OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4773/01**

MATHEMATICS (MEI)

Decision Mathematics Computation

MONDAY 18 JUNE 2007

Morning
Time: 2 hours 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

This document consists of **7** printed pages and **1** blank page.

  [Turn over

1 The number of branches on a tree in a particular year is modelled as the number of branches that were on the tree in the previous year plus new growth of p times the number that were on the tree the year before that, $0 < p < 1$.

(i) Let u_n be the number of branches on the tree in year n. Write down a recurrence relation for u_{n+2} in terms of u_{n+1}, u_n and p. [2]

(ii) The tree was bought $(n = 0)$ with 20 branches. It had 25 branches after one year $(n = 1)$. Given that $p = 0.11$, solve your recurrence relation. [8]

(iii) Construct a spreadsheet to model your recurrence relation, and use it to check your answer to part **(ii)**.

Add to your spreadsheet to show the number of branches each year as the nearest integer to that given by applying the recurrence relation to the previous two integers. Print out your spreadsheet for n from 0 to 20. Print out the formulae which you used. (Just one example of each formula will suffice.) [4]

To control the growth of the tree it is pruned each year, after the new growth has taken place. New growth is not pruned, but a proportion, r $(0 < r < 1)$, of old branches is removed. Let v_n be the number of branches on the tree in year n, after pruning.

(iv) Modify your answer to part **(i)** to produce a recurrence relation for v_{n+2} in terms of v_{n+1}, v_n, p and r. [2]

(v) Modify your spreadsheet to allow for pruning and find a value of r which will lead to about 100 branches after 20 years $(\text{using } v_0 = 20, v_1 = 25 \text{ and } p = 0.11)$. [2]

2 Six trees are to be planted, two pines, one eucalyptus, one mimosa, one jacaranda and one acacia. There are 7 locations available, at each of which one tree can be planted. The table shows which trees can be planted in each of the locations.

Location	1	2	3	4	5	6	7
Trees	pine eucalyptus	eucalyptus jacaranda	mimosa acacia	pine jacaranda	pine mimosa	mimosa acacia	acacia

(i) Draw a bipartite graph to represent this information. [2]

The gardener decides that he does not wish to use location number 2. He starts placing trees by locations prior to planting.

(ii) Show that there is no solution if location 2 is not used. [2]

The gardener now decides to reject location 7 instead of location 2. He starts by placing the eucalyptus by location 1, the jacaranda by location 2, the acacia by location 3, the first pine by location 4 and the mimosa by location 5. He then realises that he has nowhere to put the second pine.

(iii) Represent the gardener's incomplete matching on a second bipartite graph. [1]

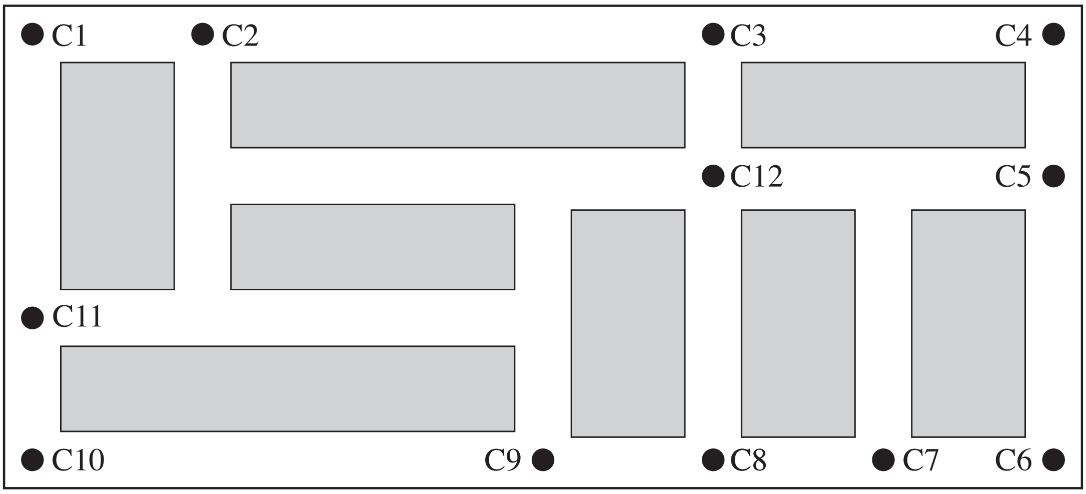
(iv) Give an alternating path, starting at the second pine and ending at location 6, of arcs taken alternately from the full bipartite graph and from the graph representing the incomplete matching. Hence give a complete matching. [3]

(v) The gardener would like to have an automatic procedure to solve similar tree-planting problems in the future. Produce an LP to solve the problem of finding a maximal matching from the information given in the original table.

Produce a print-out of your LP. Run it and produce a print-out of your results.

Interpret your results. [10]

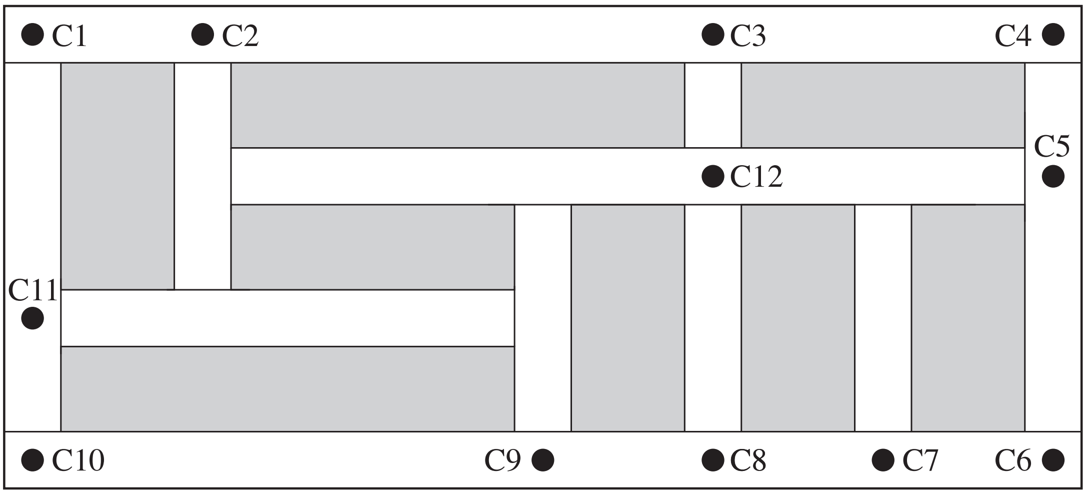
3 The builders of a shopping precinct have to decide where to place CCTV cameras. The diagram shows buildings (which are shaded), pavements, and 12 possible locations for cameras.



Cameras can be rotated to view along different directions, and all pavements must be in sight of at least one camera.

(i) By inspection select a set of 6 locations from which cameras can scan all pavements. [2]

The diagram below shows one way of splitting the pavements into rectangles.



(ii) Formulate an LP to select a minimum set of locations from which cameras can scan all of the rectangles. Produce a print-out of your formulation. [8]

(iii) Run your LP, produce a print-out of your output, and interpret the results. [3]

The costs of installing a camera depends on the location. They are listed below.

Location	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
Cost (£000)	5	2	3	5	4	1.5	2	2	5	3	4	7

(iv) Modify your LP to find the cheapest way of achieving full coverage of all pavements. [2]

(v) Run your modified LP, produce a print-out of your output, and interpret the results. [3]

[Question 4 is printed overleaf.]

4 A component in a machine has a short lifespan. It fails either after 1, 2 or 3 days, with probabilities given in the table.

Time to failure (days)	1	2	3
Probability	0.1	0.3	0.6

When a component fails it is replaced at the end of the day.

(i) Construct a look-up table to simulate the failure time for a component. Print out the formulae which you use. [3]

(ii) Set up a spreadsheet to simulate failure times for a number of components so that you can accumulate the times to failure. Simulate enough components so that the accumulated failure times exceed 16 days.

Print out your spreadsheet formulae. [3]

(iii) From your simulation in part **(ii)** record

- whether or not there was a failure on day 14,
- whether or not there was a failure on day 15,
- whether or not there was a failure on day 16,
- the total number of failures up to and including day 14.

Repeat your simulation 9 more times (10 times in total), recording information as before. Hence estimate the probability of a failure on day 14, the probability of a failure on day 15, the probability of a failure on day 16, and the expected number of failures up to and including day 14. [5]

Replacing a part when it fails costs £50, plus the cost of the component, which is £25. An alternative policy is to replace a component if it fails on its first day, and otherwise to replace it anyway, failed or not, at the end of its second day. Such a scheduled replacement costs £30 plus the cost of the component.

(iv) Simulate the operation of this scheduled replacement policy over a period of 14 days. Repeat your simulation 10 times and use your results from part **(iii)** to see whether or not this policy is cost effective. [6]

(v) How could you improve the reliability of your results? [1]

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Mark Scheme 4773
June 2007

1.

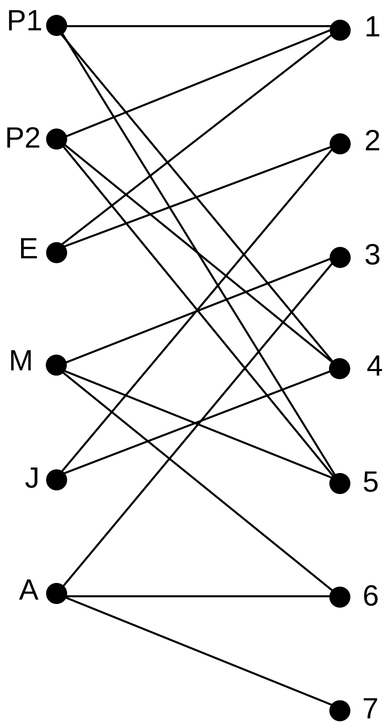
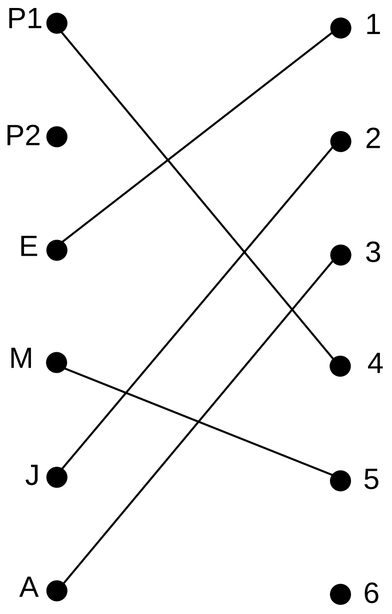
(i) $u_{n+2} = u_{n+1} + pu_n$	M1 A1
(ii) Auxiliary equation is $\lambda^2 - \lambda - 0.11 = 0$	M1 A1 M1 gen homogeneous A1 with 1.1 & –0.1 B1 case 1(u_0 = 20) +case 2(u_1 = 25) M1 simultaneous A1 22.5 and −2.5 B1 final answer
Solution is $u_n = 22.5(1.1)^n - 2.5(-0.1)^n$	
(iii)	B1 recurrence relation B1 checking formula B1 discretising
Formula: =INT(H3+B$2*H2+0.5)	B1
(iv) $v_{n+2} = (1-r)v_{n+1} + pv_n$	M1 A1

Rec rel		Formula	Int RR
20.0000	0	20.0000	20
25.0000	1	25.0000	25
27.2000	2	27.2000	27
29.9500	3	29.9500	30
32.9420	4	32.9420	33
36.2365	5	36.2365	36
39.8601	6	39.8601	40
43.8461	7	43.8461	44
48.2307	8	48.2307	48
53.0538	9	53.0538	53
58.3592	10	58.3592	58
64.1951	11	64.1951	64
70.6146	12	70.6146	70
77.6761	13	77.6761	77
85.4437	14	85.4437	85
93.9881	15	93.9881	93
103.3869	16	103.3869	102
113.7256	17	113.7256	112
125.0981	18	125.0981	123
137.6080	19	137.6080	135
151.3687	20	151.3687	149

1. (cont)

(v) Pruning $r =$ 0.026 20 25 27 29 31 33 36 39 42 45 48 52 56 60 65 70 75 81 87 94 101	B1
$r = 0.025$ to 0.027	B1

2.

(i) 	M1 A1
(ii) e.g. locations 3, 6 and 7 for only two trees, so one must be rejected. Therefore other 6 locations needed.	B1 B1
(iii) 	B1
(iv) e.g. P2 – 5 – M – 6	M1 A1
P1 P2 E M J A 4 5 1 6 2 3	A1

2 (cont).

(v) Max P11+P14+P15+P21+P24+P25+E1+E2+M3+M5+M6 +J2+J4+A3+A6+A7	M1 objective A1
st P11+P14+P15<=1 P21+P24+P25<=1 E1+E2<=1 M3+M5+M6<=1	M1 tree constraints A2 (–1 each error)
J2+J4<=1 A3+A6+A7<=1 P11+P21+E1<=1 E2+J2<=1 M3+A3<=1 P14+P24+J4<=1 P15+P25+M5<=1 M6+A6<=1 A7<=1 End	M1 location A2 constraints (–1 each error)
LP OPTIMUM FOUND AT STEP 13 OBJECTIVE FUNCTION VALUE 1) 6.000000	B1 running
VARIABLE VALUE REDUCED COST P11 0.000000 0.000000 P14 0.000000 0.000000 P15 1.000000 0.000000 P21 0.000000 0.000000 P24 1.000000 0.000000 P25 0.000000 0.000000 E1 1.000000 0.000000 E2 0.000000 0.000000 M3 0.000000 0.000000 M5 0.000000 1.000000 M6 1.000000 0.000000 J2 1.000000 0.000000 J4 0.000000 0.000000 A3 0.000000 0.000000 A6 0.000000 0.000000 A7 1.000000 0.000000	
P1 P2 E M J A 5 4 1 6 2 7	B1 interpretation

3.

(i) e.g. C2 C3 C5 C7 C9 C11	M1 A1
(ii)	
Min C1+C2+C3+C4+C5+C6+C7+C8+C9+C10+C11+C12	M1 objective
st C1+C2+C3+C4>=1	A1
C4+C5+C6>=1	
C6+C7+C8+C9+C10>=1	M1
C1+C10+C11>=1	A5 constraints
C2>=1	(–1 each error)
C3+C8+C12>=1	
C5+C12>=1	
C11>=1	
C9>=1	
C7>=1	
end	
(iii)	
LP OPTIMUM FOUND AT STEP 7	B1 running
OBJECTIVE FUNCTION VALUE	
1) 6.000000	

VARIABLE	VALUE	REDUCED COST
C1	0.000000	0.000000
C2	1.000000	0.000000
C3	0.000000	1.000000
C4	1.000000	0.000000
C5	0.000000	0.000000
C6	0.000000	0.000000
C7	1.000000	0.000000
C8	0.000000	1.000000
C9	1.000000	0.000000
C10	0.000000	0.000000
C11	1.000000	0.000000
C12	1.000000	0.000000

Use locations 2, 4, 7, 9, 11 and 12.	B1
6 cameras needed	B1
(iv) New objective:	M1
5C1+2C2+3C3+5C4+4C5+1.5C6+2C7+2C8+5C9 +3C10+4C11+7C12	A1
(v) Running	B1
Use locations 2, 5, 7, 8, 9 and 11.	B1
Cost = £19000	B1

4.

(i)	e.g. 1 0 =LOOKUP(RAND(),B1:B3,A1:A3) 2 0.1 3 0.4	B1 rand B1 probs B1 outcomes
(ii)	=LOOKUP(RAND(),\$B\$3:\$B\$5,\$A\$3:\$A\$5) + accumulation e.g. 2 2 3 5 2 7 3 10 2 12 3 15 3 18 3 21 3 24 3 27 2 29 2 31 3 34 2 36 2 38 3 41	M1 formula A1 repeats B1 accumulation
(iii)	e.g. day 14 day 15 day16 no. of replacements 0 0 1 5 1 0 0 6 0 0 1 5 0 0 1 5 1 0 0 6 0 1 0 5 0 1 0 6 1 0 1 5 0 1 0 5 1 0 1 6 0.4 0.3 0.5 5.4	M1 first run A1 B1 repetitions B1 probabilities expected no. of B1 replacements

Q4 (cont)

(iv) e.g.

				Replacements day 1	day 2
1	0	1	1	2	6
2	0.1	1	2	0	7
		2	4	0	7
		2	6	1	6
		2	8	1	6
		2	10	0	7
		2	12	1	6
		2	14	0	7
		1	15	0	7
		2	17	1	6
		2	19		
		2	21	0.6	6.5

B1 changed probabilities

B1 repetitions

B1 results

B1 averages

5.4*(50+25) = 405 versus B1

0.6*(50+25) + 6.5*(30+25) = 402.5 B1

(v) More repetitions. B1

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General Comments

There were fewer problems this year involving missing printouts. However, many candidates could usefully spend a few moments helping the examiner, and themselves, by arranging their printouts in the correct order and orientation, and checking that they are correctly labelled.

Comments on Individual Questions

1 **Recurrence relations**

It was very surprising to see so many candidates failing to produce the Excel output that was needed. It was expected that there would be many who failed correctly to "integerise" as specified, but many, even among those who succeeded with the recurrence relation algebra, failed to answer the first part of (iii) adequately.

2 **Networks**

This question was entirely on matchings.
Some candidates worked through the question without introducing vertices P1 and P2 for the two pine trees. This was possible, but was not always carried through successfully.
Not all candidates were able to mount a convincing argument in part (ii). However, in addition to the logically argued solutions, a few candidates produced an appropriate LP which showed a complete matching was not possible.
It was very noticeable that the computing work (part (v)) was done better more often than was the theoretical work. In particular, there were many candidates who were unable to tackle the alternating path in part (iv).

3 **LP modelling**

Many candidates were able to cope well with this question, and good solutions were often seen. Some candidates thought that C3 could not see all the way to C8, but this erroneous assumption was often then incorrectly implemented. A few candidates were unclear about the use of inequalities in their constraints. Marks were unnecessarily lost when candidates failed to make clear the interpretation of their LP output.

4 **Simulation**

Many candidates found this to be the most difficult of the questions. Parts (i), (ii) and (iii) were relatively easy, but were often not done efficiently. Furthermore candidates often made it **very** difficult for examiners to check what was being done.
In part (iv) few candidates made clear how they were modelling the changed distribution of failure times, probably because they were themselves confused about it.
Few correct cost comparison calculations were seen.
Most managed to pick up the final mark!