194 (3 s. f.))	 B1 B1 M1 A1	 FT their reactions Must use **their** F and R cao	4
		19		

Q 4				
(i)	**either**			
	$0.5\times20\times0.5^2+20\times9.8\times4$	M1	KE or GPE terms	
		B1	KE term	
		B1	GPE term	
	= 786.5 J	A1	cao	
	or			
	$a = 1/32$	B1		
	$T-20g=20\times 1/32$	M1	N2L. All terms present.	
	T = 196.625	A1		
	WD is $4T$ = 786.5 so 786.5 J	A1	cao	4
(ii)	$20g\times0.5=10g$ so 98 W	M1	Use of $P = Fv$ or $\Delta\text{WD}/\Delta t$	
		A1	All correct	
		A1		3
(iii)	GPE lost is $35\times9.8\times3=1029$ J	B1		
	KE gained is $0.5\times35\times(3^2-1^2)=140$ J	M1	ΔKE	
		A1	The 140 J need not be evaluated	
	so WE gives WD against friction is	M1	Use of WE equation	
	1029 – 140 = 889 J	A1	cao	5
(iv)	**either**			
	$0.5\times35\times3^2+35\times9.8\times0.1x=150x$	M1	WE equation. Allow 1 missing term. No extra terms.	
		B1	One term correct (neglect sign)	
		B1	Another term correct (neglect sign)	
		A1	All correct except allow sign errors	
	x = 1.36127… so 1.36 m (3 S. F.)	A1	cao	
	or			
	$35g\times0.1-150=35a$	M1	Use of N2L. Must have attempt at weight component. No extra terms.	
		A1	Allow sign errors, otherwise correct	
	a = – 3.3057…	A1	cao	
	$0 = 9 - 2ax$	M1	Use of appropriate *uvast* or sequence	
	x = 1.36127… so 1.36 m (3 S. F.)	A1	cao	5
		17		

Many excellent scripts were seen for this component with the vast majority of candidates able to make some progress worthy of credit on every question. Question 2 was, perhaps, less well answered than the other questions but even so most candidates obtained some credit for their work. The poor quality of diagrams hampered the progress of a substantial number of candidates on all of the questions. Presentation was satisfactory on the whole but in a few cases poor presentation led to arithmetic errors and a lack of coherence in the solutions. As has happened in previous sessions, some candidates did not appreciate the detail that was required in order to *'show'* a given answer and omitted relevant steps in the working or relevant comment in the explanation. Some candidates penalised themselves by premature approximation of answers leading to errors in accuracy on following parts.

Comments on Individual Questions

1 **Impulse and Momentum**

Many candidates gained significant credit on this question. Those that drew clear diagrams were usually more successful than those who did not.

(a)(i) This part was well done by almost all of the candidates.

(ii) Many numerically correct solutions were seen to this part but a significant minority of candidates did not produce a diagram and then usually failed to *show* that the direction of motion after impact was the same as the original direction of motion of A.

(iii) Many candidates failed to realise that impulse is a vector quantity and omitted its direction in their answer.

(iv) (A) Many of the diagrams were poor; labels were omitted; directions not indicated.

(B) This part posed few problems to the majority of candidates.

(C) Many correct solutions were seen to this part. Incorrect solutions usually arose because of sign errors in the application of Newton's experimental law.

(b) A large proportion of the candidates found this part of the question difficult. Many of them did not appreciate that the ball would move as a projectile with an initial velocity in the horizontal direction only and considered the initial motion to be at 8 m s^{-1} at 45º to the horizontal. Others failed to realise that the component of the velocity in the horizontal direction would be unchanged by the impact.

2 **Centres of Mass**

This question caused slightly more problems to candidates than other questions on the paper.

(i) Many candidates failed to give enough detail in their explanations to *show* the given answer. Some candidates introduced spurious negative signs. A small number of candidates did not appear to understand the relevance of the use of radians and offered $\frac{8\sin 90}{90} = \frac{16}{\pi}$ as an answer.

(ii) This part was well done on the whole with any errors usually arithmetic in nature.

(iii) Poor diagrams were seen on a large number of scripts. Diagrams showing the centre of mass directly below the point of suspension were in a minority and this led to errors in finding the lengths required to calculate angle α.

(iv) This part caused fewer problems than the previous part with many candidates able to obtain at least some credit for their work.

3 **Moments and Resolving**

It was pleasing to see some excellent answers to this question with many candidates gaining a substantial amount of credit.

(i) Many good responses to this part were seen with the majority of candidates able to obtain full credit.

(ii) The candidates seemed to have few problems with this part although a small minority failed to draw the diagram requested in the question.

(iii) While many good solutions to this part were seen, a sizeable number of candidates did not appreciate that the normal reaction was at right angles to the plank and drew the reactions acting vertically.

(iv) This part was not well answered. Many of the candidates failed to realise that for μ to be a minimum, the friction had to be acting at the place where the normal reaction was largest. Some candidates merely calculated a value for μ at one point and then failed to show or give a reason as to why this was the minimum value. Some complex calculations were offered by some candidates to find the frictional force with few of them appreciating that, for limiting friction, this had to be equal and opposite to the component of the weight down the plane.

4 **Work- Energy**.

As in previous sessions, those candidates who used work energy methods were on the whole more successful than those who attempted to use Newton's second law and the constant acceleration equations. It was encouraging to see a significant number of completely correct responses.

(i) This part was well done by most of the candidates.

(ii) The majority of candidates could obtain some credit for this part but a few failed to realise that the pail being raised at a steady speed implied that the force required was equal and opposite to the weight of the pail. Some utilised the force they had obtained in the previous part without realising that the conditions had changed.

(iii) Many of the candidates scored highly on this part. However, some attempted to solve the problem by applying Newton's second law and the constant acceleration equations in a vertical direction without giving any justification of their method.

(iv) A large number of completely correct responses were seen to this part. Errors, on the whole, usually arose from the omission of one term in the work energy equation or from a sign error.