OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

June 2007

3895-8/7895-8/MS/R/06

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MARK SCHEME FOR THE UNITS

Mark Scheme 4751
June 2007

Section A

1	$x > -0.6$ o.e. eg $-3/5 < x$ isw	3	M2 for $-3 < 5x$ or $x > \dfrac{3}{-5}$ or M1 for $-5x < 3$ or $k < 5x$ or $-3 < kx$ [condone ≤ for Ms]; if 0, allow SC1 for −0.6 found	3
2	$t = [\pm]\sqrt{\dfrac{2s}{a}}$ o.e.	3	B2 for t omitted or $t = \sqrt{\dfrac{s}{\frac{1}{2}a}}$ o.e. M1 for correct constructive first step in rearrangement and M1 (indep) for finding sq rt of their t^2	3
3	'If $2n$ is an even integer, then n is an odd integer' showing wrong eg 'if n is an even integer, $2n$ is an even integer'	1 1	or: $2n$ an even integer $\Rightarrow$ n an odd integer or counterexample eg $n = 2$ and $2n = 4$ seen [in either order]	2
4	$c = 6$ $k = -7$	1 2	M1 for f(2) = 0 used or for long division as far as $x^3 - 2x^2$ in working	3
5	(i) $4x^4y$ (ii) 32	2 2	M1 for two elements correct; condone y^1 M1 for $\left(\dfrac{2}{1}\right)^5$ or 2^5 soi or $\left(\dfrac{1}{32}\right)^{-1}$ or $\dfrac{1}{\frac{1}{32}}$	4
6	-720 [x^3]	4	B3 for 720; M1 for each of 3^2 and $\pm 2^3$ or $(-2x)^3$ or $(2x)^3$, and M1 for 10 or (5×4×3)/(3×2×1) or for 1 5 10 10 5 1 seen but not for $^5C_3$	4
7	$\dfrac{-5}{10}$ o.e. isw	3	M1 for $4x + 5 = 2x \times -3$ and M1 for $10x = -5$ o.e. <u>or</u> M1 for $2 + \dfrac{5}{2x} = -3$ and M1 for $\dfrac{5}{2x} = -5$ o.e.	3
8	(i) $2\sqrt{2}$ or $\sqrt{8}$ (ii) $30 - 12\sqrt{5}$	2 3	M1 for $7\sqrt{2}$ or $5\sqrt{2}$ seen M1 for attempt to multiply num. and denom. by $2 - \sqrt{5}$ and M1 (dep) for denom −1 or 4 – 5 soi or for numerator $12\sqrt{5} - 30$	5
9	(i) ±5 (ii) $y = (x - 2)^2 - 4$ or $y = x^2 - 4x$ o.e. isw	2 2	B1 for one soln M1 if y omitted or for $y = (x + 2)^2 - 4$ or $y = x^2 + 4x$ o.e.	4
10	(i) ½×$(x + 1)(2x - 3) = 9$ o.e. $2x^2 - x - 3 = 18$ or $x^2 - ½x - 3/2 = 9$ (ii) $(2x - 7)(x + 3)$ −3 and 7/2 o.e. or ft their factors base 4, height 4.5 o.e. cao	M1 A1 B1 B1 B1	for clear algebraic use of ½ bh; condone $(x + 1)(2x - 3) = 18$ allow x terms uncollected. NB ans $2x^2 - x - 21 = 0$ given NB B0 for formula or comp. sq. if factors seen, allow omission of −3 B0 if also give $b = -9$, $h = -2$	5

Section B

11	**i**	grad AC = $\frac{7-3}{3-1}$ or 4/2 o.e.[= 2]	M1	not from using − ½	
		so grad AT = – ½	M1	or ft their grad AC [for use of $m_1 m_2 = -1$]	
		eqn of AT is $y - 7 = -$ ½ $(x - 3)$	M1	or subst (3, 7) in $y = -$ ½ $x + c$ or in $2y + x = 17$; allow ft from their grad of AT, except 2 (may be AC not AT)	
		one correct constructive step towards $x + 2y = 17$ [ans given]	M1	or working back from given line to $y = -$ ½ $x + 8.5$ o.e.	4
	ii	$x + 2(2x - 9) = 17$	M1	attempt at subst for x or y or elimination	
		$5x - 18 = 17$ or $5x = 35$ o.e.	A1	allow $2.5x = 17.5$ etc	
		$x = 7$ and $y = 5$ [so (7, 5)]	B1	graphically: allow M2 for both lines correct or showing (7, 5) fits both lines	3
	iii	$(x - 1)^2 + (2x - 12)^2 = 20$	M1	subst $2x - 9$ for y [oe for x]	
		$5x^2 - 50x + 125$[= 0]	M1	rearranging to 0; condone one error	
		$(x - 5)^2 = 0$	A1	showing 5 is root and only root	
		equal roots so tangent	B1	explicit statement of condition needed (may be obtained earlier in part) or showing line is perp. to radius at point of contact	
		(5, 1)	B1	condone $x = 5$, $y = 1$	
		<u>or</u>			
		$y - 3 = -$ ½ $(x - 1)$ o.e. seen	M1	or if $y = 2x - 9$ is tgt then line through C with gradient −½ is radius	
		subst or elim. with $y = 2x - 9$	M1		
		$x = 5$	A1		
		(5,1)	B1		
		showing (5, 1) on circle	B1	or showing distance between (1, 3) and (5, 1) = $\sqrt{20}$	5

12	**i**	$4(x-3)^2-9$	4	1 for $a = 4$, 1 for $b = 3$, 2 for $c = -9$ or M1 for $27 - 4 \times 3^2$ or $\frac{27}{4} - 3^2 [= -\frac{9}{4}]$	4
	ii	min at (3, –9) or ft from (i)	B2	1 for each coord [e.g. may start again and use calculus to obtain $x = 3$]	2
	iii	$(2x-3)(2x-9)$ $x = 1.5$ or 4.5 o.e.	M1 A2	attempt at factorising or formula or use of their (i) to sq rt stage A1 for 1 correct; accept fractional equivs eg 36/8 and 12/8	3
	iv	sketch of quadratic the right way up crosses x axis at 1.5 and 4.5 or ft crosses y axis at 27	M1 A1 B1	 allow unsimplified shown on graph or in table etc; condone not extending to negative x	3
13	**i**	$2x^3 + 5x^2 + 4x - 6x^2 - 15x - 12$ 3 is root use of $b^2 - 4ac$ $5^2 - 4 \times 2 \times 4$ or -7 and [negative] implies no real root	1 B1 M1 A1	for correct interim step; allow correct long division of f(x) by $(x-3)$ to obtain $2x^2 + 5x + 4$ with no remainder allow f(3) = 0 shown or equivalents for M1 and A1 using formula or completing square	4
	ii	divn of f(x) + 22 by $x - 2$ as far as $2x^3 - 4x^2$ used $2x^2 + 3x - 5$ obtained $(2x+5)(x-1)$ 1 and -2.5 o.e. <u>or</u> $2 \times 2^3 - 2^2 - 11 \times 2 - 12$ $16 - 4 - 22 - 12$ $x = 1$ is a root obtained by factor thm $x = -2.5$ obtained as root	M1 A1 M1 A1 +A1 M1 A1 B1 B2	or inspection eg $(x-2)(2x^2 \ldots\ldots -5)$ attempt at factorising/quad. formula/ compl. sq. <u>or</u> equivs using f(x) + 22 not just stated	5
	iii	cubic right way up crossing x axis only once (3, 0) and (0, −12) shown	G1 G1 G1	must have turning points must have max and min below x axis at intns with axes or in working (indep of cubic shape); ignore other intns	3

Mark Scheme 4752
June 2007

1	(i) $-\sqrt{3}$ (ii) $\frac{5}{3}\pi$	1 2	Accept any exact form accept $\frac{5\pi}{3}$, $1\frac{2}{3}\pi$. M1 π rad = 180° used correctly	3
2	$y' = 6\times\frac{3}{2}x^{\frac{1}{2}}$ or $9x^{\frac{1}{2}}$ o.e. $y'' = \frac{9}{2}x^{-\frac{1}{2}}$ o.e. $\sqrt{36} = 6$ used interim step to obtain $\frac{3}{4}$	2 1 M1 A1	1 if one error in coeff or power, or extra term f.t. their y′ only if fractional power f.t. their y″ www answer given	5
3	(i) $y = 2f(x)$ (ii) $y = f(x - 3)$	2 2	1 if 'y=' omitted [penalise only once] M1 for $y = kf(x)$, $k > 0$ M1 for $y = f(x + 3)$ or $y = f(x - k)$	4
4	(i) 11 27 or ft from their 11 (ii) 20	1 1 2	 M1 for 1 × 2 + 2× 3 + 3 × 4 soi, or 2,6,12 identified, or for substituting n = 3 in standard formulae	4
5	$\theta = 0.72$ o.e 13.6 [cm]	2 3	M1 for $9 = \frac{1}{2} \times 25 \times \theta$ No marks for using degrees unless attempt to convert B2 ft for 10 + 5 × their θ or for 3.6 found or M1 for $s = 5\theta$ soi	5
6	(i) $\log_a 1 = 0$, $\log_a a = 1$ (ii) showing both sides equivalent	1+1 3	NB, if not identified, accept only in this order M1 for correct use of 3rd law and M1 for correct use of 1st or 2nd law. Completion www A1. Condone omission of *a*.	5
7	(i) curve with increasing gradient any curve through (0, 1) marked (ii) 2.73	G1 G1 3	correct shape in both quadrants M1 for $x \log 3 = \log 20$ (or $x=\log_3 20$) and M1 for $x = \log 20 \div \log 3$ or B2 for other versions of 2.726833.. or B1 for other answer 2.7 to 2.8	5
8	(i) $2(1 - \sin^2\theta) + 7\sin\theta = 5$ (ii) $(2\sin\theta - 1)(\sin\theta - 3)$ $\sin\theta = \frac{1}{2}$ 30° and 150°	1 M1 DM1 A1 A1	for $\cos^2\theta + \sin^2\theta = 1$ o.e. used 1st and 3rd terms in expansion correct f.t. factors B1,B1 for each solution obtained by any valid method, ignore extra solns outside range, 30°, 150° plus extra soln(s) scores 1	5

9	**i**	$y' = 6x^2 - 18x + 12$ $= 12$ $y = 7$ when $x = 3$ tgt is $y - 7 = 12(x - 3)$ verifying $(-1, -41)$ on tgt	M1 M1 B1 M1 A1	condone one error subst of $x = 3$ in their y' f.t. their y and y' or B2 for showing line joining (3, 7) and (−1, −41) has gradient 12	5
	ii	$y' = 0$ soi quadratic with 3 terms $x = 1$ or 2 $y = 3$ or 2	M1 M1 A1 A1	Their y' Any valid attempt at solution or A1 for (1, 3) and A1 for (2,2) marking to benefit of candidate	4
	iii	cubic curve correct orientation touching x- axis only at (0.2,0) max and min correct curve crossing y axis only at −2	G1 G1 G1	 f.t.	3
10	**i**	970 [m]	4	M3 for attempt at trap rule ½×10×(28+22+2[19+14+11+12+16]) M2 with 1 error, M1 with 2 errors. Or M3 for 6 correct trapezia, M2 for 4 correct trapezia, M1 for 2 correct trapezia.	4
	ii	concave curve or line of traps is above curve (19+14+11 +11+12+16) × 10 830 to 880 incl.[m]	1 M1 A1	Accept suitable sketch M1 for 3 or more rectangles with values from curve.	3
	iii	$t = 10$, $v_{\text{model}} = 19.5$ difference = 0.5 compared with 3% of 19 = 0.57	B1 B1f.t.	or $\frac{0.5}{19} \times 100 \approx 2.6$	2
	iv	$28t - ½\, t^2 + 0.005t^3$ o.e. value at 60 [− value at 0] 960	M1 M1 A1	2 terms correct, ignore + c ft from integrated attempt with 3 terms	3
11	**ai**	13	1		1
	aii	120	2	M1 for attempt at AP formula ft their a, d or for 3 + 5 + …+ 21	2
	bi	$\frac{125}{1296}$	2	M1 for $\frac{1}{6} \times \left(\frac{5}{6}\right)^3$	2
	ii	$a = 1/6$, $r = 5/6$ s.o.i. $S_\infty = \frac{\frac{1}{6}}{1 - \frac{5}{6}}$ o.e.	1+1 1	If not specified, must be in right order	3
	iii	$\left(\frac{5}{6}\right)^{n-1} < 0.006$ $(n-1)\log_{10}\left(\frac{5}{6}\right) < \log_{10} 0.006$ $n - 1 > \frac{\log_{10} 0.006}{\log_{10}\left(\frac{5}{6}\right)}$ $n_{\min} = 30$ Or $\log(1/6) + \log(5/6)^{n-1} < \log 0.001$ $(n-1)\log(5/6) < \log(0.001/(1/6))$	M1 M1 DM1 B1 M1 M1	 condone omission of base, but not brackets NB change of sign must come at correct place	4

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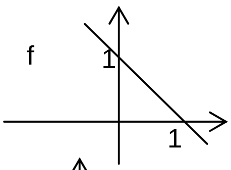
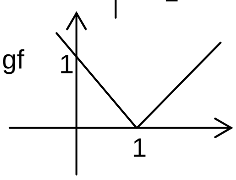
Section A

1 (i) $\frac{1}{2}(1 + 2x)^{-1/2} \times 2$ $= \dfrac{1}{\sqrt{1+2x}}$	M1 B1 A1 [3]	chain rule $\frac{1}{2}u^{-1/2}$ or $\frac{1}{2}(1 + 2x)^{-1/2}$ oe, but must resolve $\frac{1}{2} \times 2 = 1$
(ii) $y = \ln(1 - e^{-x})$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1-e^{-x}}.(-e^{-x})(-1)$ $= \dfrac{e^{-x}}{1-e^{-x}}$ $= \dfrac{1}{e^{x}-1}$ *	M1 B1 A1 E1 [4]	chain rule $\dfrac{1}{1-e^{-x}}$ or $\dfrac{1}{u}$ if substituting $u = 1 - e^{-x}$ $\times(-e^{-x})(-1)$ or e^{-x} www (may imply $\times e^{x}$ top and bottom)
2 gf$(x) = \lvert 1 - x \rvert$ f  gf 	B1 B1 B1 [3]	 intercepts must be labelled line must extend either side of each axis condone no labels, but line must extend to left of y axis
3(i) Differentiating implicitly: $(4y+1)\dfrac{dy}{dx} = 18x$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{18x}{4y+1}$ When $x = 1, y = 2, \dfrac{dy}{dx} = \dfrac{18}{9} = 2$	M1 A1 M1 A1cao [4]	$(4y+1)\dfrac{dy}{dx} = \ldots$ allow $4y+1\dfrac{dy}{dx} = \ldots$ condone omitted bracket if intention implied by following line. $4y\dfrac{dy}{dx}+1$ M1 A0 substituting $x = 1, y = 2$ into their derivative (provided it contains x's and y's). Allow unsupported answers.
(ii) $\dfrac{dy}{dx} = 0$ when $x = 0$ $\Rightarrow 2y^2 + y = 1$ $\Rightarrow 2y^2 + y - 1 = 0$ $\Rightarrow (2y - 1)(y + 1) = 0$ $\Rightarrow y = \frac{1}{2}$ or $y = -1$ So coords are $(0, \frac{1}{2})$ and $(0, -1)$	B1 M1 A1 A1 [4]	$x = 0$ from their numerator = 0 (must have a denominator) Obtaining correct quadratic and attempt to factorise or use quadratic formula $y = \dfrac{-1 \pm \sqrt{1-4\times-2}}{4}$ cao allow unsupported answers provided quadratic is shown

4(i) $T = 25 + ae^{-kt}$. When $t = 0$, $T = 100$ $\Rightarrow$ $100 = 25 + ae^0$ $\Rightarrow$ $a = 75$ When $t = 3$, $T = 80$ $\Rightarrow$ $80 = 25 + 75e^{-3k}$ $\Rightarrow$ $e^{-3k} = 55/75$ $\Rightarrow$ $-3k = \ln(55/75)$, $k = -\ln(55/75) / 3$ $= 0.1034$	M1 A1 M1 M1 A1cao [5]	substituting $t = 0$ and $T = 100$ into their equation (even if this is an incorrect version of the given equation) substituting $t = 3$ and $T = 80$ into (their) equation taking lns correctly at any stage 0.1 or better or $-\frac{1}{3}\ln(\frac{55}{75})$ o.e. if final answer
(ii) (A) $T = 25 + 75e^{-0.1034\times5}$ $= 69.72$ (B) 25°C	M1 A1 B1cao [3]	substituting $t = 5$ into their equation 69.5 to 70.5, condone inaccurate rounding due to value of k.
5 $n = 1$, $n^2 + 3n + 1 = 5$ prime $n = 2$, $n^2 + 3n + 1 = 11$ prime $n = 3$, $n^2 + 3n + 1 = 19$ prime $n = 4$, $n^2 + 3n + 1 = 29$ prime $n = 5$, $n^2 + 3n + 1 = 41$ prime $n = 6$, $n^2 + 3n + 1 = 55$ not prime so statement is false	M1 E1 [2]	One or more trials shown finding a counter-example – must state that it is not prime.
6 (i) $-\pi/2 < \arctan x < \pi/2$ $\Rightarrow$ $-\pi/4 < f(x) < \pi/4$ $\Rightarrow$ range is $-\pi/4$ to $\pi/4$	M1 A1cao [2]	$\pi/4$ or $-\pi/4$ or 45 seen not $\leq$
(ii) $y = ½ \arctan x$ $x \leftrightarrow y$ $x = ½ \arctan y$ $\Rightarrow$ $2x = \arctan y$ $\Rightarrow$ $\tan 2x = y$ $\Rightarrow$ $y = \tan 2x$ *either* $\frac{dy}{dx} = 2\sec^2 2x$	M1 A1cao M1 A1cao	$\tan(\arctan y$ or $x) = y$ or x derivative of tan is $\sec^2$ used
or $y = \frac{\sin 2x}{\cos 2x} \Rightarrow \frac{dy}{dx} = \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$ $= \frac{2}{\cos^2 2x}$	M1 A1cao	quotient rule (need not be simplified but mark final answer)
When $x = 0$, $dy/dx = 2$	B1 [5]	www
(iii) So gradient of $y = ½ \arctan x$ is ½ .	B1ft [1]	ft their '2', but not 1 or 0 or ∞

Section B

7(i) Asymptote when $1 + 2x^3 = 0$ $\Rightarrow$ $2x^3 = -1$ $\Rightarrow$ $x = -\frac{1}{\sqrt[3]{2}}$ $= -0.794$	M1 A1 A1cao [3]	 oe, condone $\pm\frac{1}{\sqrt[3]{2}}$ if positive root is rejected must be to 3 s.f.
(ii) $\frac{dy}{dx} = \frac{(1+2x^3).2x - x^2.6x^2}{(1+2x^3)^2}$ $= \frac{2x+4x^4-6x^4}{(1+2x^3)^2}$ $= \frac{2x-2x^4}{(1+2x^3)^2}$ * $dy/dx = 0$ when $2x(1 - x^3) = 0$ $\Rightarrow$ $x = 0, y = 0$ or $x = 1$, $y = 1/3$	M1 A1 E1 M1 B1 B1 B1 B1 [8]	Quotient or product rule: (udv–vdu M0) $2x(1+2x^3)^{-1} + x^2(-1)(1+2x^3)^{-2}.6x^2$ allow one slip on derivatives correct expression – condone missing bracket if if intention implied by following line derivative = 0 $x = 0$ or 1 – allow unsupported answers $y = 0$ and 1/3 SC–1 for setting denom = 0 or extra solutions (e.g. $x = -1$)
(iii) $A = \int_0^1 \frac{x^2}{1+2x^3} dx$	M1	Correct integral and limits – allow $\int_1^0$
either $= \left[\frac{1}{6}\ln(1+2x^3)\right]_0^1$ $= \frac{1}{6}\ln 3$ *	M1 A1 M1 E1	$k\ln(1 + 2x^3)$ $k = 1/6$ substituting limits dep previous M1 www
or let $u = 1 + 2x^3 \Rightarrow du = 6x^2dx$ $\Rightarrow$ $A = \int_1^3 \frac{1}{6}.\frac{1}{u} du$ $= \left[\frac{1}{6}\ln u\right]_1^3$ $= \frac{1}{6}\ln 3$ *	 M1 A1 M1 E1 [5]	 $\frac{1}{6u}$ $\frac{1}{6}\ln u$ substituting correct limits (but must have used substitution) www

8 (i) $x \cos 2x = 0$ when $x = 0$ or $\cos 2x = 0$ $\Rightarrow \quad 2x = \pi/2$ $\Rightarrow \quad x = \frac{1}{4}\pi$ $\Rightarrow$ P is $(\pi/4, 0)$	M1 M1 A1 [3]	$\cos 2x = 0$ or $x = \frac{1}{2}\cos^{-1}0$ $x = 0.785..$ or 45 is M1 M1 A0
(ii) $f(-x) = -x\cos(-2x)$ $= -x\cos 2x$ $= -f(x)$ Half turn symmetry about O.	M1 E1 B1 [3]	$-x\cos(-2x)$ $= -x\cos 2x$ Must have two of: rotational, order 2, about O, (half turn = rotational order 2)
(iii) $f'(x) = \cos 2x - 2x\sin 2x$	M1 A1 [2]	product rule
(iv) $f'(x) = 0 \Rightarrow \cos 2x = 2x\sin 2x$ $\Rightarrow \quad 2x\dfrac{\sin 2x}{\cos 2x} = 1$ $\Rightarrow \quad x\tan 2x = \frac{1}{2}$ *	 M1 E1 [2]	 $\dfrac{\sin}{\cos} = \tan$ www
(v) $f'(0) = \cos 0 - 2.0.\sin 0 = 1$ $f''(x) = -2\sin 2x - 2\sin 2x - 4x\cos 2x$ $= -4\sin 2x - 4x\cos 2x$ $\Rightarrow \quad f''(0) = -4\sin 0 - 4.0.\cos 0 = 0$	B1ft M1 A1 E1 [4]	allow ft on (their) product rule expression product rule on $(2)x\sin 2x$ correct expression – mark final expression www
(vi) Let $u = x$, $dv/dx = \cos 2x$ $\Rightarrow v = \frac{1}{2}\sin 2x$ $\int_0^{\pi/4} x\cos 2x\,dx = \left[\frac{1}{2}x\sin 2x\right]_0^{\pi/4} - \int_0^{\pi/4}\frac{1}{2}\sin 2x\,dx$ $= \frac{\pi}{8} + \left[\frac{1}{4}\cos 2x\right]_0^{\pi/4}$ $= \frac{\pi}{8} - \frac{1}{4}$ Area of region enclosed by curve and x-axis between $x = 0$ and $x = \pi/4$	M1 A1 A1 M1 A1 B1 [6]	Integration by parts with $u = x$, $dv/dx = \cos 2x$ $\left[\frac{1}{4}\cos 2x\right]$ - sign consistent with their previous line substituting limits – dep using parts www or graph showing correct area – condone P for $\pi/4$.

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Section A

1 $\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 3$ $\Rightarrow R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$ $\tan\alpha = 3 \Rightarrow \alpha = 71.57°$ $\sqrt{10}\sin(\theta - 71.57°) = 1$ $\Rightarrow \theta - 71.57° = \sin^{-1}(1/\sqrt{10})$ $\theta - 71.57° = 18.43°, 161.57°$ $\Rightarrow \theta = 90°,$ $233.1°$	M1 B1 M1 A1 M1 B1 A1 [7]	equating correct pairs oe ft www cao (71.6°or better) oe ft R, α www and no others in range (MR-1 for radians)
2 Normal vectors are $\begin{pmatrix}2\\3\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\Rightarrow \begin{pmatrix}2\\3\\4\end{pmatrix}.\begin{pmatrix}1\\-2\\1\end{pmatrix} = 2-6+4=0$ $\Rightarrow$ planes are perpendicular.	B1 B1 M1 E1 [4]	
3 **(i)** $y = \ln x \Rightarrow x = e^y$ $\Rightarrow V = \int_0^2 \pi x^2 dy$ $= \int_0^2 \pi(e^y)^2 dy = \int_0^2 \pi e^{2y} dy$ *	B1 M1 E1 [3]	
(ii) $\int_0^2 \pi e^{2y} dy = \pi\left[\frac{1}{2}e^{2y}\right]_0^2$ $= ½\,\pi(e^4 - 1)$	B1 M1 A1 [3]	½ e^{2y} substituting limits in $k\pi e^{2y}$ or equivalent, but must be exact and evaluate e^0 as 1.
4 $x = \frac{1}{t} - 1 \Rightarrow \frac{1}{t} = x+1$ $\Rightarrow t = \frac{1}{x+1}$ $\Rightarrow y = \frac{2+\frac{1}{x+1}}{1+\frac{1}{x+1}} = \frac{2x+2+1}{x+1+1} = \frac{2x+3}{x+2}$	M1 A1 M1 E1	Solving for t in terms of x or y Subst their t which must include a fraction, clearing subsidiary fractions/ changing the subject oe www
or $\frac{3+2x}{2+x} = \frac{3+\frac{2-2t}{t}}{2+\frac{1-t}{t}}$ $= \frac{3t+2-2t}{2t+1-t}$ $= \frac{t+2}{t+1} = y$	M1 A1 M1 E1 [4]	substituting for x or y in terms of t clearing subsidiary fractions/changing the subject

5 $\mathbf{r} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1-\lambda\\2+2\lambda\\-1+3\lambda\end{pmatrix}$		
When $x = -1$, $1 - \lambda = -1$, $\Rightarrow \lambda = 2$	M1	Finding λ or μ
$\Rightarrow y = 2 + 2\lambda = 6$, $z = -1 + 3\lambda = 5$		
$\Rightarrow$ point lies on first line	E1	checking other two coordinates
$\mathbf{r} = \begin{pmatrix}0\\6\\3\end{pmatrix} + \mu\begin{pmatrix}1\\0\\-2\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}\mu\\6\\3-2\mu\end{pmatrix}$		
When $x = -1$, $\mu = -1$,		
$\Rightarrow y = 6$, $z = 3 - 2\mu = 5$	E1	checking other two co-ordinates
$\Rightarrow$ point lies on second line		
Angle between $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ is θ, where	M1	Finding angle between correct vectors
$\cos\theta = \dfrac{-1\times1+2\times0+3\times-2}{\sqrt{14}.\sqrt{5}}$	M1	use of formula
$= -\dfrac{7}{\sqrt{70}}$	A1	$\pm\dfrac{7}{\sqrt{70}}$
$\Rightarrow \theta = 146.8°$		
$\Rightarrow$ acute angle is 33.2°	A1cao [7]	Final answer must be acute angle
6(i) $A \approx 0.5[\dfrac{(1.1696+1.0655}{2}+1.1060]$	M1	Correct expression for trapezium rule
= 1.11 (3 s.f.)	A1 cao [2]	
(ii) $(1+e^{-x})^{1/2} = 1+\dfrac{1}{2}e^{-x}+\dfrac{\frac{1}{2}.-\frac{1}{2}}{2!}(e^{-x})^2+...$	M1 A1	Binomial expansion with p = ½ Correct coeffs
$\approx 1+\dfrac{1}{2}e^{-x}-\dfrac{1}{8}e^{-2x}$ *	E1 [3]	
(iii) $I = \int_1^2 (1+\dfrac{1}{2}e^{-x}-\dfrac{1}{8}e^{-2x})dx$		
$= \left[x-\dfrac{1}{2}e^{-x}+\dfrac{1}{16}e^{-2x}\right]_1^2$	M1	integration
$= (2-\dfrac{1}{2}e^{-2}+\dfrac{1}{16}e^{-4})-(1-\dfrac{1}{2}e^{-1}+\dfrac{1}{16}e^{-2})$	A1	substituting limits into correct expression
= 1.9335 − 0.8245		
= 1.11 (3 s.f.)	A1 [3]	

Section B

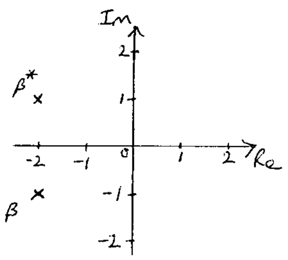
7 (a) (i) $P_{\max} = \dfrac{2}{2-1} = 2$ $P_{\min} = \dfrac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \dfrac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \quad \dfrac{dP}{dt} = -2(2-\sin t)^{-2}.-\cos t$ $= \dfrac{2\cos t}{(2-\sin t)^2}$ $\dfrac{1}{2}P^2\cos t = \dfrac{1}{2}\dfrac{4}{(2-\sin t)^2}\cos t$ $= \dfrac{2\cos t}{(2-\sin t)^2} = \dfrac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\ldots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\dfrac{1}{P(2P-1)} = \dfrac{A}{P} + \dfrac{B}{2P-1}$ $= \dfrac{A(2P-1)+BP}{P(2P-1)}$ $\Rightarrow \quad 1 = A(2P-1) + BP$ $P = 0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = ½ \Rightarrow 1 = A.0 + ½B \Rightarrow B = 2$ So $\dfrac{1}{P(2P-1)} = -\dfrac{1}{P} + \dfrac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\dfrac{dP}{dt} = \dfrac{1}{2}(2P-P^2)\cos t$ $\Rightarrow \quad \int \dfrac{1}{2P^2-P}dP = \int \dfrac{1}{2}\cos t\,dt$ $\Rightarrow \quad \int (\dfrac{2}{2P-1} - \dfrac{1}{P})dP = \int \dfrac{1}{2}\cos t\,dt$ $\Rightarrow \quad \ln(2P-1) - \ln P = ½\sin t + c$ When $t = 0$, $P = 1$ $\Rightarrow \quad \ln 1 - \ln 1 = ½\sin 0 + c \Rightarrow c = 0$ $\Rightarrow \ln(\dfrac{2P-1}{P}) = \dfrac{1}{2}\sin t$ *	M1 A1 A1 B1 E1 [5]	separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) ½ $\sin t$ finding constant = 0
(iii) $P_{\max} = \dfrac{1}{2-e^{1/2}} = 2.847$ $P_{\min} = \dfrac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

8 (i) $\frac{dy}{dx}=\frac{10\cos\theta+10\cos 2\theta}{-10\sin\theta-10\sin 2\theta}$ $= -\frac{\cos\theta+\cos 2\theta}{\sin\theta+\sin 2\theta}$ * When $\theta=\pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3+\cos 2\pi/3}{\sin\pi/3+\sin 2\pi/3}$ $= 0$ as $\cos\pi/3 = ½$, $\cos 2\pi/3 = -½$ At A $x = 10\cos\pi/3 + 5\cos 2\pi/3$ $= 2½$ $y = 10\sin\pi/3 + 5\sin 2\pi/3 = 15\sqrt{3}/2$	M1 E1 B1 M1 A1 A1 [6]	$dy/d\theta \div dx/d\theta$ or solving $\cos\theta+\cos 2\theta=0$ substituting $\pi/3$ into x or y 2½ $15\sqrt{3}/2$ (condone 13 or better)
(ii) $x^2+y^2=(10\cos\theta+5\cos 2\theta)^2+(10\sin\theta+5\sin 2\theta)^2$ $= 100\cos^2\theta+100\cos\theta\cos 2\theta+25\cos^2 2\theta$ $+100\sin^2\theta+100\sin\theta\sin 2\theta+25\sin^2 2\theta$ $= 100 + 100\cos(2\theta - \theta) + 25$ $= 125 + 100\cos\theta$ *	B1 M1 DM1 E1 [4]	 expanding $\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = \cos(2\theta - \theta)$ or substituting for $\sin 2\theta$ and $\cos 2\theta$
(iii) Max $\sqrt{125+100} = 15$ min $\sqrt{125-100} = 5$	B1 B1 [2]	
(iv) $2\cos^2\theta+2\cos\theta - 1 = 0$ $\cos\theta = \frac{-2\pm\sqrt{12}}{4} = \frac{-2\pm 2\sqrt{3}}{4}$ At B, $\cos\theta = \frac{-1+\sqrt{3}}{2}$ $OB^2 = 125 + 50(-1 + \sqrt{3}) = 75 + 50\sqrt{3} = 161.6\ldots$ $\Rightarrow$ $OB = \sqrt{161.6\ldots} = 12.7$ (m)	 M1 A1 M1 A1 [4]	 quadratic formula or θ=68.53° or 1.20radians, correct root selected or OB=$10\sin\theta+5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao

Paper B Comprehension

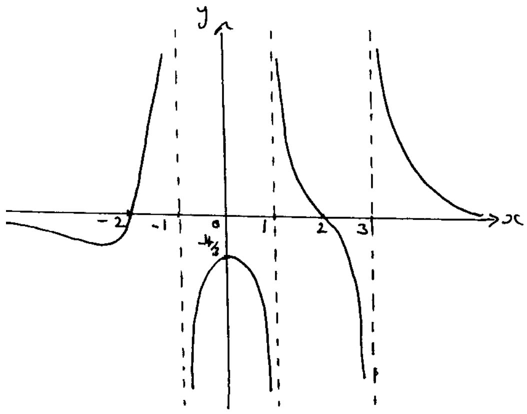
1)	M $(a\pi, 2a)$, $\theta=\pi$ N $(4a\pi, 0)$, $\theta=4\pi$	B1 B1	
2)	Compare the equations with equations given in text, $x = a\theta - b\sin\theta, y = b\cos\theta$	M1	Seeing a=7, b=0.25
	Wavelength = $2\pi a = 14\pi$ (≈ 44) Height = $2b$ = 0.5	A1 B1	
3i)	Wavelength = 20 $\Rightarrow a = \frac{10}{\pi}$ (=3.18…) Height = 2 $\Rightarrow b = 1$	B1 B1	
ii)	In this case, the ratio is observed to be 12:8 Trough length : Peak length = $\pi a + 2b : \pi a - 2b$ and this is $(10 + 2 \times 1) : (10 - 2 \times 1)$ So the curve is consistent with the parametric equations	B1 M1 A1	 substituting
4i)	$x = a\theta$, $y = b\cos\theta$ is the sine curve V and $x = a\theta - b\sin\theta, y = b\cos\theta$ is the curtate cycloid U. The sine curve is above mid-height for half its wavelength (or equivalent)	B1	
ii)	$d = a\theta-(a\theta-b\sin\theta)$ $\theta=\pi/2$, $d=\left(\frac{\pi a}{2}\right)-\left(\frac{\pi a}{2}-b\right)=b$	M1 E1	Subtraction Using $\theta=\pi/2$
iii)	Because b is small compared to a, the two curves are close together.	M1 E1	Comparison attempted Conclusion
5)	Measurements on the diagram give Wavelength $\approx$ 3.5cm, Height $\approx$ 0.8cm $\frac{\text{Wavelength}}{\text{Height}} \approx \frac{3.5}{0.8} = 4.375$ Since 4.375 < 7, the wave will have become unstable and broken.	B1 M1 E1	measurements/reading ratio [18]

Mark Scheme 4755
June 2007

Section A			
1(i)	$\mathbf{M}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1 **[2]**	Attempt to find determinant
1(ii)	20 square units	B1 **[1]**	$2\times$ their determinant
2	$\lvert z-(3-2\mathrm{j})\rvert = 2$	B1 B1 B1 **[3]**	$z \pm (3-2\mathrm{j})$ seen radius = 2 seen Correct use of modulus
3	$x^3-4=(x-1)\left(Ax^2+Bx+C\right)+D$ $\Rightarrow x^3-4=Ax^3+(B-A)x^2+(C-B)x-C+D$ $\Rightarrow A=1, B=1, C=1, D=-3$	M1 B1 B1 B1 B1 **[5]**	Attempt at equating coefficients or long division (may be implied) For A = 1 B1 for each of B, C and D
4(i)		B1 B1 **[2]**	One for each correctly shown. s.c. B1 if not labelled correctly but position correct
4(ii)	$\alpha\beta=(1-2\mathrm{j})(-2-\mathrm{j})=-4+3\mathrm{j}$	M1 A1 **[2]**	Attempt to multiply
4(iii)	$\frac{\alpha+\beta}{\beta}=\frac{(\alpha+\beta)\beta^*}{\beta\beta^*}=\frac{\alpha\beta^*+\beta\beta^*}{\beta\beta^*}=\frac{5\mathrm{j}+5}{5}=\mathrm{j}+1$	M1 A1 A1 **[3]**	Appropriate attempt to use conjugate, or other valid method 5 in denominator or correct working consistent with their method All correct

5	**Scheme A** $w = 3x \Rightarrow x = \frac{w}{3}$	B1	Substitution. For substitution $x = 3w$ give B0 but then follow through for a maximum of 3 marks
	$\Rightarrow \left(\frac{w}{3}\right)^3 + 3\left(\frac{w}{3}\right)^2 - 7\left(\frac{w}{3}\right) + 1 = 0$	M1	Substitute into cubic
	$\Rightarrow w^3 + 9w^2 - 63w + 27 = 0$	A3	Correct coefficients consistent with x^3 coefficient, minus 1 each error
	OR	A1 **[6]**	Correct cubic equation c.a.o.
	Scheme B $\alpha + \beta + \gamma = -3$ $\alpha\beta + \alpha\gamma + \beta\gamma = -7$ $\alpha\beta\gamma = -1$	M1	Attempt to find sums and products of roots (at least two of three)
	Let new roots be k, l, m then $k + l + m = 3(\alpha + \beta + \gamma) = -9 = \frac{-B}{A}$ $kl + km + lm = 9(\alpha\beta + \alpha\gamma + \beta\gamma) = -63 = \frac{C}{A}$	M1	Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation
	$klm = 27\alpha\beta\gamma = -27 = \frac{-D}{A}$	A3	Correct coefficients consistent with x^3 coefficient, minus 1 each error
	$\Rightarrow \omega^3 + 9\omega^2 - 63\omega + 27 = 0$	A1 **[6]**	Correct cubic equation c.a.o.
6(i)	$\frac{1}{r+2} - \frac{1}{r+3} = \frac{r+3-(r+2)}{(r+2)(r+3)} = \frac{1}{(r+2)(r+3)}$	M1 A1 **[2]**	Attempt at common denominator
6(ii)	$\sum_{r=1}^{50} \frac{1}{(r+2)(r+3)} = \sum_{r=1}^{50} \left[\frac{1}{r+2} - \frac{1}{r+3}\right]$	M1	Correct use of part (i) (may be implied)
	$= \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + ...$	M1,	First two terms in full
	$+ \left(\frac{1}{51} - \frac{1}{52}\right) + \left(\frac{1}{52} - \frac{1}{53}\right)$	M1	Last two terms in full (allow in terms of n)
	$= \frac{1}{3} - \frac{1}{53} = \frac{50}{159}$	A1 **[4]**	Give B4 for correct without working Allow 0.314 (3s.f.)

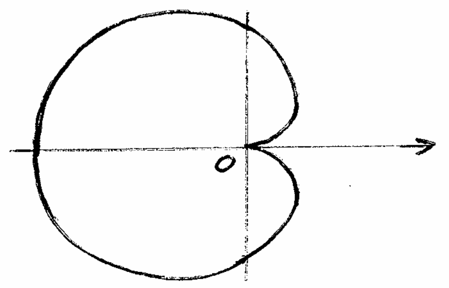
7	$\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$ $n = 1$, LHS = RHS = 1 Assume true for $n = k$ Next term is 3^k Add to both sides $\text{RHS} = \frac{3^k - 1}{2} + 3^k$ $= \frac{3^k - 1 + 2\times 3^k}{2}$ $= \frac{3\times 3^k - 1}{2}$ $= \frac{3^{k+1} - 1}{2}$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$. Since it is true for $k = 1$, it is true for $k = 1, 2, 3$ and so true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	 Assuming true for k Attempt to add 3^k to RHS c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
8(i)	$(2,\ 0)$, $(-2,\ 0)$, $\left(0,\ \dfrac{-4}{3}\right)$	B1 B1 B1 **[3]**	1 mark for each s.c. B2 for $2,\ -2,\ \dfrac{-4}{3}$
8(ii)	$x=3,\ x=-1,\ x=1, y=0$	B4 **[4]**	Minus 1 for each error
8(iii)	Large positive *x*, $y \to 0^+$, approach from above (e.g. consider $x=100$) Large negative *x*, $y \to 0^-$, approach from below (e.g. consider $x=-100$)	B1 B1 M1 **[3]**	Direction of approach must be clear for each B mark Evidence of method required
8(iv)	Curve 4 branches correct Asymptotes correct and labelled Intercepts labelled 	B2 B1 B1 **[4]**	Minus 1 each error, min 0

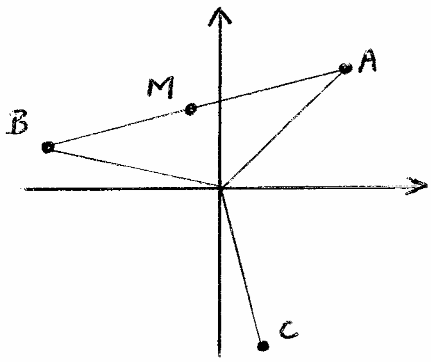
9(i)	$x = 1 - 2\mathrm{j}$	B1 **[1]**	
9(ii)	Complex roots occur in conjugate pairs. A cubic has three roots, so one must be real. Or, valid argument involving graph of a cubic or behaviour for large positive and large negative x.	E1 **[1]**	
9(iii)	**Scheme A**		
	$(x-1-2\mathrm{j})(x-1+2\mathrm{j}) = x^2 - 2x + 5$ $(x-\alpha)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$ comparing constant term: $-5\alpha = 15 \Rightarrow \alpha = -3$	M1 A1 A1(ft) M1 M1	Attempt to use factor theorem Correct factors Correct quadratic(using their factors) Use of factor involving real root Comparing constant term
	So real root is $x = -3$	A1(ft)	From their quadratic
	$(x+3)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$ $\Rightarrow x^3 + x^2 - x + 15 = x^3 + Ax^2 + Bx + 15$ $\Rightarrow A = 1, B = -1$	M1 M1 A1 **[9]**	Expand LHS Compare coefficients 1 mark for both values
	OR **Scheme B**		
	Product of roots $= -15$ $(1+2\mathrm{j})(1-2\mathrm{j}) = 5$	M1 A1 M1 A1	Attempt to use product of roots Product is –15 Multiplying complex roots
	$\Rightarrow 5\alpha = -15$	A1	
	$\Rightarrow \alpha = -3$	A1	c.a.o.
	Sum of roots $= -A$ $\Rightarrow -A = 1 + 2j + 1 - 2j - 3 = -1 \Rightarrow A = 1$	M1	Attempt to use sum of roots
	Substitute root $x = -3$ into cubic $(-3)^3 + (-3)^2 - 3B + 15 = 0 \Rightarrow B = -1$	M1	Attempt to substitute, or to use sum
	A = 1 and B = -1	A1 **[9]**	c.a.o.
	OR **Scheme C**		
	$\alpha = -3$	6	As scheme A, or other valid method
	$(1+2\mathrm{j})^3 + A(1+2\mathrm{j})^2 + B(1+2\mathrm{j}) + 15 = 0$	M1	Attempt to substitute root
	$\Rightarrow A(-3+4\mathrm{j}) + B(1+2\mathrm{j}) + 4 - 2\mathrm{j} = 0$ $\Rightarrow -3A + B + 4 = 0$ and $4A + 2B - 2 = 0$	M1	Attempt to equate real and imaginary parts, or equivalent.
	$\Rightarrow A = 1$ and $B = -1$	A1 **[9]**	c.a.o.

Section B (continued)			
10(i)	$\mathbf{AB}=\begin{pmatrix}1&-2&k\\2&1&2\\3&2&-1\end{pmatrix}\begin{pmatrix}-5&-2+2k&-4-k\\8&-1-3k&-2+2k\\1&-8&5\end{pmatrix}$ $=\begin{pmatrix}k-21&0&0\\0&k-21&0\\0&0&k-21\end{pmatrix}$ $n=21$	M1 A1 **[2]**	Attempt to multiply matrices (can be implied)
10(ii)	$\mathbf{A}^{-1}=\frac{1}{k-21}\begin{pmatrix}-5&-2+2k&-4-k\\8&-1-3k&-2+2k\\1&-8&5\end{pmatrix}$ $k\neq 21$	M1 M1 A1 A1 **[4]**	Use of **B** Attempt to use their answer to (i) Correct inverse Accept n in place of 21 for full marks
10 (iii)	**Scheme A** $\frac{1}{-20}\begin{pmatrix}-5&0&-5\\8&-4&0\\1&-8&5\end{pmatrix}\begin{pmatrix}1\\12\\3\end{pmatrix}=\frac{1}{-20}\begin{pmatrix}-20\\-40\\-80\end{pmatrix}=\begin{pmatrix}1\\2\\4\end{pmatrix}$ $x=1,\ y=2,\ z=4$ OR **Scheme B** Attempt to eliminate 2 variables Substitute in their value to attempt to find others $x=1,\ y=2,\ z=4$	M1 M1 A3 **[5]** M1 M1 A3 **[5]**	Attempt to use inverse Their inverse with $k=1$ One for each correct (ft) s.c. award 2 marks only for $x=1,\ y=2,\ z=4$ with no working.
			Section B Total: 36
			Total: 72

Mark Scheme 4756
June 2007

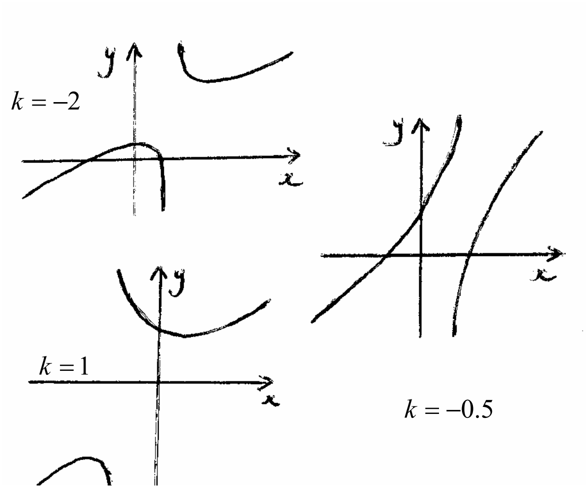
1(a)(i)	 	B2 **2**	Must include a sharp point at O and have infinite gradient at $\theta = \pi$ Give B1 for r increasing from zero for $0 < \theta < \pi$, or decreasing to zero for $-\pi < \theta < 0$
(ii)	Area is $\int \frac{1}{2} r^2 \, d\theta = \int_0^{\frac{1}{2}\pi} \frac{1}{2} a^2 (1 - \cos\theta)^2 \, d\theta$ $= \frac{1}{2} a^2 \int_0^{\frac{1}{2}\pi} \left(1 - 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta) \right) d\theta$ $= \frac{1}{2} a^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta \right]_0^{\frac{1}{2}\pi}$ $= \frac{1}{2} a^2 (\frac{3}{4}\pi - 2)$	M1 A1 B1 B1B1 ft B1 **6**	For integral of $(1 - \cos\theta)^2$ For a correct integral expression including limits *(may be implied by later work)* Using $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ Integrating $a + b\cos\theta$ and $k\cos 2\theta$ Accept $0.178a^2$
(b)	Put $x = 2\sin\theta$ Integral is $\int_0^{\frac{1}{6}\pi} \frac{1}{(4 - 4\sin^2\theta)^{\frac{3}{2}}} (2\cos\theta) \, d\theta$ $= \int_0^{\frac{1}{6}\pi} \frac{2\cos\theta}{8\cos^3\theta} \, d\theta = \int_0^{\frac{1}{6}\pi} \frac{1}{4}\sec^2\theta \, d\theta$ $= \left[\frac{1}{4}\tan\theta \right]_0^{\frac{1}{6}\pi}$ $= \frac{1}{4} \times \frac{1}{\sqrt{3}} = \frac{1}{4\sqrt{3}}$	M1 A1 M1 A1 ag **4**	or $x = 2\cos\theta$ Limits not required For $\int \sec^2\theta \, d\theta = \tan\theta$ *SR* If $x = 2\tanh u$ is used M1 for $\frac{1}{4}\sinh(\frac{1}{2}\ln 3)$ A1 for $\frac{1}{8}(\sqrt{3} - \frac{1}{\sqrt{3}}) = \frac{1}{4\sqrt{3}}$ (max 2 / 4)
(c)(i)	$f'(x) = \frac{-2}{\sqrt{1 - 4x^2}}$	B2 **2**	Give B1 for any non-zero real multiple of this (or for $\frac{-2}{\sin y}$ etc)
(ii)	$f'(x) = -2(1 - 4x^2)^{-\frac{1}{2}}$ $= -2(1 + 2x^2 + 6x^4 + ...)$ $f(x) = C - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$ $f(0) = \frac{1}{2}\pi \Rightarrow C = \frac{1}{2}\pi$ $f(x) = \frac{1}{2}\pi - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$	M1 A1 M1 A1 **4**	Binomial expansion (3 terms, $n = -\frac{1}{2}$) Expansion of $(1 - 4x^2)^{-\frac{1}{2}}$ correct (accept unsimplified form) Integrating series for $f'(x)$ Must obtain a non-zero x^5 term C not required

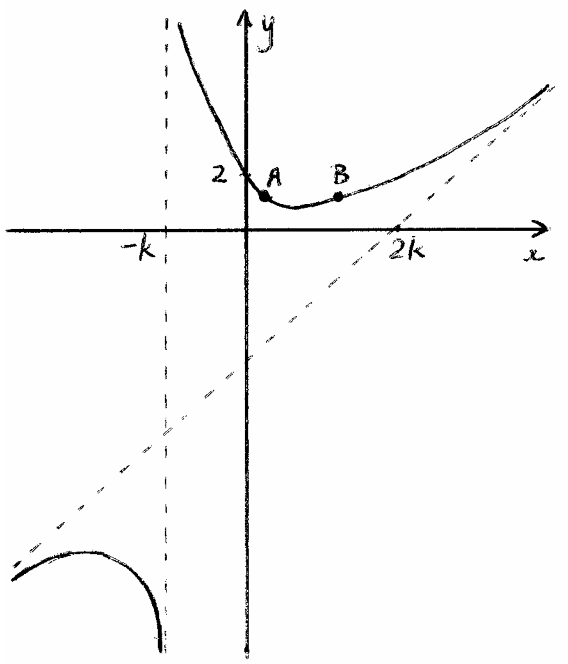
	OR by repeated differentiation Finding $f^{(5)}(x)$ M1 Evaluating $f^{(5)}(0)$ $(=-288)$ M1 $f'(x)=-2-4x^2-12x^4+\dots$ A1 ft $f(x)=\frac{1}{2}\pi-2x-\frac{4}{3}x^3-\frac{12}{5}x^5+\dots$ A1		Must obtain a non-zero value ft from (c)(i) when B1 given

2 (a)	$(\cos\theta + \mathrm{j}\sin\theta)^5$ $= c^5 + 5\mathrm{j}c^4 s - 10c^3 s^2 - 10\mathrm{j}c^2 s^3 + 5cs^4 + \mathrm{j}s^5$ Equating imaginary parts $\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$ $= 5(1-s^2)^2 s - 10(1-s^2)s^3 + s^5$ $= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$ $= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$	M1 M1 A1 M1 A1 ag **5**	
(b)(i)	$\|-2+2\mathrm{j}\| = \sqrt{8}$, $\arg(-2+2\mathrm{j}) = \frac{3}{4}\pi$ $r = \sqrt{2}$ $\theta = \frac{1}{4}\pi$ $\theta = \frac{11}{12}\pi,\ -\frac{5}{12}\pi$	B1B1 B1 ft B1 ft M1 A1 **6**	Accept 2.8; 2.4, 135° (Implies B1 for $\sqrt{8}$) One correct (Implies B1 for $\frac{3}{4}\pi$) Adding or subtracting $\frac{2}{3}\pi$ Accept $\theta = \frac{1}{4}\pi + \frac{2}{3}k\pi,\ k = 0, 1, -1$
(ii)		B2 **2**	Give B1 for two of B, C, M in the correct quadrants Give B1 ft for all four points in the correct quadrants
(iii)	$\|w\| = \frac{1}{2}\sqrt{2}$ $\arg w = \frac{1}{2}(\frac{1}{4}\pi + \frac{11}{12}\pi) = \frac{7}{12}\pi$	B1 ft B1 **2**	Accept 0.71 Accept 1.8
(iv)	$\|w^6\| = (\frac{1}{2}\sqrt{2})^6 = \frac{1}{8}$ $\arg(w^6) = 6 \times \frac{7}{12}\pi = \frac{7}{2}\pi$ $w^6 = \frac{1}{8}(\cos\frac{7}{2}\pi + \mathrm{j}\sin\frac{7}{2}\pi)$ $= -\frac{1}{8}\mathrm{j}$	M1 A1 ft A1 **3**	Obtaining either modulus or argument Both correct (ft) Allow from $\arg w = \frac{1}{4}\pi$ etc
			SR If B, C interchanged on diagram (ii) B1 (iii) B1 B1 for $-\frac{1}{12}\pi$ (iv) M1A1A1

3 (i)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (3-\lambda)[(3-\lambda)(-4-\lambda)-4]$ $-5[5(-4-\lambda)+4]+2[-10-2(3-\lambda)]$ $=(3-\lambda)(-16+\lambda+\lambda^2)-5(-16-5\lambda)+2(-16+2\lambda)$ $=-48+19\lambda+2\lambda^2-\lambda^3+80+25\lambda-32+4\lambda$ $=48\lambda+2\lambda^2-\lambda^3$ Characteristic equation is $\lambda^3-2\lambda^2-48\lambda=0$	M1 A1 M1 A1 ag **4**	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$ Any correct form Simplification
(ii)	$\lambda(\lambda-8)(\lambda+6)=0$ Other eigenvalues are $8,\ -6$ When $\lambda=8$, $3x+5y+2z=8x$ $(\ 5x+3y-2z=8y\ )$ $2x-2y-4z=8z$ $y=x$ and $z=0$; eigenvector is $\begin{pmatrix}1\\1\\0\end{pmatrix}$ When $\lambda=-6$, $3x+5y+2z=-6x$ $5x+3y-2z=-6y$ $y=-x$, $z=-2x$; eigenvector is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$	M1 A1 M1 M1 A1 M1 M1 A1 **8**	Solving to obtain a non-zero value Two independent equations Obtaining a non-zero eigenvector *($-5x+5y+2z=8x$ etc can earn M0M1)* Two independent equations Obtaining a non-zero eigenvector
(iii)	$\mathbf{P}=\begin{pmatrix}1&1&1\\-1&1&-1\\1&0&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}0&0&0\\0&8&0\\0&0&-6\end{pmatrix}^2$ $=\begin{pmatrix}0&0&0\\0&64&0\\0&0&36\end{pmatrix}$	B1 ft M1 A1 **3**	B0 if **P** is clearly singular Order must be consistent with **P** when B1 has been earned
(iv)	$\mathbf{M}^3-2\mathbf{M}^2-48\mathbf{M}=\mathbf{0}$ $\mathbf{M}^3=2\mathbf{M}^2+48\mathbf{M}$ $\mathbf{M}^4=2\mathbf{M}^3+48\mathbf{M}^2$ $=2(2\mathbf{M}^2+48\mathbf{M})+48\mathbf{M}^2$ $=52\mathbf{M}^2+96\mathbf{M}$	M1 M1 A1 **3**	

4 (a)	$\int_0^1 \frac{1}{\sqrt{9x^2+16}}\,\mathrm{d}x = \left[\frac{1}{3}\operatorname{arsinh}\frac{3x}{4} \right]_0^1$ $= \frac{1}{3}\operatorname{arsinh}\frac{3}{4}$ $= \frac{1}{3}\ln\left(\frac{3}{4} + \sqrt{\frac{9}{16}+1} \right)$ $= \frac{1}{3}\ln 2$	M1 A1 A1 M1 A1 **5**	For arsinh or for any sinh substitution For $\frac{3}{4}x$ or for $3x = 4\sinh u$ For $\frac{1}{3}$ or for $\int \frac{1}{3}\mathrm{d}u$
	OR M2 $\left[\frac{1}{3}\ln(3x + \sqrt{9x^2+16}\,) \right]_0^1$ A1A1 $= \frac{1}{3}\ln 8 - \frac{1}{3}\ln 4$ $= \frac{1}{3}\ln 2$ A1		For $\ln(kx + \sqrt{k^2x^2 + \ldots}\,)$ [Give M1 for $\ln(ax + \sqrt{bx^2 + \ldots}\,)$] or $\frac{1}{3}\ln(x + \sqrt{x^2 + \frac{16}{9}}\,)$
(b)(i)	$2\sinh x\cosh x = 2 \times \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})\frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x})$ $= \frac{1}{2}(\mathrm{e}^{2x} - \mathrm{e}^{-2x})$ $= \sinh 2x$	M1 A1 **2**	$(\mathrm{e}^x - \mathrm{e}^{-x})(\mathrm{e}^x + \mathrm{e}^{-x}) = (\mathrm{e}^{2x} - \mathrm{e}^{-2x})$ For completion
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 20\sinh x - 6\sinh 2x$ For stationary points, $20\sinh x - 12\sinh x\cosh x = 0$ $4\sinh x(5 - 3\cosh x) = 0$ $\sinh x = 0$ or $\cosh x = \frac{5}{3}$ $x = 0, \quad y = 17$ $x = (\pm)\ln\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1} \right) = \ln 3$ $y = 10\left(3 + \frac{1}{3}\right) - \frac{3}{2}\left(9 + \frac{1}{9}\right) = \frac{59}{3}$ $x = -\ln 3, \quad y = \frac{59}{3}$	B1B1 M1 A1 A1 ag A1 ag B1 **7**	When exponential form used, give B1 for any 2 terms correctly differentiated Solving $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to obtain a value of $\sinh x$, $\cosh x$ or e^x (or $x = 0$ stated) Correctly obtained Correctly obtained *The last A1A1 ag can be replaced by B1B1 ag for a full verification*
(iii)	$\left[20\sinh x - \frac{3}{2}\sinh 2x \right]_{-\ln 3}^{\ln 3}$ $= \left\{ 10\left(3 - \frac{1}{3}\right) - \frac{3}{4}\left(9 - \frac{1}{9}\right) \right\} \times 2$ $= \left(\frac{80}{3} - \frac{20}{3} \right) \times 2 = 40$	B1B1 M1 A1 ag **4**	When exponential form used, give B1 for any 2 terms correctly integrated Exact evaluation of $\sinh(\ln 3)$ and $\sinh(2\ln 3)$

5 (i)		B1 B1 B1 B1 B1 B1 **6**	Maximum on LH branch and minimum on RH branch Crossing axes correctly Two branches with positive gradient Crossing axes correctly Maximum on LH branch and minimum on RH branch Crossing positive y-axis and minimum in first quadrant
(ii)	$y=\dfrac{(x+k)(x-2k)+2k^2+2k}{x+k}$ $=x-2k+\dfrac{2k(k+1)}{x+k}$ Straight line when $2k(k+1)=0$ $k=0,\quad k=-1$	M1 A1 (ag) B1B1 **4**	Working in either direction For completion
(iii)(A)	Hyperbola	B1 **1**	
(B)	$x=-k$ $y=x-2k$	B1 B1 **2**	

(iv)		B1 B1 B1 B1 B1	 **5**	Asymptotes correctly drawn Curve approaching asymptotes correctly (both branches) Intercept 2 on y-axis, and not crossing the x-axis Points A and B marked, with minimum point between them Points A and B at the same height ($y = 1$)

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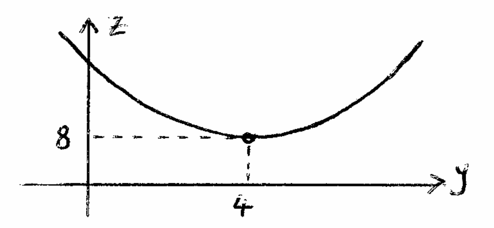
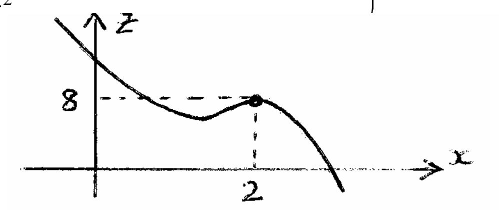
1 (i)	$\mathbf{d}_K = \begin{pmatrix}8\\-1\\-14\end{pmatrix}\times\begin{pmatrix}6\\2\\-5\end{pmatrix}=\begin{pmatrix}33\\-44\\22\end{pmatrix}$ $\left[\ =11\begin{pmatrix}3\\-4\\2\end{pmatrix}\ \right]$ $\mathbf{d}_L = \begin{pmatrix}8\\-1\\-14\end{pmatrix}\times\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}15\\-20\\10\end{pmatrix}$ $\left[\ =5\begin{pmatrix}3\\-4\\2\end{pmatrix}\ \right]$ Hence K and L are parallel For a point on K, $z=0,\ x=3,\ y=4$ i.e. $(3,\ 4,\ 0)$ For a point on L, $z=0,\ x=6,\ y=28$ i.e. $(6,\ 28,\ 0)$	M1* A1* A1 M1*A1* A1*	Finding direction of K or L One direction correct ** These marks can be earned anywhere in the question* Correctly shown Finding one point on K or L *or* $(6,\ 0,\ 2)$ *or* $(0,\ 8,\ -2)$ *etc* *Or* $(27,\ 0,\ 14)$ *or* $(0,\ 36,\ -4)$ *etc*
	$\left[\begin{pmatrix}6\\28\\0\end{pmatrix}-\begin{pmatrix}3\\4\\0\end{pmatrix}\right]\times\begin{pmatrix}3\\-4\\2\end{pmatrix}=\begin{pmatrix}3\\24\\0\end{pmatrix}\times\begin{pmatrix}3\\-4\\2\end{pmatrix}=\begin{pmatrix}48\\-6\\-84\end{pmatrix}$ Distance is $\dfrac{\sqrt{48^2+6^2+84^2}}{\sqrt{3^2+4^2+2^2}}=\dfrac{\sqrt{9396}}{\sqrt{29}}=18$	M1 M1 A1 **9**	For $(\mathbf{b}-\mathbf{a})\times\mathbf{d}$ Correct method for finding distance
	OR $\begin{pmatrix}6+3\lambda-3\\28-4\lambda-4\\2\lambda\end{pmatrix}\cdot\begin{pmatrix}3\\-4\\2\end{pmatrix}=0$ M1 $-87+29\lambda=0,\quad \lambda=3$ M1 Distance is $\sqrt{12^2+12^2+6^2}=18$ A1		For $(\mathbf{b}+\lambda\mathbf{d}-\mathbf{a}).\mathbf{d}=0$ Finding λ, and the magnitude
(ii)	Distance from $(3, 4, 0)$ to R is $\left\lvert\dfrac{2\times3+4-0-40}{\sqrt{2^2+1^2+1^2}}\right\rvert$ $=\dfrac{30}{\sqrt6}=\dfrac{30\sqrt6}{6}=5\sqrt6$	M1A1 ft A1 ag **3**	
(iii)	K, M intersect if $1+5\lambda=3+3\mu$ (1) $-4-4\lambda=4-4\mu$ (2) $3\lambda=2\mu$ (3) Solving (2) and (3): $\lambda=4,\ \mu=6$ Check in (1): LHS $=1+20=21$, RHS $=3+18=21$ Hence K, M intersect, at $(21,\ -20,\ 12)$	M1 A1 ft M1M1 M1A1 A1 **7**	At least 2 eqns, different parameters Two equations correct Intersection correctly shown *Can be awarded after M1A1M1M0M0*
	OR M meets P when M1 $8(1+5\lambda)-(-4-4\lambda)-14(3\lambda)=20$ A1 M meets Q when $6(1+5\lambda)+2(-4-4\lambda)-5(3\lambda)=26$ A1 Both equations have solution $\lambda=4$ A1 Point is on P, Q and M; hence on K and M M2 Point of intersection is $(21,\ -20,\ 12)$ A1		Intersection of M with **both** P and Q

(iv)	$\left[\begin{pmatrix}6\\28\\0\end{pmatrix}-\begin{pmatrix}1\\-4\\0\end{pmatrix}\right]\cdot\left[\begin{pmatrix}3\\-4\\2\end{pmatrix}\times\begin{pmatrix}5\\-4\\3\end{pmatrix}\right]=\begin{pmatrix}5\\32\\0\end{pmatrix}\cdot\begin{pmatrix}-4\\1\\8\end{pmatrix}=12$	M1A1 ft M1	For evaluating $\mathbf{d}_L \times \mathbf{d}_M$ For $(\mathbf{b}-\mathbf{c}).(\mathbf{d}_L \times \mathbf{d}_M)$
	Distance is $\dfrac{12}{\sqrt{4^2+1^2+8^2}}=\dfrac{12}{9}=\dfrac{4}{3}$	A1 ft A1 **5**	Numerical expression for distance

2 (i)	$\frac{\partial z}{\partial x} = y^2 - 8xy - 6x^2 + 54x - 36$ $\frac{\partial z}{\partial y} = 2xy - 4x^2$	B2 B1 **3**	Give B1 for 3 terms correct
(ii)	At stationary points, $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ When $x = 0$, $y^2 - 36 = 0$ $y = \pm 6$; points $(0,\ 6,\ 20)$ and $(0,\ -6,\ 20)$ When $y = 2x$, $4x^2 - 16x^2 - 6x^2 + 54x - 36 = 0$ $-18x^2 + 54x - 36 = 0$ $x = 1,\ 2$ Points $(1,\ 2,\ 5)$ and $(2,\ 4,\ 8)$	M1 M1 A1A1 M1 M1A1 A1 **8**	If A0, give A1 for $y = \pm 6$ or $y = 2,\ 4$ *A0 if any extra points given*
(iii)	When $x = 2$, $z = 2y^2 - 16y + 40$  When $y = 4$, $z = -2x^3 + 11x^2 - 20x + 20$ $\left(\frac{\mathrm{d}^2 z}{\mathrm{d}x^2} = -12x + 22 = -2 \text{ when } x = 2 \right)$  The point is a minimum on one section and a maximum on the other; so it is neither a maximum nor a minimum	B1 B1 B1 B1 B1 B1 **6**	'Upright' parabola $(2,\ 4,\ 8)$ identified as a minimum (in the first quadrant) 'Negative cubic' curve $(2,\ 4,\ 8)$ identified as a stationary point Fully correct (unambiguous minimum and maximum)
(iv)	Require $\frac{\partial z}{\partial x} = -36$ and $\frac{\partial z}{\partial y} = 0$ When $x = 0$, $y^2 - 36 = -36$ $y = 0$; point $(0,\ 0,\ 20)$ When $y = 2x$, $4x^2 - 16x^2 - 6x^2 + 54x - 36 = -36$ $-18x^2 + 54x = 0$ $x = 0,\ 3$ $x = 0$ gives $(0,\ 0,\ 20)$ same as above $x = 3$ gives $(3,\ 6,\ -7)$	M1 M1 A1 M1 M1 A1 A1 **7**	$\frac{\partial z}{\partial x} = 36$ can earn all M marks Solving to obtain x (or y) *or stating 'no roots' if appropriate (e.g. when +36 has been used)*

3 (i)	$1+\left(\frac{dy}{dx}\right)^2=1+\left(x-\frac{1}{4x}\right)^2$	M1	
	$=1+x^2-\frac{1}{2}+\frac{1}{16x^2}=x^2+\frac{1}{2}+\frac{1}{16x^2}$		
	$=\left(x+\frac{1}{4x}\right)^2$	A1	
	Arc length is $\int_1^a \left(x+\frac{1}{4x}\right)dx$	M1	For $\int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
	$=\left[\frac{1}{2}x^2+\frac{1}{4}\ln x\right]_1^a$	M1	
	$=\frac{1}{2}a^2+\frac{1}{4}\ln a-\frac{1}{2}$	A1 ag **5**	
(ii)	Curved surface area is $\int 2\pi x\,ds$	M1	
	$=\int_1^4 2\pi x\left(x+\frac{1}{4x}\right)dx$	A1 ft	Any correct integral form (including limits)
	$=2\pi\left[\frac{1}{3}x^3+\frac{1}{4}x\right]_1^4$	M1 A1	for $\frac{1}{3}x^3+\frac{1}{4}x$
	$=\frac{87\pi}{2}$ (≈ 137)	A1 **5**	
(iii)	$\rho=\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}=\frac{\left(a+\frac{1}{4a}\right)^3}{1+\frac{1}{4a^2}}$	B1	any form, in terms of x or a
		B1	any form, in terms of x or a
	$=\frac{a\left(a+\frac{1}{4a}\right)^3}{a+\frac{1}{4a}}=a\left(a+\frac{1}{4a}\right)^2$	M1	Formula for ρ or κ
		A1	ρ or κ correct, in any form, in terms of x or a
		A1 ag **5**	
(iv)	At $(1,\ \frac{1}{2})$, $\rho=(\frac{5}{4})^2=\frac{25}{16}$		
	$\frac{dy}{dx}=1-\frac{1}{4}=\frac{3}{4}$, so $\hat{\mathbf{n}}=\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}$	M1 A1	Finding gradient Correct normal vector (not necessarily unit vector); may be in terms of x
	$\mathbf{c}=\begin{pmatrix}1\\ \frac{1}{2}\end{pmatrix}+\frac{25}{16}\begin{pmatrix}-\frac{3}{5}\\ \frac{4}{5}\end{pmatrix}$	M1	OR *M2A1 for obtaining equation of normal line at a general point and differentiating partially*
	Centre of curvature is $\left(\frac{1}{16},\ \frac{7}{4}\right)$	A1A1 **5**	

(v)	Differentiating partially w.r.t. p $0 = x^2 - 2p\ln x$ $p = \dfrac{x^2}{2\ln x}$ and $y = \dfrac{x^4}{2\ln x} - \dfrac{x^4}{4\ln x}$ $y = \dfrac{x^4}{4\ln x}$	M1 A1 M1 A1 **4**	

4 (i)	By Lagrange's theorem, a proper subgroup has order 2 or 5 A group of prime order is cyclic Hence every proper subgroup is cyclic	M1 A1 M1 A1 **4**	Using Lagrange *(need not be mentioned explicitly)* or equivalent For completion
(ii)	e.g. $2^2=4,\ 2^3=8,\ 2^4=5,\ 2^5=10,$ $2^6=9,\ 2^7=7,\ 2^8=3,\ 2^9=6,\ 2^{10}=1$ 2 has order 10, hence M is cyclic	M1 A1 A1 A1 **4**	Considering order of an element Identifying an element of order 10 (2, 6, 7 or 8) Fully justified For conclusion (can be awarded after M1A1A0)
(iii)	$\{1,\ 10\}$ $\{1,\ 3,\ 4,\ 5,\ 9\}$	B1 B2 **3**	Ignore $\{1\}$ and M Deduct 1 mark (from B1B2) for each (proper) subgroup given in excess of 2
(iv)	E is the identity A, C, G, I are rotations B, D, F, H, J are reflections	B1 M1 A1 A1 **4**	Considering elements of order 2 *(or equivalent) Implied by four of B, D, F, H, J in the same set* Give A1 if one element is in the wrong set; or if two elements are interchanged
(v)	P and M are not isomorphic M is abelian, P is non-abelian	B1 B1 **2**	Valid reason e.g. M has one element of order 2 P has more than one
(vi)	see table below	B3 **3**	Give B2 for 7 correct B1 for 4 correct
(vii)	$\{E, B\}, \{E, D\}, \{E, F\}, \{E, H\}, \{E, J\}$ $\{E, A, C, G, I\}$	M1 A1 ft B2 cao **4**	Ignore $\{E\}$ and P *Subgroups of order 2* Using elements of order 2 (allow two errors/omissions) Correct or ft. A0 if any others given *Subgroups of order greater than 2* Deduct 1 mark (from B2) for each extra subgroup given

Table for (vi):

	A	B	C	D	E	F	G	H	I	J
Order	5	2	5	2	1	2	5	2	5	2

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 0 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0.6 & 0.7 \\ 0.8 & 0.1 & 0 & 0 \\ 0.2 & 0.9 & 0 & 0 \end{pmatrix}$	B2	**2**	Give B1 for two columns correct
(ii)	$\mathbf{P}^4 = \begin{pmatrix} 0.3366 & 0.3317 & 0 & 0 \\ 0.6634 & 0.6683 & 0 & 0 \\ 0 & 0 & 0.3366 & 0.3317 \\ 0 & 0 & 0.6634 & 0.6683 \end{pmatrix}$	B2		Give B1 for two non-zero elements correct to at least 2dp
	$\mathbf{P}^7 = \begin{pmatrix} 0 & 0 & 0.3334 & 0.3333 \\ 0 & 0 & 0.6666 & 0.6667 \\ 0.3335 & 0.3333 & 0 & 0 \\ 0.6665 & 0.6667 & 0 & 0 \end{pmatrix}$	B2	**4**	Give B1 for two non-zero elements correct to at least 2dp
(iii)	$\mathbf{P}^7 \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.1000 \\ 0.2000 \\ 0.2334 \\ 0.4666 \end{pmatrix}$ P(8th letter is C) $= 0.233$	M1 A1	**2**	Using $\mathbf{P}^7$ (or $\mathbf{P}^8$) and initial probs
(iv)	$0.1000 \times 0.3366 + 0.2000 \times 0.6683$ $+ 0.2334 \times 0.3366 + 0.4666 \times 0.6683$ $= 0.558$	M1 M1 A1 ft A1	**4**	Using probabilities for 8th letter Using diagonal elements from $\mathbf{P}^4$
(v)(A)	$\mathbf{P}^n \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix} \approx \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.2333 \\ 0.4667 \\ 0.1 \\ 0.2 \end{pmatrix}$	M1		Approximating $\mathbf{P}^n$ when n is large and even
	P($(n+1)$ th letter is $A) = 0.233$	A1		
(B)	$\mathbf{P}^n \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.3 \\ 0.2 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2333 \\ 0.4667 \end{pmatrix}$	M1		Approximating $\mathbf{P}^n$ when n is large and odd
	P($(n+1)$ th letter is $A) = 0.1$	A1	**4**	
(vi)	$\mathbf{Q} = \begin{pmatrix} 0 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0.6 & 0.6 \\ 0.8 & 0.1 & 0 & 0 \\ 0.2 & 0.9 & 0 & 0.1 \end{pmatrix}$	B1	**1**	

(vii)	$\mathbf{Q}^n \to \begin{pmatrix} 0.1721 & 0.1721 & 0.1721 & 0.1721 \\ 0.3105 & 0.3105 & 0.3105 & 0.3105 \\ 0.1687 & 0.1687 & 0.1687 & 0.1687 \\ 0.3487 & 0.3487 & 0.3487 & 0.3487 \end{pmatrix}$ Probabilities are 0.172, 0.310, 0.169, 0.349	M1 M1 A2 **4**	Considering $\mathbf{Q}^n$ for large n OR at least two eqns for equilib probs Probabilities from equal columns OR solving to obtain equilib probs Give A1 for two correct
(viii)	$0.3487 \times 0.1 \times 0.1$ $= 0.0035$	M1M1 A1 **3**	Using 0.3487 and 0.1

Post-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix} 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0.1 & 0.9 \\ 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \end{pmatrix}$	B2	**2**	Give B1 for two rows correct
(ii)	$\mathbf{P}^4=\begin{pmatrix} 0.3366 & 0.6634 & 0 & 0 \\ 0.3317 & 0.6683 & 0 & 0 \\ 0 & 0 & 0.3366 & 0.6634 \\ 0 & 0 & 0.3317 & 0.6683 \end{pmatrix}$	B2		Give B1 for two non-zero elements correct to at least 2dp
	$\mathbf{P}^7=\begin{pmatrix} 0 & 0 & 0.3335 & 0.6665 \\ 0 & 0 & 0.3333 & 0.6667 \\ 0.3334 & 0.6666 & 0 & 0 \\ 0.3333 & 0.6667 & 0 & 0 \end{pmatrix}$	B2	**4**	Give B1 for two non-zero elements correct to at least 2dp
(iii)	$\begin{pmatrix}0.4 & 0.3 & 0.2 & 0.1\end{pmatrix}\mathbf{P}^7$ $=\begin{pmatrix}0.1000 & 0.2000 & 0.2334 & 0.4666\end{pmatrix}$ P(8th letter is C) $=0.233$	M1 A1	**2**	Using $\mathbf{P}^7$ (or $\mathbf{P}^8$) and initial probs
(iv)	$0.1000\times0.3366\ +\ 0.2000\times0.6683$ $+\ 0.2334\times0.3366\ +\ 0.4666\times0.6683$ $=0.558$	M1 M1A1 ft A1	**4**	Using probabilities for 8th letter Using diagonal elements from $\mathbf{P}^4$
(v)(A)	$\mathbf{u}\,\mathbf{P}^n\approx\begin{pmatrix}0.4 & 0.3 & 0.2 & 0.1\end{pmatrix}\begin{pmatrix} \frac13 & \frac23 & 0 & 0 \\ \frac13 & \frac23 & 0 & 0 \\ 0 & 0 & \frac13 & \frac23 \\ 0 & 0 & \frac13 & \frac23 \end{pmatrix}$ $=\begin{pmatrix}0.2333 & 0.4667 & 0.1 & 0.2\end{pmatrix}$ P($(n+1)$ th letter is A) $=0.233$	M1 A1		Approximating $\mathbf{P}^n$ when n is large and even
(B)	$\mathbf{u}\,\mathbf{P}^n\approx\begin{pmatrix}0.4 & 0.3 & 0.2 & 0.1\end{pmatrix}\begin{pmatrix} 0 & 0 & \frac13 & \frac23 \\ 0 & 0 & \frac13 & \frac23 \\ \frac13 & \frac23 & 0 & 0 \\ \frac13 & \frac23 & 0 & 0 \end{pmatrix}$ $=\begin{pmatrix}0.1 & 0.2 & 0.2333 & 0.4667\end{pmatrix}$ P($(n+1)$ th letter is A) $=0.1$	M1 A1	**4**	Approximating $\mathbf{P}^n$ when n is large and odd
(vi)	$\mathbf{Q}=\begin{pmatrix} 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0.1 & 0.9 \\ 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.6 & 0 & 0.1 \end{pmatrix}$	B1	**1**	

(vii)	$\mathbf{Q}^n \to \begin{pmatrix} 0.1721 & 0.3105 & 0.1687 & 0.3487 \\ 0.1721 & 0.3105 & 0.1687 & 0.3487 \\ 0.1721 & 0.3105 & 0.1687 & 0.3487 \\ 0.1721 & 0.3105 & 0.1687 & 0.3487 \end{pmatrix}$ Probabilities are 0.172, 0.310, 0.169, 0.349	M1 M1 A2 **4**	Considering $\mathbf{Q}^n$ for large n OR at least two eqns for equilib probs Probabilities from equal rows OR solving to obtain equilib probs Give A1 for two correct
(viii)	$0.3487 \times 0.1 \times 0.1$ $= 0.0035$	M1M1 A1 **3**	Using 0.3487 and 0.1

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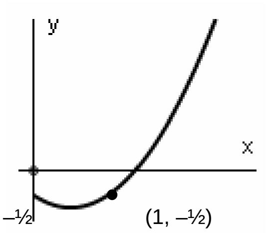
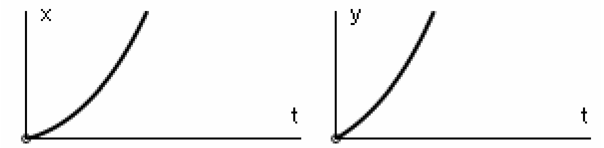
1(i)	$\lambda^2+4\lambda+29=0$	M1	Auxiliary equation	
		M1	Solve for complex roots	
	$\lambda=-2\pm5j$	A1		
	CF $y=e^{-2t}\left(A\cos5t+B\sin5t\right)$	F1	CF for their roots (if complex, must be exp/trig form)	
	PI $y=a\cos t+b\sin t$	B1	Correct form for PI	
	$\dot{y}=-a\sin t+b\cos t, \ddot{y}=-a\cos t-b\sin t$	M1	Differentiate twice	
	$-a\cos t-b\sin t+4\left(-a\sin t+b\cos t\right)$ $+29\left(a\cos t+b\sin t\right)=3\cos t$	M1	Substitute	
	$4b+28a=3$ $-4a+28b=0$	M1	Compare coefficients (both sin and cos)	
	$a=0.105$	M1	Solve for two coefficients	
	$b=0.015$	A1	Both	
	$y=e^{-2t}\left(A\cos5t+B\sin5t\right)+0.105\cos t+0.015\sin t$	F1	GS = PI + CF (with two arbitrary constants)	11
(ii)	$t=0, y=0\Rightarrow 0=A+0.105$	M1	Use condition on y	
	$\Rightarrow A=-0.105$	F1		
	$\dot{y}=-2e^{-2t}\left(A\cos5t+B\sin5t\right)$ $+e^{-2t}\left(-5A\sin5t+5B\cos5t\right)-0.105\sin t+0.015\cos t$	M1	Differentiate (product rule)	
	$t=0,\dot{y}=0\Rightarrow 0=-2A+5B+0.015$	M1	Use condition on $\dot{y}$	
	$\Rightarrow B=-0.045$			
	$y=-e^{-2t}\left(0.105\cos5t+0.045\sin5t\right)+0.105\cos t+0.015\sin t$	A1	cao	
	For large t, $y\approx0.105\cos t+0.015\sin t$	M1	Ignore decaying terms	
	amplitude $\approx\sqrt{0.105^2+0.015^2}\approx0.106$	M1	Calculate amplitude from solution of this form	
		A1	cao	8
(iii)	$y(10\pi)\approx0.105$	B1	Their a from PI, provided GS of correct form	
	$\dot{y}(10\pi)\approx0.015$	B1	Their b from PI, provided GS of correct form	2
(iv)	$y=e^{-2t}\left(C\cos5t+D\sin5t\right)$	F1	Correct or follows previous CF Must not use same arbitrary constants as before	
	oscillations	B1		
	with decaying amplitude (or tends to zero)	B1	Must indicate that y approaches zero, not that $y\approx0$ for $t>10\pi$	3

2(i)	$\frac{dy}{dx} - \frac{2}{x}y = \frac{1}{x} + x^{n-1}$	M1	Rearrange
	$I = \exp\left(\int -\frac{2}{x}dx\right)$	M1	Attempt IF
	$= \exp(-2\ln x)$	M1	Integrate to get $k\ln x$
	$= x^{-2}$	A1	Simplified form of IF
	$\frac{d}{dx}\left(yx^{-2}\right) = x^{-3} + x^{n-3}$	M1	Multiply both sides by IF and recognise derivative
	$yx^{-2} = -\frac{1}{2}x^{-2} + \frac{1}{n-2}x^{n-2} + A$	M1 A1	Integrate RHS including constant
	$y = -\frac{1}{2} + \frac{1}{n-2}x^n + Ax^2$	F1	Their integral (with constant) divided by IF
			8
(ii)	From solution, $x \to 0 \Rightarrow y \to -\frac{1}{2}$	B1	Limit consistent with their solution
	From DE, $x = 0 \Rightarrow 0 - 2y = 1$	M1	Use DE with $x = 0$
	$\Rightarrow y = -\frac{1}{2}$	E1	Correctly deduced
			3
(iii)	$y = -\frac{1}{2}, x = 1 \Rightarrow -\frac{1}{2} = -\frac{1}{2} + \frac{1}{n-2} + A$	M1	Use condition
	$\Rightarrow A = -\frac{1}{n-2}$		
	$y = -\frac{1}{2} + \frac{1}{n-2}\left(x^n - x^2\right)$	F1	Consistent with their GS and given condition
	$n = 1, y = -\frac{1}{2} - x + x^2$		
		B1 B1	Shape for $x > 0$ consistent with their solution (provided not y = constant) Through $\left(1, -\frac{1}{2}\right)$ or (0, their value from (ii))
			4
(iv)	$\frac{d}{dx}\left(yx^{-2}\right) = x^{-3} + x^{-1}$	M1 F1	Use result from (i) or attempt to solve from scratch Follow work in (i)
	$yx^{-2} = -\frac{1}{2}x^{-2} + \ln x + B$	M1	Integrate
		A1	RHS (accept repeated error in first term from (i))
	$y = -\frac{1}{2} + x^2 \ln x + Bx^2$	M1	Divide by IF, including constant (here or later)
	$y(1) = -\frac{1}{2} + B$	M1	Use condition at $x = 1$
	$y(2) = -\frac{1}{2} + 4\ln 2 + 4B$	M1	Use condition at $x = 2$
	$y(1) = y(2) \Rightarrow 3B = -4\ln 2 \Rightarrow B = -\frac{4}{3}\ln 2$	M1	Equate and solve
	$y = -\frac{1}{2} + x^2\left(\ln x - \frac{4}{3}\ln 2\right)$	A1	cao
			9

3(i)	$\int y^{-\frac{1}{2}}\,\mathrm{d}y = \int -k\left(1+0.1\cos 25t\right)\mathrm{d}t$	M1	Separate	
	$2y^{\frac{1}{2}} = -k\left(t+0.004\sin 25t\right)+c$	M1 A1 A1	Integrate LHS RHS (condone no constant)	
	$t=0, y=1 \Rightarrow c=2$	M1 F1	Use condition (must have constant)	
	$y=\left(1-\frac{1}{2}k\left(t+0.004\sin 25t\right)\right)^2$	M1 A1	Rearrange, dealing properly with constant cao	8
(ii)	$t=1, y=0.5 \Rightarrow 2\left(0.5\right)^{\frac{1}{2}} = -k\left(1+0.004\sin 25\right)+2$	M1	Substitute	
	$\Rightarrow k \approx 0.586$	E1	Calculate k (must be from correct solution)	
	$t=2 \Rightarrow y=\left(1-\frac{1}{2}\times 0.586\left(2+0.004\sin 50\right)\right)^2 \approx 0.172$	M1 A1	Substitute cao	4
(iii)	solution curve on insert	M1 A1 A1	Reasonable attempt at curve From (0,1) and decreasing Curve broadly in line with tangent field	
	tank empty after 3.0 minutes	F1	Answer must be consistent with their curve	4
(iv)	$x(0.1)=1+0.1\left(-0.6446\right)$	M1 A1	 –0.6446	
	= 0.93554	E1	Must be clearly shown	
	$x(0.2)=0.93554+0.1\left(-0.51985\right)$	M1 A1	 –0.51985	
	= 0.88356	A1	awrt 0.884	6
(v)	$y<0.01 \Rightarrow \sqrt{y}<0.1 \Rightarrow \sqrt{y}+0.1\cos 25t<0$ for some t	M1	Consider size of $\sqrt{y}$ and sign of $\sqrt{y}+0.1\cos 25t$	
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t}>0$ for some values of t	E1	Complete argument	2

4(i)	$\ddot{x} = -5\dot{x} + 4\dot{y} - 2\,\mathrm{e}^{-2t}$	M1	Differentiate	
	$= -5\dot{x} + 4\left(-9x + 7y + 3\,\mathrm{e}^{-2t}\right) - 2\,\mathrm{e}^{-2t}$	M1	Substitute for $\dot{y}$	
	$= -5\dot{x} - 36x + \frac{28}{4}\left(\dot{x} + 5x - \mathrm{e}^{-2t}\right) + 10\,\mathrm{e}^{-2t}$	M1 M1	y in terms of $x, \dot{x}$ Substitute for y	
	$\ddot{x} - 2\dot{x} + x = 3\,\mathrm{e}^{-2t}$	E1		5
(ii)	$\lambda^2 - 2\lambda + 1 = 0$	M1	Auxiliary equation	
	$\lambda = 1$ (repeated)	A1		
	CF $x = (A + Bt)\mathrm{e}^{t}$	F1	CF for their roots	
	PI $x = a\,\mathrm{e}^{-2t}$	B1	Correct form for PI	
	$\dot{x} = -2a\,\mathrm{e}^{-2t}, \ddot{x} = 4a\,\mathrm{e}^{-2t}$	M1	Differentiate twice	
	$4a\,\mathrm{e}^{-2t} - 2\left(-2\,\mathrm{e}^{-2t}\right) + a\,\mathrm{e}^{-2t} = 3\,\mathrm{e}^{-2t}$	M1	Substitute and compare	
	$a = \frac{1}{3}$	A1		
	GS $x = \frac{1}{3}\mathrm{e}^{-2t} + (A + Bt)\mathrm{e}^{t}$	F1	GS = PI + CF (with two arbitrary constants)	8
(iii)	$y = \frac{1}{4}\left(\dot{x} + 5x - \mathrm{e}^{-2t}\right)$	M1	y in terms of $x, \dot{x}$	
	$= \frac{1}{4}\left(-\frac{2}{3}\mathrm{e}^{-2t} + B\,\mathrm{e}^{t} + (A + Bt)\mathrm{e}^{t} + \frac{5}{3}\mathrm{e}^{-2t} + 5(A + Bt)\mathrm{e}^{t} - \mathrm{e}^{-2t}\right)$	M1 F1	Differentiate x $\dot{x}$ follows their x (but must use product rule)	
	$y = \frac{1}{4}\mathrm{e}^{t}\left(6A + B + 6Bt\right)$	A1	cao	4
(iv)	$\frac{1}{3} + A = 0$	M1	Condition on x	
	$\frac{1}{4}(6A + B) = 0$	M1	Condition on y	
	$A = -\frac{1}{3}, B = 2$			
	$x = \frac{1}{3}\mathrm{e}^{-2t} + \left(2t - \frac{1}{3}\right)\mathrm{e}^{t}$ $y = 3t\,\mathrm{e}^{t}$	A1	Both solutions correct	
	$t = 0 \Rightarrow \dot{x} = 1, \dot{y} = 3$	B1	Both values correct	
		B1 B1 B1	x through origin and consistent with their solution for large t (but not linear) y through origin and consistent with their solution for large t (but not linear) Gradient of both curves at origin consistent with their values of $\dot{x}, \dot{y}$	7

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Q1				
(i)	$\rightarrow\ 40 - P\cos 60 = 0$	M1	For any resolution in an equation involving *P*. Allow for *P* = 40 cos 60 or *P* = 40 cos 30 or *P* = 40 sin 60 or *P* = 40 sin 30	
		A1	Correct equation	
	$P = 80$	A1	cao	3
(ii)	$\downarrow\quad Q + P\cos 30 = 120$	M1	Resolve vert. All forces present. Allow sin ↔ cos No extra forces. Allow wrong signs.	
	$Q = 40\left(3-\sqrt{3}\right)$ = 50.7179… so 50.7 (3 s. f.)	A1	cao	2
				5

Q2				
(i)	Straight lines connecting (0, 10), (10, 30),	B1	Axes with labels (words or letter). Scales indicated. Accept no arrows.	
	(25, 40) and (45, 40)	B1	Use of straight line segments and horiz section	
		B1	All correct with salient points clearly indicated	3
(ii)	$0.5(10+30)\times 10 + 0.5(30+40)\times 15 + 40\times 20$	M1	Attempt at area(s) or use of appropriate *uvast*	
		M1	Evidence of attempt to find whole area	
	= 200 + 525 + 800 = 1525	A1	cao	3
(iii)	$0.5\times 40\times T = 1700 - 1525$	M1	Equating triangle area to 1700 – **their** (ii)	
	so $20T = 175$ and $T = 8.75$	F1	(1700 – **their** (ii))/20. Do not award for – ve answer.	2
				8

Q3				
(i)	String light and pulley smooth	E1	Accept pulley smooth alone	1
(ii)	$5g$ (49) N thrust	M1	Three forces in equilibrium. Allow sign errors.	
		B1	for $15g$ (147) N used as a tension	
		A1	$5g$ (49) N thrust. Accept $\pm 5g$ (49). Ignore diagram. [Award SC2 for $\pm 5g$ (49) N without 'thrust' and SC3 if it is]	3
				4

Q4				
(i)	$P - 800 = 20000 \times 0.2$ $P = 4800$	M1 A1 A1	N2L. Allow $F = mga$. Allow wrong or zero resistance. No extra forces. Allow sign errors. If done as 1 equn need m = 20 000. If A and B analysed separately, must have 2 equns with 'T'. N2L correct.	3
(ii)	New accn $4800 - 2800 = 20000a$ $a = 0.1$	M1 A1	$F = ma$. Finding new accn. No extra forces. Allow 500 N but not 300 N omitted. Allow sign errors. FT **their** P	2
(iii)	$T - 2500 = 10000 \times 0.1$ $T = 3500$ so 3500 N	M1 A1	N2L with new a. Mass 10000. All forces present for A or B except allow 500 N omitted on A. No extra forces cao	2
				7

Q5				
	Take F +ve up the plane $F + 40\cos 35 = 100\sin 35$ $F = 24.5915\ldots$ so 24.6 N (3 s. f.) up the plane	M1 B1 A1 A1	Resolve // plane (or horiz or vert). All forces present. At least one resolved. Allow sin $\leftrightarrow$ cos and sign errors. Allow $100g$ used. Either $\pm 40\cos 35$ or $\pm 100\sin 35$ or equivalent seen Accept $\pm$ 24.5915… or $\pm$90.1237... even if inconsistent or wrong signs used. 24.6 N up the plane (specified or from diagram) or equiv all obtained from consistent and correct working.	4
				4

Q6				
(i)	$(-\mathbf{i}+16\mathbf{j}+72\mathbf{k})+(-80\mathbf{k})=8\mathbf{a}$	M1	Use of N2L. All forces present.	
	$\mathbf{a}=\left(-\frac{1}{8}\mathbf{i}+2\mathbf{j}-\mathbf{k}\right)\text{ m s}^{-2}$	E1	Need at least the **k** term clearly derived	2
(ii)	$\mathbf{r}=4(\mathbf{i}-4\mathbf{j}+3\mathbf{k})+0.5\times16\left(-\frac{1}{8}\mathbf{i}+2\mathbf{j}-\mathbf{k}\right)$	M1	Use of appropriate **uvas***t* or integration (twice)	
		A1	Correct substitution (or limits if integrated)	
	$= 3\mathbf{i} + 4\mathbf{k}$	A1		3
(iii)	$\sqrt{3^2+4^2}=5$ so 5 m	B1	FT **their** (ii) even if it not a displacement. Allow surd form	1
(iv)	$\arctan\frac{4}{3}$	M1	Accept $\arctan\frac{3}{4}$. FT **their** (ii) even if not a displacement. Condone sign errors. (May use arcsin4/5 or equivalent. FT **their** (ii) and (iii) even if not displacement. Condone sign errors)	
	= 53.130… so 53.1° (3 s. f.)	A1	cao	2
				8

Q7				
(i)	8 m s^{-1} (in the negative direction)	B1	Allow $\pm$ and no direction indicated	1
(ii)	$(t+2)(t-4)=0$ so $t = -2$ or 4	M1 A1	Equating v to zero and solving or subst If subst used then both must be clearly shown	2
(iii)	$a = 2t - 2$ $a = 0$ when $t = 1$ $v(1) = 1 - 2 - 8 = -9$ so 9 m s^{-1} in the negative direction $(1, -9)$	M1 A1 F1 A1 B1	Differentiating Correct Accept -9 but not 9 without comment FT	5
(iv)	$\int_1^4 (t^2 - 2t - 8)\ \mathrm{d}x$ $= \left[\frac{t^3}{3} - t^2 - 8t\right]_1^4$ $= \left(\frac{64}{3} - 16 - 32\right) - \left(\frac{1}{3} - 1 - 8\right)$ $= -18$ distance is 18 m	M1 A1 M1 A1 A1	Attempt at integration. Ignore limits. Correct integration. Ignore limits. Attempt to sub correct limits and subtract Limits correctly evaluated. Award if -18 seen but no need to evaluate Award even if -18 not seen. Do not award for -18. cao	5
(v)	$2 \times 18 = 36$ m	F1	Award for $2\times$ **their** (iv).	1
(vi)	$\int_4^5 (t^2 - 2t - 8)\ \mathrm{d}x = \left[\frac{t^3}{3} - t^2 - 8t\right]_4^5$ $= \left(\frac{125}{3} - 25 - 40\right) - \left(-\frac{80}{3}\right) = 3\frac{1}{3}$ so $3\frac{1}{3} + 18 = 21\frac{1}{3}$ m	M1 A1 A1	$\int_4^5$ attempted or, otherwise, complete method seen. Correct substitution Award for $3\frac{1}{3}$ + **their** (positive) (iv)	3
				17

Q8				
(i)	$y = 25\sin\theta t + 0.5\times(-9.8)t^2$	M1	Use of $s = ut + \frac{1}{2}at^2$ .Accept sin, cos, 0.96, 0.28, ± 9.8, ± 10, $u = 25$ and derivation of -4.9 not clear.	
	$= 7t - 4.9t^2$	E1	Shown including deriv of – 4.9. Accept $25\sin\theta t = 7t$ WW	
	$x = 25\cos\theta t = 25\times 0.96t = 24t$	B1	Accept $25\times 0.96t$ or $25\cos\theta t$ seen WW	3
(ii)	$0 = 7^2 - 19.6s$	M1	Accept sequence of *uvast*. Accept *u*=24 but not 25. Allow $u \leftrightarrow v$ and ± 9.8 and ± 10	
	$s = 2.5$ so 2.5 m	A1	+ve answer obtained by correct manipulation.	2
(iii)	Need $7t - 4.9t^2 = 1.25$ so $4.9t^2 - 7t + 1.25 = 0$	M1	Equate *y* to **their** (ii)/2 or equivalent.	
		M1	Correct sub into quad formula of their 3 term quadratic being solved (i.e. allow manipulation errors before using the formula).	
	$t = 0.209209\ldots$ and $1.219361\ldots$	A1	Both. cao. [Award M1 A1 for two correct roots WW]	
	need $24\times$ (1.219… - 0.209209…) $= 24\times 1.01...$ so 24.2 m (3 s.f.)	B1	FT **their** roots (only if both positive)	4
(iv) (A)	$\dot{y} = 7 - 9.8t$	M1	Attempt at $\dot{y}$. Accept sign errors and $u = 24$ but not 25	
	$\dot{y}(1.25) = 7 - 9.8\times 1.25 = -5.25$ m s^{-1}	A1		
(B)	Falling as velocity is negative	E1	Reason must be clear. FT **their** $\dot{y}$ even if not a velocity Could use an argument involving time.	
(C)	Speed is $\sqrt{24^2 + (-5.25)^2}$	M1	Use of Pythag and 24 or 7 with **their** $\dot{y}$	
	= 24.5675… so 24.6 m s^{-1} (3 s. f.)	A1	cao	5

(v)	$y = 7t - 4.9t^2,\ x = 24t$	M1	Elimination of t	
	so $y = \frac{7x}{24} - 4.9\left(\frac{x}{24}\right)^2$	A1	Elimination correct. Condone wrong notation with interpretation correct for the problem.	
	$y = \frac{7x}{24} - 4.9 \times \frac{x^2}{576} = \frac{0.7x}{576}(240 - 7x)$	E1	If not wrong accept as long as $24^2 = 576$ seen. Condone wrong notation with interpretation correct for the problem.	
	either Need $y = 0$	M1		
	so $x = 0$ or $\frac{240}{7}$ so $\frac{240}{7}$ m	A1	Accept $x = 0$ not mentioned. Condone $0 \le X \le \frac{240}{7}$.	
	or	B1	Time of flight $^{10}\!/_{7}$ s	
		B1	Range $^{240}\!/_{7}$ m. Condone $0 \le X \le \frac{240}{7}$.	5
				19

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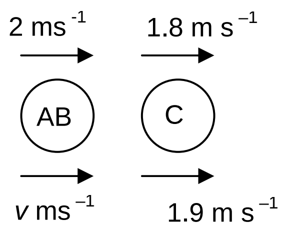
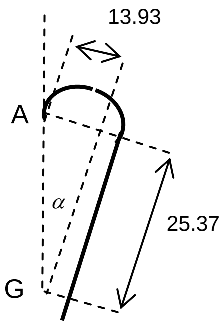
Q 1				
(a) (i)	Impulse has magnitude $2 \times 9 = 18$ N s speed is $\frac{18}{6} = 3 \text{ m s}^{-1}$.	B1 B1		2
(ii)	PCLM $\rightarrow$ $3 \times 6 - 1 \times 2 = 8v$ $v = 2$ so 2 m s^{-1} in orig direction of A	M1 A1 E1	Use of PCLM + combined mass RHS All correct Must justify direction (diag etc)	3
(iii)	$\rightarrow \quad 2 \times 2 - 2 \times -1 = 6$ N s	M1 A1	Attempted use of $m\mathbf{v}$ - $m\mathbf{u}$ for 6 N s dir specified (accept diag)	2
(iv) (A)		B1	Accept masses not shown	1
(B)	PCLM $\rightarrow$ $2 \times 8 + 10 \times 1.8 = 8v + 10 \times 1.9$ $v = 1.875$	M1 A1 A1	PCLM. All terms present Allow sign errors only	3
(C)	NEL $\frac{1.9 - 1.875}{1.8 - 2} = -e$ so $e = 0.125$	M1 A1 F1	Use of NEL with **their** v Any form. FT **their** v FT **their** v (only for $0 < e \leq 1$)	3
(b)	Using $v^2 = u^2 + 2as$ $v = \sqrt{2 \times 10 \times 9.8} = 14$ rebounds at $14 \times \frac{4}{7}$ $= 8 \text{ m s}^{-1}$ No change to the horizontal component Since both horiz and vert components are 8 m s^{-1} the angle is 45°	B1 M1 F1 B1 A1	Allow ± 14 Using **their** *vertical* component FT from **their** 14. Allow $\pm$ Need not be explicitly stated cao	5
		19		

Q 2				
(i)	$\theta = \dfrac{\pi}{2}$ gives CG $= \dfrac{8\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \dfrac{16}{\pi}$ $\left(-\dfrac{16}{\pi}, 8\right)$ justified	B1 E1 E1		3
(ii)	$(8\pi + 72)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 8\pi\begin{pmatrix}-\frac{16}{\pi}\\ 8\end{pmatrix} + 72\begin{pmatrix}36\\ 0\end{pmatrix}$ $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}25.3673...\\ 2.06997...\end{pmatrix} = \begin{pmatrix}25.37\\ 2.07\end{pmatrix}$ (4 s. f.)	M1 B1 A1 A1 E1 E1	Method for c.m. Correct mass of 8π or equivalent 1st RHS term correct 2nd RHS term correct [If separate cpts award the A1s for x- and y- cpts correct on RHS]	6
(iii)	 $\tan\alpha = \dfrac{13.93}{25.37}$ α = 28.7700… so 28.8° (3 s. f.)	B1 M1 M1 A1 A1	General position and angle (lengths need not be shown) Angle or complement attempted. arctan or equivalent. Attempt to get 16 – 2.0699… Obtaining 13.93… cao Accept use of 2.0699… but not 16. cao	5
(iv)	c. w. moments about A $12 \times 13.93 - 16F = 0$ so F = 10.4475…	M1 A1 A1	[FT use of 2.0699…] Moments about any point, all forces present (1.5525... if 2.0699… used)	3
		17		

Q 3				
(i)	Moments c.w. about B $200 \times 0.6 - 0.8R_A = 0$ $R_A = 150$ so 150 N Resolve or moments $R_B = 50$ so 50 N	M1 A1 M1 F1	Accept about any point. Allow sign errors.	4
(ii)	Moments c.w. about D $-0.8R_C + 1.2 \times 200 = 0$ $R_C = 300 \uparrow$ Resolve or moments $R_D = 100 \downarrow$	M1 A1 M1 A1 E1	Or equiv. Accept about any point. All terms present. No extra terms. Allow sign errors. Neglect direction Or equiv. All terms present. No extra terms. Allow sign errors. Neglect direction Both directions clearly shown (on diag)	5
(iii)	Moments c.w. about P $0.4 \times 200\cos\alpha - 0.8R_Q = 0$ $R_Q = 96$ so 96 N resolve perp to plank $R_P = 200\cos\alpha + R_Q$ $R_P = 288$ so 288 N	M1 A1 A1 M1 A1 A1	Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working] Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working]	6
(iv)	Need one with greatest normal reaction So at P Resolve parallel to the plank $F = 200\sin\alpha$ so $F = 56$ $\mu = \frac{F}{R}$ $= \frac{56}{288} = \frac{7}{36}$ (= 0.194 (3 s. f.))	B1 B1 M1 A1	FT their reactions Must use **their** F and R cao	4
		19		

Q 4				
(i)	**either** $0.5\times 20\times 0.5^2 + 20\times 9.8\times 4$ $= 786.5$ J **or** $a = \frac{1}{32}$ $T - 20g = 20\times \frac{1}{32}$ $T = 196.625$ WD is $4T = 786.5$ so 786.5 J	 M1 B1 B1 A1 B1 M1 A1 A1	 KE or GPE terms KE term GPE term cao N2L. All terms present. cao	 4
(ii)	$20g\times 0.5 = 10g$ so 98 W	M1 A1 A1	Use of $P = Fv$ or $\frac{\Delta \text{WD}}{\Delta t}$ All correct	 3
(iii)	GPE lost is $35\times 9.8\times 3 = 1029$ J KE gained is $0.5\times 35\times(3^2 - 1^2) = 140$ J so WE gives WD against friction is 1029 – 140 = 889 J	B1 M1 A1 M1 A1	 Δ KE The 140 J need not be evaluated Use of WE equation cao	 5
(iv)	**either** $0.5\times 35\times 3^2 + 35\times 9.8\times 0.1x = 150x$ $x = 1.36127\ldots$ so 1.36 m (3 S. F.) **or** $35g\times 0.1 - 150 = 35a$ $a = -3.3057\ldots$ $0 = 9 - 2ax$ $x = 1.36127\ldots$ so 1.36 m (3 S. F.)	 M1 B1 B1 A1 A1 M1 A1 A1 M1 A1	 WE equation. Allow 1 missing term. No extra terms. One term correct (neglect sign) Another term correct (neglect sign) All correct except allow sign errors cao Use of N2L. Must have attempt at weight component. No extra terms. Allow sign errors, otherwise correct cao Use of appropriate *uvast* or sequence cao	 5
		17		

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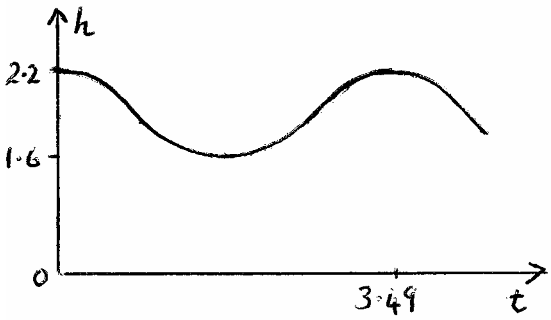
1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$ [Pressure] $= \mathrm{M\,L^{-1}\,T^{-2}}$	B1 B1 B1 B1 B1 **5**	*(Deduct 1 mark if answers given as* $\mathrm{m\,s^{-1}}$, $\mathrm{m\,s^{-2}}$, $\mathrm{kg\,m\,s^{-2}}$ *etc)*
(ii)	$[P] = \mathrm{M\,L^{-1}\,T^{-2}}$ $[\frac{1}{2}\rho v^2] = (\mathrm{M\,L^{-3}})(\mathrm{L\,T^{-1}})^2$ $= \mathrm{M\,L^{-1}\,T^{-2}}$ $[\rho g h] = (\mathrm{M\,L^{-3}})(\mathrm{L\,T^{-2}})(\mathrm{L}) = \mathrm{M\,L^{-1}\,T^{-2}}$ All 3 terms have the same dimensions	M1 A1 A1 E1 **4**	Finding dimensions of 2nd or 3rd term Allow e.g. 'Equation is dimensionally consistent' following correct work
(b)(i)		M1 A1 **2**	For a 'cos' curve (starting at the highest point) Approx correct values marked on both axes
(ii)	Period $\frac{2\pi}{\omega} = 3.49$ $\omega = 1.8$ $h = 1.9 + 0.3\cos 1.8t$	M1 A1 M1 F1 **4**	Accept $\frac{2\pi}{3.49}$ For $h = c + a\cos/\sin$ with either $c = \frac{1}{2}(1.6 + 2.2)$ or $a = \frac{1}{2}(2.2 - 1.6)$
(iii)	When $h = 1.7$, float is 0.2 m below centre Acceleration is $\omega^2 x = 1.8^2 \times 0.2$ $= 0.648\ \mathrm{m\,s^{-2}}$ upwards	M1A1 A1 cao **3**	Award M1 if there is at most one error
	OR When $h = 1.7$, $\cos 1.8t = -\frac{2}{3}$ ($1.8t = 2.30$, $t = 1.28$) Acceleration $\ddot{h} = -0.3 \times 1.8^2 \cos 1.8t$ M1 $= -0.3 \times 1.8^2 \times (-\frac{2}{3})$ A1 $= 0.648\ \mathrm{m\,s^{-2}}$ upwards A1 cao		

2 (i)	$R\cos 60 = 0.4\times 9.8$ Normal reaction is 7.84 N	M1 A1 **2**	Resolving vertically (e.g. $R\sin 60 = mg$ is M1A0 $R = mg\cos 60$ is M0)
(ii)	$R\sin 60 = 0.4\times\dfrac{v^2}{2.7\sin 60}$ Speed is $6.3\ \mathrm{m\,s^{-1}}$	M1 M1 A1 A1 cao **4**	Horizontal equation of motion Acceleration $\dfrac{v^2}{r}$ (M0 for $\dfrac{v^2}{2.7}$)
	OR $R\sin 60 = 0.4\times(2.7\sin 60)\omega^2$ M1 A1 $\omega = 2.694$ $v = (2.7\sin 60)\omega$ M1 Speed is $6.3\ \mathrm{m\,s^{-1}}$ A1 cao		Horizontal equation of motion or $R = 0.4\times 2.7\times\omega^2$ For $v = r\omega$ (M0 for $v = 2.7\omega$)
(iii)	By conservation of energy, $\frac{1}{2}\times 0.4\times(9^2 - v^2) = 0.4\times 9.8\times(2.7 + 2.7\cos\theta)$ $81 - v^2 = 52.92 + 52.92\cos\theta$ $v^2 = 28.08 - 52.92\cos\theta$	M1 A1 A1 **3**	Equation involving KE and PE Any (reasonable) correct form e.g. $v^2 = 81 - 52.92(1+\cos\theta)$
(iv)	$R + 0.4\times 9.8\cos\theta = 0.4\times\dfrac{v^2}{2.7}$ $R + 3.92\cos\theta = \dfrac{0.4}{2.7}(28.08 - 52.92\cos\theta)$ $R + 3.92\cos\theta = 4.16 - 7.84\cos\theta$ $R = 4.16 - 11.76\cos\theta$	M1 A1 M1 A1 E1 **5**	Radial equation with 3 terms Substituting expression for v^2 *SR* If θ is taken to the downward vertical, maximum marks are: M1A0A0 in (iii) M1A1M1A1E0 in (iv)
(v)	Leaves surface when $R = 0$ $\cos\theta = \dfrac{4.16}{11.76}$ $v^2 = 28.08 - 52.92\times\dfrac{4.16}{11.76}$ $(= 9.36)$ Speed is $3.06\ \mathrm{m\,s^{-1}}$	M1 A1 M1 A1 cao **4**	*Dependent on previous M1* or using $mg\cos\theta = \dfrac{mv^2}{r}$

3 (i)	Tension is $637 \times 0.1 = 63.7$ N Energy is $\frac{1}{2} \times 637 \times 0.1^2$ $= 3.185$ J	B1 M1 A1 **3**	
(ii)	Let θ be angle between RA and vertical $\cos\theta = \frac{5}{13}$ ($\theta = 67.4°$) $T\cos\theta = mg$ $63.7 \times \frac{5}{13} = m \times 9.8$ Mass of ring is 2.5 kg	B1 M1 A1 E1 **4**	 Resolving vertically
(iii)	 Loss of PE is $2.5 \times 9.8 \times (0.9 - 0.5)$ EE at lowest point is $\frac{1}{2} \times 637 \times 0.3^2$ $(= 28.665)$ By conservation of energy, $2.5 \times 9.8 \times 0.4 + \frac{1}{2} \times 2.5u^2 = \frac{1}{2} \times 637 \times 0.3^2 - 3.185$ $9.8 + 1.25u^2 = 25.48$ $u^2 = 12.544$ $u = 3.54$	M1 A1 M1 A1 M1 F1 A1 cao **7**	Considering PE or PE at start and finish Award M1 if not more than one error Equation involving KE, PE and EE
(iv)	From lowest point to level of A, Loss of EE is 28.665 Gain in PE is $2.5 \times 9.8 \times 0.9 = 22.05$ Since $28.665 > 22.05$, Ring will rise above level of A	 M1 M1 M1 A1 cao **4**	 EE at 'start' and at level of A PE at 'start' and at level of A (For M2 it must be the same 'start') Comparing EE and PE *(or equivalent,* *e.g.* $\frac{1}{2}mu^2 + 3.185 = mg \times 0.5 + \frac{1}{2}mv^2$ *)* Fully correct derivation
			SR If 637 is used as modulus, maximum marks are: (i) B0M1A0 (ii) B1M1A1E0 (iii) M1A1M1A1M1F1A0 (iv) M1M1M1A0

4 (a)	Area is $\int_0^2 x^3\,dx = \left[\tfrac{1}{4}x^4\right]_0^2 = 4$	B1	
	$\int x\,y\,dx = \int_0^2 x^4\,dx$	M1	
	$= \left[\tfrac{1}{5}x^5\right]_0^2 = 6.4$	A1	
	$\bar{x} = \dfrac{6.4}{4} = 1.6$	A1	
	$\int \tfrac{1}{2}y^2\,dx = \int_0^2 \tfrac{1}{2}x^6\,dx$	M1	Condone omission of $\tfrac{1}{2}$
	$= \left[\tfrac{1}{14}x^7\right]_0^2 = \dfrac{64}{7}$	A1	
	$\bar{y} = \dfrac{\int \tfrac{1}{2}y^2\,dx}{\int y\,dx}$	M1	
	$= \dfrac{\tfrac{64}{7}}{4} = \dfrac{16}{7}$	A1 **8**	Accept 2.3 from correct working
(b)(i)	Volume is $\int \pi y^2\,dx = \int_1^2 \pi(4 - x^2)\,dx$	M1	*π may be omitted throughout*
	$= \pi\left[4x - \tfrac{1}{3}x^3\right]_1^2 = \tfrac{5}{3}\pi$	A1	For $\tfrac{5}{3}$
	$\int \pi x y^2\,dx = \int_1^2 \pi x(4 - x^2)\,dx$	M1	
	$= \pi\left[2x^2 - \tfrac{1}{4}x^4\right]_1^2 = \tfrac{9}{4}\pi$	A1	For $\tfrac{9}{4}$
	$\bar{x} = \dfrac{\int \pi x y^2\,dx}{\int \pi y^2\,dx}$	M1	
	$= \dfrac{\tfrac{9}{4}\pi}{\tfrac{5}{3}\pi} = \dfrac{27}{20} = 1.35$	E1 **6**	Must be fully correct
(ii)	Height of solid is $h = 2\sqrt{3}$ $T\,h = mg \times 0.35$ $F = T = 0.101mg, \quad R = mg$	B1 M1 F1	Taking moments
	Least coefficient of friction is $\dfrac{F}{R} = 0.101$	A1 **4**	Must be fully correct (e.g. A0 if $m = \tfrac{5}{3}\pi$ is used)

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1(i)	$x = \text{PB}$	M1	May be implied	
	$x = \sqrt{a^2 + y^2}$	A1		
	$V = \frac{1}{2}kx^2 - mgy$	M1	EPE term	
		M1	GPE term	
	$= \frac{1}{2}k\left(a^2 + y^2\right) - mgy$	A1	cao	
				5
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}y} = ky - mg$	M1	Differentiate their V	
	equilibrium $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}y} = 0$	B1	Seen or implied	
	$\Rightarrow y = \frac{mg}{k}$	A1	cao	
	$\frac{\mathrm{d}^2V}{\mathrm{d}y^2} = k > 0$	M1	Consider sign of V'' (or V' either side)	
	$\Rightarrow$ stable	E1	Complete argument	
				5
(iii)	$R = T\sin P\hat{B}A = k \cdot PB \cdot \frac{a}{PB}$	M1	Use Hooke's law and resolve	
	$= ka$	A1		
				2
2(i)	$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = 0 \Rightarrow mv$ constant	M1	Or no external forces $\Rightarrow$ momentum conserved, or attempt using δ terms.	
	hence $mv = m_0u$	A1		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = k$	B1	$\frac{\mathrm{d}m}{\mathrm{d}t} = k$ seen	
	$\Rightarrow m = m_0 + kt$	B1	$m_0 + kt$ stated or clearly used as mass	
	$v = \frac{m_0u}{m} = \frac{m_0u}{m_0 + kt}$	E1	Complete argument (dependent on all previous marks and $m_0 + kt$ derived, not just stated)	
	$x = \int \frac{m_0u}{m_0 + kt}\,\mathrm{d}t$	M1	Integrate v	
	$= \frac{m_0u}{k}\ln\left(m_0 + kt\right) + A$	A1	cao	
	$x = 0, t = 0 \Rightarrow A = -\frac{m_0u}{k}\ln m_0$	M1	Use condition	
	$x = \frac{m_0u}{k}\ln\left(\frac{m_0 + kt}{m_0}\right)$	A1	cao	
				9
(ii)	$v = \frac{1}{2}u \Rightarrow m_0 + kt = 2m_0$	M1	Attempt to calculate value of m or t	
	$\Rightarrow x = \frac{m_0u}{k}\ln\left(\frac{2m_0}{m_0}\right)$	M1	Substitute their m or t into x	
	$\Rightarrow x = \frac{m_0u}{k}\ln 2$	F1	$t = \frac{m_0}{k}$ or $m = 2m_0$ in their x	
				3

3(i)	$I = \int_{-a}^{a} \rho x^2 \, dx$	M1 A1	Set up integral Or equivalent	
	$\rho = \frac{m}{2a}$	M1	Use mass per unit length in integral or I	
	$I = \frac{m}{2a}\left[\frac{1}{3}x^3\right]_{-a}^{a}$	M1	Integrate	
	$= \frac{1}{6}ma^2 - -\frac{1}{6}ma^2$	M1	Use limits	
	$\frac{1}{3}ma^2$	E1	Complete argument	
				6
(ii)	$I_{\text{rod}} = \frac{1}{3}\times 1.2\times 0.4^2 + 1.2\times 0.4^2$	M1 A1	Use $\frac{1}{3}ma^2$ or $\frac{4}{3}ma^2$ Rod term(s) all correct	
	$I_{\text{sphere}} = \frac{2}{5}\times 2\times 0.1^2 + 2\times 0.9^2$	M1 M1 A1	Use formula for sphere Use parallel axis theorem Sphere terms all correct	
	$I = I_{\text{rod}} + I_{\text{sphere}} = 1.884$	M1 A1	Add moment of inertia for rod and sphere cao	
				7
(iii)	$\frac{1}{2}I\dot{\theta}^2 - 1.2g\times 0.4\cos\theta - 2g\times 0.9\cos\theta$ $= -1.2g\times 0.4\cos\alpha - 2g\times 0.9\cos\alpha$	M1 M1 M1 A1	Use energy KE term Reasonable attempt at GPE terms All terms correct (but ignore signs)	
	$\dot{\theta}^2 = \frac{4.56g}{1.884}(\cos\theta - \cos\alpha)$	M1 F1	Rearrange Only follow an incorrect I	
				6
(iv)	$2\dot{\theta}\ddot{\theta} = \frac{4.56g}{1.884}(-\sin\theta\,\dot{\theta})$ or $I\ddot{\theta} = -1.2g\times 0.4\sin\theta - 2g\times 0.9\sin\theta$	M1	Differentiate, or use moment $= I\ddot{\theta}$	
		F1	Equation for $\ddot{\theta}$ (only follow their I or $\dot{\theta}^2$)	
	$\sin\theta \approx \theta \Rightarrow \ddot{\theta} = -11.86\theta$	M1	Use small angle approximation (in terms of θ)	
	i.e. SHM	E1	All correct (for their I) and make conclusion	
	$T \approx \frac{2\pi}{\sqrt{11.86}} \approx 1.82$	F1	$\frac{2\pi}{\text{their } \omega}$	
				5

4(i)	$2v\dfrac{\mathrm{d}v}{\mathrm{d}x}=2-8v^2$	M1 A1	N2L	
	$\displaystyle\int\frac{v}{1-4v^2}\,\mathrm{d}v=\int\mathrm{d}x$	M1	Separate	
	$-\frac{1}{8}\ln\left\|1-4v^2\right\|=x+c_1$	A1	LHS	
	$x=0, v=0\Rightarrow c_1=0$	M1	Use condition	
	$1-4v^2=\mathrm{e}^{-8x}$	M1	Rearrange	
	$v^2=\frac{1}{4}\left(1-\mathrm{e}^{-8x}\right)$	E1	Complete argument	
				7
(ii)	$F=2-8v^2=2-2\left(1-\mathrm{e}^{-8x}\right)$	M1	Substitute given v^2 into F	
	$=2\,\mathrm{e}^{-8x}$	A1	cao	
	Work done $=\int_0^2 F\,\mathrm{d}x$	M1	Set up integral of F	
	$=\int_0^2 2\,\mathrm{e}^{-8x}\,\mathrm{d}x$	A1	cao	
	$=\left[-\frac{1}{4}\mathrm{e}^{-8x}\right]_0^2$	M1	Integrate	
	$=\frac{1}{4}\left(1-\mathrm{e}^{-16}\right)$	A1	Accept $\frac{1}{4}$ or 0.25 from correct working	
				6
(iii)	$2\dfrac{\mathrm{d}v}{\mathrm{d}t}=2-8v^2$	M1	N2L	
	$\displaystyle\frac{1}{4}\int\frac{1}{\frac{1}{4}-v^2}\,\mathrm{d}v=\int\mathrm{d}t$	M1	Separate	
	$\frac{1}{4}\ln\left\|\dfrac{\frac{1}{2}+v}{\frac{1}{2}-v}\right\|=t+c_2$	A1	LHS	
	$t=0, v=0\Rightarrow c_2=0$	M1	Use condition	
	$\dfrac{\frac{1}{2}+v}{\frac{1}{2}-v}=\mathrm{e}^{4t}$	M1	Rearrange (remove log)	
	$1+2v=\mathrm{e}^{4t}\left(1-2v\right)$			
	$2v\left(1+\mathrm{e}^{4t}\right)=\mathrm{e}^{4t}-1$	M1	Rearrange (v in terms of t)	
	$v=\frac{1}{2}\left(\dfrac{\mathrm{e}^{4t}-1}{\mathrm{e}^{4t}+1}\right)=\frac{1}{2}\left(\dfrac{1-\mathrm{e}^{-4t}}{1+\mathrm{e}^{-4t}}\right)$	E1	Complete argument	
				7
(v)	$t=1\Rightarrow v=0.4820$ $t=2\Rightarrow v=0.4997$ Impulse $=mv_2-mv_1$ $=0.0353$	B1 B1 M1 A1	 Use impulse-momentum equation Accept anything in interval [0.035, 0.036]	4

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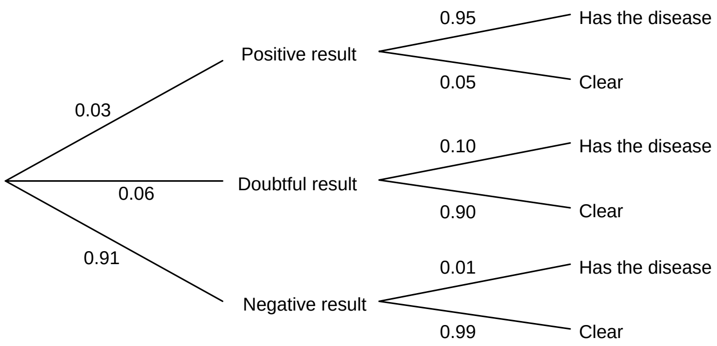
Q1 (i)	$\binom{8}{4}$ ways to select $= 70$	M1 for $\binom{8}{4}$ A1 CAO	**2**
(ii)	$4! = 24$	B1 CAO	**1**
		TOTAL	**3**
Q2 (i)	<table><tr><td>Amount</td><td>0- <20</td><td>20- <50</td><td>50- <100</td><td>100- <200</td></tr><tr><td>Frequency</td><td>800</td><td>480</td><td>400</td><td>200</td></tr></table>	B1 for amounts B1 for frequencies	**2**
(ii)	Total ≈ $10\times800+35\times480+75\times400+150\times200=£84800$	M1 for their midpoints × their frequencies A1 CAO	**2**
		TOTAL	**4**
Q3 (i)	Mean $=\dfrac{3026}{56}=54.0$ $S_{xx}=178890-\dfrac{3026^2}{56}=15378$ $s=\sqrt{\dfrac{15378}{55}}=16.7$	B1 for mean M1 for attempt at S_{xx} A1 CAO	**3**
(ii)	$\bar{x}+2s=54.0+2\times16.7=87.4$ So 93 is an outlier	M1 for their $\bar{x}$ +2×their s A1 FT for 87.4 and comment	**2**
(iii)	New mean $=1.2\times54.0-10=54.8$ New $s=1.2\times16.7=20.1$	B1 FT M1A1 FT	**3**
		TOTAL	**8**
Q4 (i)	(A) P(at least one) $=\dfrac{36}{50}=\dfrac{18}{25}=0.72$ (B) P(exactly one) $=\dfrac{9+6+5}{50}=\dfrac{20}{50}=\dfrac{2}{5}=0.4$	B1 aef M1 for (9+6+5)/50 A1 aef	**3**
(ii)	P(not paper | aluminium) $=\dfrac{13}{24}$	M1 for denominator 24 or 24/50 or 0.48 A1 CAO	**2**
(iii)	P(one kitchen waste) $=2\times\dfrac{18}{50}\times\dfrac{32}{49}=\dfrac{576}{1225}=0.470$	M1 for both fractions M1 for 2 × product of both, or sum of 2 pairs A1	**3**
		TOTAL	**8**

Q5 (i)	11^{th} value is 4, 12^{th} value is 4 so median is 4 Interquartile range = 5 – 2 = 3	B1 M1 for either quartile A1 CAO	**3**
(ii)	No, not valid any two valid reasons such as : • the sample is only for two years, which may not be representative • the data only refer to the local area, not the whole of Britain • even if decreasing it may have nothing to do with global warming • more days with rain does not imply more total rainfall • a five year timescale may not be enough to show a long term trend	B1 E1 E1	**3**
		TOTAL	**6**
Q6 (i)	Either P(all 4 correct) $= \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{35}$ or P(all 4 correct) $= \frac{1}{{}^7C_4} = \frac{1}{35}$	M1 for fractions, or ${}^7C_4$ seen A1 **NB answer given**	**2**
(ii)	$E(X) = 1 \times \frac{4}{35} + 2 \times \frac{18}{35} + 3 \times \frac{12}{35} + 4 \times \frac{1}{35} = \frac{80}{35} = 2\frac{2}{7} = 2.29$ $E(X^2) = 1 \times \frac{4}{35} + 4 \times \frac{18}{35} + 9 \times \frac{12}{35} + 16 \times \frac{1}{35} = \frac{200}{35} = 5.714$ $\text{Var}(X) = \frac{200}{35} - \left(\frac{80}{35}\right)^2 = \frac{24}{49}$ = 0.490 (to 3 s.f.)	M1 for Σrp (at least 3 terms correct) A1 CAO M1 for Σx^2p (at least 3 terms correct) M1*dep* for – their $E(X)^2$ A1 FT their E(X) provided Var(X) > 0	**5**
		TOTAL	**7**

	Section B		
Q7 (i)		G1 probabilities of result G1 probabilities of disease G1 probabilities of clear G1 labels	**4**
(ii)	P(negative and clear) $= 0.91 \times 0.99$ $= 0.9009$	M1 for their 0.91×0.99 A1 CAO	**2**
(iii)	P(has disease) $= 0.03 \times 0.95 + 0.06 \times 0.10 + 0.91 \times 0.01$ $= 0.0285 + 0.006 + 0.0091$ $= 0.0436$	M1 three products M1*dep* sum of three products A1 FT their tree	**3**
(iv)	P(negative | has disease) $= \frac{\text{P(negative } \textit{and} \text{ has disease)}}{\text{P(has disease)}} = \frac{0.0091}{0.0436} = 0.2087$	M1 for their 0.01×0.91 or 0.0091 on its own or as numerator M1 *indep* for their 0.0436 as denominator A1 FT their tree	**3**
(v)	Thus the test result is not very reliable. A relatively large proportion of people who have the disease will test negative.	E1 FT for idea of 'not reliable' or 'could be improved', etc E1 FT	**2**
(vi)	P(negative or doubtful and declared clear) $=0.91+0.06\times0.10\times0.02+0.06\times0.90\times1$ $=0.91+0.00012+0.054=0.96412$	M1 for their 0.91 + M1 for either triplet M1 for second triplet A1 CAO	**4**
		TOTAL	**18**

Q8 (i)	$X \sim \mathrm{B}(17, 0.2)$ $\mathrm{P}(X \geq 4) = 1 - \mathrm{P}(X \leq 3)$ $= 1 - 0.5489 = 0.4511$	B1 for 0.5489 M1 for 1 – their 0.5489 A1 CAO	**3**
(ii)	$\mathrm{E}(X) = np = 17 \times 0.2 = 3.4$	M1 for product A1 CAO	**2**
(iii)	$\mathrm{P}(X = 2) = 0.3096 - 0.1182 = 0.1914$ $\mathrm{P}(X = 3) = 0.5489 - 0.3096 = 0.2393$ $\mathrm{P}(X = 4) = 0.7582 - 0.5489 = 0.2093$ So 3 applicants is most likely	B1 for 0.2393 B1 for 0.2093 A1 CAO *dep* on both B1s	**3**
(iv)	(*A*) Let p = probability of a randomly selected maths graduate applicant being successful (for population) H_0: $p = 0.2$ H_1: $p > 0.2$ (*B*) H_1 has this form as the suggestion is that mathematics graduates are <u>more</u> likely to be successful.	B1 for definition of p in context B1 for H_0 B1 for H_1 E1	**4**
(v)	Let $X \sim \mathrm{B}(17, 0.2)$ $\mathrm{P}(X \geq 6) = 1 - \mathrm{P}(X \leq 5) = 1 - 0.8943 = 0.1057 > 5\%$ $\mathrm{P}(X \geq 7) = 1 - \mathrm{P}(X \leq 6) = 1 - 0.9623 = 0.0377 < 5\%$ So critical region is $\{7,8,9,10,11,12,13,14,15,16,17\}$	B1 for 0.1057 B1 for 0.0377 M1 for at least one comparison with 5% A1 CAO for critical region *dep* on M1 and at least one B1	**4**
(vi)	Because $\mathrm{P}(X \geq 6) = 0.1057 > 10\%$ Either: comment that 6 is still outside the critical region Or comparison $\mathrm{P}(X \geq 7) = 0.0377 < 10\%$	E1 E1	**2**
		TOTAL	**18**

Mark Scheme 4767
June 2007

Question 1

(i)	$X \sim N(11, 3^2)$ $P(X < 10) = P\left(Z < \frac{10-11}{3}\right)$ $= P(Z < -0.333)$ $= \Phi(-0.333) = 1 - \Phi(0.333)$ $= 1 - 0.6304 = 0.3696$	M1 for standardizing M1 for use of tables with their z-value M1 *dep* for correct tail A1CAO (must include use of differences)	**4**
(ii)	P(3 of 8 less than ten) $= \binom{8}{3} \times 0.3696^3 \times 0.6304^5 = 0.2815$	M1 for coefficient M1 for $0.3696^3 \times 0.6304^5$ A1 FT (min 2sf)	**3**
(iii)	$\mu = np = 100 \times 0.3696 = 36.96$ $\sigma^2 = npq = 100 \times 0.3696 \times 0.6304 = 23.30$ $Y \sim N(36.96, 23.30)$ $P(Y \geq 50) = P\left(Z > \frac{49.5 - 36.96}{\sqrt{23.30}}\right)$ $= P(Z > 2.598) = 1 - \Phi(2.598) = 1 - 0.9953$ $= 0.0047$	M1 for Normal approximation with correct (FT) parameters B1 for continuity corr. M1 for standardizing and using correct tail A1 **CAO (**FT 50.5 or omitted CC)	**4**
(iv)	H_0: $\mu = 11$; H_1: $\mu > 11$ Where μ denotes the mean time taken by the new hairdresser	B1 for H_0, as seen. B1 for H_1, as seen. B1 for definition of μ	**3**
(v)	Test statistic $= \frac{12.34 - 11}{3/\sqrt{25}} = \frac{1.34}{0.6}$ $= 2.23$ 5% level 1 tailed critical value of z = 1.645 2.23 > 1.645, so significant. There is sufficient evidence to reject H_0 It is reasonable to conclude that the new hairdresser does take longer on average than other staff.	M1 must include $\sqrt{25}$ A1 (FT their μ) B1 for 1.645 M1 for sensible comparison leading to a conclusion A1 for conclusion in words in context (FT their μ)	**5**
			19

Question 2

<table>
<tr>
<td>(i)</td>
<td>
<table>
<tr><td>x</td><td>2.61</td><td>2.73</td><td>2.87</td><td>2.96</td><td>3.05</td><td>3.14</td><td>3.17</td><td>3.24</td><td>3.76</td><td>4.1</td></tr>
<tr><td>y</td><td>3.2</td><td>2.6</td><td>3.5</td><td>3.1</td><td>2.8</td><td>2.7</td><td>3.4</td><td>3.3</td><td>4.4</td><td>4.1</td></tr>
<tr><td>Rank x</td><td>10</td><td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td></tr>
<tr><td>Rank y</td><td>6</td><td>10</td><td>3</td><td>7</td><td>8</td><td>9</td><td>4</td><td>5</td><td>1</td><td>2</td></tr>
<tr><td>d</td><td>4</td><td>-1</td><td>5</td><td>0</td><td>-2</td><td>-4</td><td>0</td><td>-2</td><td>1</td><td>-1</td></tr>
<tr><td>d^2</td><td>16</td><td>1</td><td>25</td><td>0</td><td>4</td><td>16</td><td>0</td><td>4</td><td>1</td><td>1</td></tr>
</table>
$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 68}{10 \times 99}$$
= 0.588 (to 3 s.f.) [*allow* 0.59 to 2 s.f.]
</td>
<td>M1 for ranking (allow all ranks reversed)

M1 for d^2

A1 for $\Sigma d^2 = 68$

M1 for method for r_s
A1 f.t. for $|r_s| < 1$
NB No ranking scores zero</td>
<td>5</td>
</tr>
<tr>
<td>(ii)</td>
<td>H_0: no association between x and y
H_1: positive association between x and y
Looking for positive association (one–tail test): critical value at 5% level is 0.5636
Since 0.588> 0.5636, there is sufficient evidence to reject H_0,
i.e. conclude that there is positive association between true weight x and estimated weight y.</td>
<td>B1 for H_0, in context.
B1 for H_1, in context.
NB H_0 H_1 <u>not</u> ito ρ
B1 for ± 0.5636
M1 for sensible comparison with c.v., provided $|r_s| < 1$
A1 for conclusion in words & in context, f.t. their r_s and sensible cv</td>
<td>5</td>
</tr>
<tr>
<td>(iii)</td>
<td>$\Sigma x = 31.63$, $\Sigma y = 33.1$, $\Sigma x^2 = 101.92$, $\Sigma y^2 = 112.61$, $\Sigma xy = 106.51$.
$$S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x \Sigma y = 106.51 - \tfrac{1}{10} \times 31.63 \times 33.1$$
$$= 1.8147$$
$$S_{xx} = \Sigma x^2 - \frac{1}{n}\left(\Sigma x\right)^2 = 101.92 - \tfrac{1}{10} \times 31.63^2 = 1.8743$$
$$S_{yy} = \Sigma y^2 - \frac{1}{n}\left(\Sigma y\right)^2 = 112.61 - \tfrac{1}{10} \times 33.1^2 = 3.049$$
$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{1.8147}{\sqrt{1.8743 \times 3.049}} = 0.759$$
</td>
<td>M1 for method for S_{xy}

M1 for method for at least one of S_{xx} or S_{yy}

A1 for at least one of S_{xy}, S_{xx}, S_{yy} correct.

M1 for structure of r

A1 (awrt 0.76)</td>
<td>5</td>
</tr>
<tr>
<td>(iv)</td>
<td>Use of the PMCC is better since it takes into account not just the ranking but the actual value of the weights. Thus it has more information than Spearman's and will therefore provide a more discriminatory test.

Critical value for rho = 0.5494
PMCC is very highly significant whereas Spearman's is only just significant.</td>
<td>E1 for has values, not just ranks
E1 for contains more information
Allow alternatives.
B1 for a cv
E1 dep</td>
<td>4</td>
</tr>
<tr>
<td></td><td></td><td></td><td>19</td>
</tr>
</table>

Question 3

(i)	(A) $P(X = 1) = 0.1712 - 0.0408 = 0.1304$ *OR* $= e^{-3.2}\dfrac{3.2^1}{1!} = 0.1304$ (B) $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.8946$ $= 0.1054$	M1 for tables A1 (2 s.f. WWW) M1 A1	**4**
(ii)	(A) $\lambda = 3.2 \div 5 = 0.64$ $P(X = 1) = e^{-0.64}\dfrac{0.64^1}{1!} = 0.3375$ (B) P(exactly one in each of 5 mins) $= 0.3375^5 = 0.004379$	B1 for mean (SOI) M1 for probability A1 B1 (FT to at least 2 s.f.)	**4**
(iii)	Mean no. of calls in 1 hour = $12 \times 3.2 = 38.4$ Using Normal approx. to the Poisson, $X \sim N(38.4, 38.4)$ $P(X \leq 45.5) = P\left(Z \leq \dfrac{45.5 - 38.4}{\sqrt{38.4}}\right)$ $= P(Z \leq 1.146) = \Phi(1.146) = 0.874$ (3 s.f.)	B1 for Normal approx. with correct parameters (SOI) B1 for continuity corr. M1 for probability using correct tail A1 CAO, (but FT 44.5 or omitted CC)	**4**
(iv)	(A) Suitable arguments for/against each assumption: (B) Suitable arguments for/against each assumption:	E1, E1 E1, E1	**4**
			16

Question 4

(i)

H_0: no association between age group and sex;
H_1: some association between age group and sex;

B1 (in context)

Expected		Sex: Male	Sex: Female	**Row totals**
Age group	Under 40	81.84	42.16	**124**
	40 – 49	73.92	38.08	**112**
	50 and over	42.24	21.76	**64**
Column totals		**198**	**102**	**300**

M1 A1 for expected values (to 2dp)

Contribution to test statistic		Sex: Male	Sex: Female
Age group	Under 40	1.713	3.325
	40 – 49	0.059	0.114
	50 and over	2.255	4.378

M1 for valid attempt at $(O–E)^2/E$

M1dep for summation — **6**

$X^2 = 11.84$

A1CAO for X^2 — **4**

Refer to Ξ_2^2
Critical value at 5% level = 5.991
Result is significant
There is some association between age group and sex .

B1 for 2 deg of f
B1 CAO for cv
B1 dep on their cv & X^2
E1 (conclusion in context)

NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final E1

(ii)

The analysis suggests that there are more females in the under 40 age group and less in the 50 and over age group than would be expected if there were no association.	E1	
The reverse is true for males.	E1	**3**
Thus these data do support the suggestion.	E1dep (on at least one of the previous E1s)	

(iii)

Binomial(300, 0.03) soi	B1 CAO	
$n = 300$, $p = 0.03$ so		
EITHER: use Poisson approximation to Binomial with	*EITHER:* B1 for Poisson	
$\lambda = np = 9$	B1dep for Poisson(9)	
Using tables: $P(X \geq 12) = 1 - P(X \leq 11)$	M1 for using tables to find $1 - P(X \leq 11)$	
$= 1 - 0.8030 = 0.197$	A1	
OR: use Normal approximation N(9, 8.73)	*OR:* B1 for Normal	
	B1dep for parameters	**5**
$P(X > 11.5) = P\left(Z > \dfrac{11.5-9}{\sqrt{8.73}}\right)$	M1 for using tables with correct tail (cc not required for M1)	
$= P(Z > 0.846)) = 1 - 0.8012 = 0.199$	A1	
		18

Mark Scheme 4768
June 2007

Q1	$\mathrm{f}(t) = kt^3(2-t) \qquad 0 < t \le 2$			
(i)	$\int_0^2 kt^3(2-t)\mathrm{d}t = 1$ $\therefore \left[k\left(\frac{2t^4}{4} - \frac{t^5}{5}\right)\right]_0^2 = 1$ $\therefore k\left(8 - \frac{32}{5}\right) - 0 = 1$ $\therefore k \times \frac{8}{5} = 1 \qquad \therefore k = \frac{5}{8}$	M1 E1	Integral of f(t), including limits (possibly implied later), equated to 1. Convincingly shown. Beware printed answer.	 2
(ii)	$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{5}{8}\left(6t^2 - 4t^3\right) = 0$ $\therefore 6t^2 - 4t^3 = 0$ $\therefore 2t^2(3-2t) = 0$ $\therefore t = (0 \text{ or})\ \frac{3}{2}$	M1 A1	Differentiate and set equal to zero. c.a.o.	 2
(iii)	$\mathrm{E}(T) = \int_0^2 \frac{5}{8}t^4(2-t)\mathrm{d}t$ $= \left[\frac{5}{8}\left(\frac{2t^5}{5} - \frac{t^6}{6}\right)\right]_0^2 = \frac{5}{8} \times \left(\frac{64}{5} - \frac{64}{6}\right) = \frac{4}{3}$ $\mathrm{E}(T^2) = \int_0^2 \frac{5}{8}t^5(2-t)\mathrm{d}t$ $= \left[\frac{5}{8}\left(\frac{2t^6}{6} - \frac{t^7}{7}\right)\right]_0^2 = \frac{5}{8} \times \left(\frac{128}{6} - \frac{128}{7}\right) = \frac{40}{21}$ $\mathrm{Var}(T) = \frac{40}{21} - \left(\frac{4}{3}\right)^2 = \frac{120-112}{63} = \frac{8}{63}$	M1 A1 M1 M1 A1	Integral for E(T) including limits (which may appear later). Integral for E(T^2) including limits (which may appear later). Convincingly shown. Beware printed answer.	 5
(iv)	$\overline{T} \sim \mathrm{N}\left(\frac{4}{3}, \frac{8}{63n}\right)$	B1 B1 B1	Normal distribution. Mean. ft c's E(T). Correct variance.	 3

(v)	$n = 100, \quad \bar{t} = \frac{145 \cdot 2}{100} = 1 \cdot 452,$ $s_{n-1}^2 = \frac{223 \cdot 41 - 100 \times 1 \cdot 452^2}{99} = 0 \cdot 12707$	B1	Both mean and variance. Accept sd = 0·3565	
	CI is given by 1·452 ± 1·96 $\times \frac{0 \cdot 3565}{\sqrt{100}}$	M1 B1 M1	ft c's $\bar{t}$ ± . ft c's s_{n-1}.	
	= 1·452 ± 0·0698 = (1·382, 1·522)	A1	c.a.o. Must be expressed as an interval.	
	Since E(*T*) (= 4/3) lies outside this interval it seems the model may not be appropriate.	E1		6
				18

Q2	$Ca \sim N(60{\cdot}2,\ 5{\cdot}2^2)$ $Co \sim N(33{\cdot}9,\ 6{\cdot}3^2)$ $L \sim N(52{\cdot}4,\ 4{\cdot}9^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.	
(i)	$P(Co < 40) = P(Z < \frac{40-33{\cdot}9}{6{\cdot}3} = 0{\cdot}9683)$ $= 0{\cdot}8336$	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	3
(ii)	Want $P(L > Ca)$ i.e. $P(L - Ca > 0)$ $L - Ca \sim N(52{\cdot}4 - 60{\cdot}2 = -7{\cdot}8,$ $4{\cdot}9^2 + 5{\cdot}2^2 = 51{\cdot}05)$ $P(\text{this} > 0) = P(Z > \frac{0-(-7{\cdot}8)}{\sqrt{51{\cdot}05}} = 1{\cdot}0917)$ $= 1 - 0{\cdot}8625 = 0{\cdot}1375$	M1 B1 B1 A1	Allow $Ca - L$ provided subsequent work is consistent. Mean. Variance. Accept sd = $\sqrt{51{\cdot}05}$= 7·1449... c.a.o.	4
(iii)	Want $P(Ca_1 + Ca_2 + Ca_3 + Ca_4 > 225)$ $Ca_1 + \ldots \sim N(60{\cdot}2 + 60{\cdot}2 + 60{\cdot}2 + 60{\cdot}2 = 240{\cdot}8,$ $5{\cdot}2^2 + 5{\cdot}2^2 + 5{\cdot}2^2 + 5{\cdot}2^2 = 108{\cdot}16)$ $P(\text{this} > 225) = P(Z > \frac{225-240{\cdot}8}{\sqrt{108{\cdot}16}} = -1{\cdot}519)$ $= 0{\cdot}9356$ Must assume that the weeks are independent of each other.	M1 B1 B1 A1 B1	Mean. Variance. Accept sd=$\sqrt{108{\cdot}16}$=10·4. c.a.o.	5
(iv)	$R \sim N(0{\cdot}05\times60{\cdot}2 + 0{\cdot}1\times33{\cdot}9 + 0{\cdot}2\times52{\cdot}4 = 16{\cdot}88,$ $0{\cdot}05^2\times5{\cdot}2^2 + 0{\cdot}1^2\times6{\cdot}3^2 + 0{\cdot}2^2\times4{\cdot}9^2 = 1{\cdot}4249)$ $P(R > 20) = P(Z > \frac{20-16{\cdot}88}{\sqrt{1{\cdot}4249}} = 2{\cdot}613)$ $= 1 - 0{\cdot}9955 = 0{\cdot}0045$	M1 A1 M1 M1 A1 A1	Mean. For $0{\cdot}05^2$ etc. For $\times5{\cdot}2^2$ etc. Accept sd = $\sqrt{1{\cdot}4249}$ = 1·1937. c.a.o.	6
				18

Q3				
(a) (i)	$H_0 : \mu_D = 0$ $H_1 : \mu_D > 0$ Where μ_D is the (population) mean reduction in absenteeism. Must assume Normality … … of differences.	B1 B1 B1 B1	Both. Accept alternatives e.g. $\mu_D < 0$ for H_1, or $\mu_A - \mu_B$ etc provided adequately defined. Allow absence of "population" if correct notation μ is used, but do NOT allow "$\bar{X}=...$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a population mean. Hypotheses in words only must include "population".	4
(ii)	Differences (reductions) (before – after) 1·7, 0·7, 0·6, –1·3, 0·1, –0·9, 0·6, –0·7, 0·4, 2·7, 0·9 $\bar{x} = 0\cdot4364$, $s_{n-1} = 1\cdot1518$ ($s_{n-1}{}^2 = 1\cdot3265$) Test statistic is $\dfrac{0\cdot4364 - 0}{\left(\dfrac{1\cdot1518}{\sqrt{11}}\right)}$ $= 1\cdot256(56…)$ Refer to t_{10}. Upper 5% point is 1·812. 1·256 < 1·812, ∴ Result is not significant. Seems there has been no reduction in mean absenteeism.	 B1 M1 A1 M1 A1 E1 E1	Allow "after – before" if consistent with alternatives above. Do not allow $s_n = 1\cdot098$ ($s_n{}^2 = 1\cdot205$). Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: $0 \pm$ (c's 1·812) $\times \frac{1\cdot1518}{\sqrt{11}}$ (= –0·6293, 0·6293) for subsequent comparison with $\bar{x}$. (Or $\bar{x} \pm$ (c's 1·812) $\times \frac{1\cdot1518}{\sqrt{11}}$ (= –0·1929, 1·0657) for comparison with 0.) c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{x}$ scores M1A0, but ft. No ft from here if wrong. No ft from here if wrong. For alternative H_1 expect –1·812 unless it is clear that absolute values are being used. ft only c's test statistic. ft only c's test statistic. Special case: (t_{11} and 1·796) can score 1 of these last 2 marks if either form of conclusion is given.	7

(b)	For "days lost after" $\bar{x} = 4 \cdot 6182$, $s_{n-1} = 1{\cdot}4851$ ($s_{n-1}{}^2 = 2{\cdot}2056$) CI is given by $4{\cdot}6182 \pm$ $2{\cdot}228$ $\times \dfrac{1 \cdot 4851}{\sqrt{11}}$ $= 4{\cdot}6182 \pm 0{\cdot}9976 = (3{\cdot}620(6), 5{\cdot}615(8))$ Assume Normality of population of "days lost after". Since 3·5 lies outside the interval it seems that the target has not been achieved.	B1 M1 B1 M1 A1 E1 E1	Do not allow $s_n = 1{\cdot}4160$ ($s_n{}^2 = 2{\cdot}0051$). ft c's $\bar{x}$ ±. ft c's s_{n-1}. c.a.o. Must be expressed as an interval. ZERO if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{10} is OK.	7
				18

Q4				
(i)				

Obs	21	24	12	15	13	9	6
Exp	26·53	17·22	20·25	11·00	10·94	8·74	5·32

		M1 A1	Probabilities × 100. All Expected frequencies correct.	
	$\therefore X^2 = \frac{(21-26\cdot53)^2}{26\cdot53} + \text{etc}$	M1		
	= 1·1527 + 2·6695 + 3·3611 + 1·4545 + 0·3879 + 0·0077+ 0·0869	A1	At least 4 values correct.	
	= 9·1203	A1		
	d.o.f. = 7 – 1 = 6			
	Refer to χ_6^2.	M1	No ft from here if wrong.	
	Upper 5% point is 12·59	A1	No ft from here if wrong.	
	9·1203 < 12·59 ∴ Result is not significant.	E1	ft only c's test statistic.	
	Evidence suggests the model fits the data at the 5% level.	E1	ft only c's test statistic.	9
(ii)				

Data	Diff = data –124	Rank of |diff|
239	115	9
77	–47	3
179	55	4
221	97	7
100	–24	2
312	188	10
52	–72	5
129	5	1
236	112	8
42	–82	6

		M1 M1 A1	For differences. For ranks of |difference|. All correct. ft from here if ranks wrong.	
	$W_- = 3 + 2 + 5 + 6 = 16$	B1	Or $W_+ = 9 + 4 + 7 + 10 + 1 + 8 = 39$	
	Refer to Wilcoxon single sample (/paired) tables for $n = 10$.	M1	No ft from here if wrong.	
	Lower two-tail 10% point is … … 10.	M1A1	Or, if 39 used, upper point is 45. No ft from here if wrong.	
	16 > 10 ∴ Result is not significant.	E1	Or 39 < 45. ft only c's test statistic.	
	Seems there is no evidence against the median length being 124.	E1	ft only c's test statistic.	9
				18

Mark Scheme 4769
June 2007

1)	$f(x) = \frac{1}{\theta} \quad 0 \le x \le \theta$			
(i)	$E[X] = \frac{\theta}{2}$	B1	Write-down, or by symmetry, or by integration.	
	$E[2\overline{X}] = 2E[\overline{X}] = 2E[X]$ $= \theta$ $\therefore$ unbiased	M1 A1 E1		4
(ii)	$\sum x = 2.3 \quad \therefore \bar{x} = \frac{2.3}{5} = 0.46 \quad \therefore 2\bar{x} = 0.92$	B1		
	But we know $\theta \ge 1$ $\therefore$ estimator can give nonsense answers, i.e. essentially useless	E1 E2	(E1, E1)	4
(iii)	$Y = \max\{X_i\}$, $g(y) = \frac{ny^{n-1}}{\theta^n} \quad 0 \le y \le \theta$			
	MSE $(kY) = E[(kY-\theta)^2] =$	M1		
	$E[k^2Y^2 - 2k\theta Y + \theta^2] =$			
	$k^2E[Y^2] - 2k\theta E[Y] + \theta^2$	1	BEWARE PRINTED ANSWER	
	$\frac{d\text{MSE}}{dk} =$	M1		
	$2kE[Y^2] - 2\theta E[Y] = 0$	M1		
	for $k = \frac{\theta E[Y]}{E[Y^2]}$	A1		
	$\frac{d^2\text{MSE}}{dk^2} = 2E[Y^2] > 0 \quad \therefore$ this is a minimum	M1		
	$E[Y] = \int_0^\theta \frac{ny^n}{\theta^n} dy = \frac{n}{\theta^n}\frac{\theta^{n+1}}{n+1} = \frac{n\theta}{n+1}$	M1 A1		
	$E[Y^2] = \int_0^\theta \frac{ny^{n+1}}{\theta^n} dy = \frac{n}{\theta^n}\frac{\theta^{n+2}}{n+2} = \frac{n\theta^2}{n+2}$	M1 A1		
	$\therefore$ minimising $k = \theta \frac{n\theta}{n+1}\frac{n+2}{n\theta^2} = \frac{n+2}{n+1}$	M1 A1		12
(iv)	With this k, kY is always greater than the sample maximum	E2	(E1 E1)	
	So it does not suffer from the disadvantage in part (ii)	E2	(E1 E1)	4

2(i)	$G(t) = E[t^X] = \sum_{x=0}^{n} \binom{n}{x} (pt)^x (1-p)^{n-x}$ $= [(1-p)+pt]^n$ $= (q+pt)^n$	M1 2 1	 Available as B2 for write-down or as 1+1 for algebra	4
(ii)	$\mu = G'(1) \quad G'(t) = np(q+pt)^{n-1}$ $G'(1) = np \times 1 = np$ $\sigma^2 = G''(1) + \mu - \mu^2$ $G''(t) = n(n-1)p^2(q+pt)^{n-2}$ $G''(1) = n(n-1)p^2$ $\therefore \sigma^2 = n^2p^2 - np^2 + np - n^2p^2$ $= -np^2 + np = npq$	1 1 1 1 M1 1		6
(iii)	$Z = \frac{X-\mu}{\sigma}$ Mean 0, Variance 1	B1	For <u>BOTH</u>	1
(iv)	$M(\theta) = G(e^\theta) = (q+pe^\theta)^n$ $Z = aX + b$ with: $a = \frac{1}{\sigma} = \frac{1}{\sqrt{npq}}$ and $b = -\frac{\mu}{\sigma} = -\sqrt{\frac{np}{q}}$ $M_Z(\theta) = e^{b\theta} M_X(a\theta)$ $\therefore M_Z(\theta) = e^{-\sqrt{\frac{np}{q}}\theta} \left(q + pe^{\frac{1}{\sqrt{npq}}\theta} \right)^n =$ $\left(qe^{-\frac{p\theta}{\sqrt{npq}}} + pe^{\frac{1-p}{\sqrt{npq}}\theta} \right)^n$	1 M1 1 1 1	 BEWARE PRINTED ANSWER	5
(v)	$M_Z(\theta) = (q - \frac{qp\theta}{\sqrt{npq}} + \frac{qp^2\theta^2}{2npq} +$ terms in $n^{-3/2}$, n^{-2}, ………….. + $p + \frac{pq\theta}{\sqrt{npq}} + \frac{pq^2\theta^2}{2npq} + \cdots\cdots)^n =$ $(1 + \frac{\theta^2}{2n} + \cdots\cdots)^n \to$ $e^{\theta^2/2}$	M1 M1 1 1	For expansion of exponential terms For indication that these can be neglected as $n \to \infty$. Use of result given in question	4

(vi)	N(0,1) Because $e^{\theta^2/2}$ is the mgf of N(0,1) and the relationship between distributions and their mgfs is unique	1 E1 E1		3
(vii)	"Unstandardising", $\mathrm{N}(\mu,\sigma^2)$ ie $\mathrm{N}(np,npq)$	1	Parameters need to be given.	1

3(i)	$H_0: \mu_A = \mu_B$ $H_1: \mu_A \neq \mu_B$ Where μ_A, μ_B are the population means Test statistic $\dfrac{26.4 - 25.38}{\sqrt{\dfrac{2.45}{7} + \dfrac{1.40}{5}}} =$ $\dfrac{1.02}{\sqrt{0.63} = 0.7937} = 1.285$ Refer to N(0,1) Double-tailed 5% point is 1.96 Not significant No evidence that the population means differ	1 1 M1 M1 M1 A1 1 1 1 1	Do NOT allow $\overline{X} = \overline{Y}$ or similar Accept absence of "population" if correct notation μ is used. Hypotheses stated verbally <u>must</u> include the word "population". Numerator Denominator two separate terms correct No FT if wrong No FT if wrong	10
(ii)	CI (for $\mu_A - \mu_B$) is $1.02 \pm$ $1.645 \times$ $0.7937 =$ $1.02 \pm 1.3056 =$ (– 0.2856, 2.3256)	M1 B1 M1 A1 cao	Zero out of 4 if not N(0,1)	4
(iii)	H_0 is accepted if $-1.96 <$ test statistic < 1.96 i.e. if $-1.96 < \dfrac{\bar{x} - \bar{y}}{0.7937} < 1.96$ i.e. if $-1.556 < \bar{x} - \bar{y} < 1.556$ In fact, $\overline{X} - \overline{Y} \sim \mathrm{N}(2, 0.7937^2)$ So we want $\mathrm{P}(-1.556 < \mathrm{N}(2, 0.7937^2) < 1.556) =$ $\mathrm{P}\left(\dfrac{-1.556 - 2}{0.7937} < \mathrm{N}(0,1) < \dfrac{1.556 - 2}{0.7937}\right) =$ $\mathrm{P}(-4.48 < \mathrm{N}(0,1) < -0.5594) = 0.2879$	M1 M1 A1 M1 M1 M1 A1 cao	SC1 Same wrong test can get M1,M1,A0. SC2 Use of 1.645 gets 2 out of 3. BEWARE PRINTED ANSWER Standardising	7
(iv)	Wilcoxon would give protection if assumption of Normality is wrong. Wilcoxon could not really be applied if underlying variances are indeed different. Wilcoxon would be less powerful (worse Type II error behaviour) with such small samples if Normality is correct.	E1 E1 E1		3

<table>
<tr><td>4 (i)</td><td>There might be some consistent source of plot-to-plot variation that has inflated the residual and which the design has failed to cater for.</td><td>E2</td><td>E1 – Some reference to extra variation.
E1 – Some indication of a reason.</td><td>2</td></tr>
<tr><td>(ii)</td><td>Variation between the fertilisers should be compared with experimental error.

If the residual is inflated so that it measures more than experimental error, the comparison of between - fertilisers variation with it is less likely to reach significance.</td><td>E1

E2</td><td>

(E1, E1)</td><td>3</td></tr>
<tr><td>(iii)</td><td>Randomised blocks
<table><tr><td>C
B
A
D
E</td><td>.
.
.</td><td>.
.
.</td></tr></table>SPECIAL CASE: Latin Square $\frac{2}{4}$ (1, E1)</td><td>1
E1
E1
E1</td><td>Blocks (strips) clearly correctly oriented w.r.t. fertiliser gradient.

All fertilisers appear in a block.

Different (random) arrangements in the blocks.</td><td>4</td></tr>
<tr><td>(iv)</td><td>Totals are: 95.0 123.2 86.8 130.2 67.4
(each from sample of size 4)
Grand total 502.6
"Correction factor" CF = $\frac{502.6^2}{20} = 12630.338$
Total SS = 13610.22 – CF = 979.882

Between fertilisers SS =
$\frac{95.0^2}{4} + \ldots + \frac{67.4^2}{4} - \text{CF} =$
13308.07 – CF = 677.732

Residual SS (by subtraction) =
979.882 – 677.732=302.15
<table><tr><th>Source of variation</th><th>SS</th><th>df</th><th>MS</th><th>MS Ratio</th></tr><tr><td>Between fertiliser</td><td>677.732</td><td>4</td><td>169.433</td><td>8.41</td></tr><tr><td>Residual</td><td>302.15</td><td>15</td><td>20.143</td><td></td></tr><tr><td>Total</td><td>979.882</td><td>19</td><td></td><td></td></tr></table>Refer to $F_{4, 15}$
-upper 5% point is 3.06
Significant
- seems effects of fertilisers are not all the same</td><td>M1
M1
A1

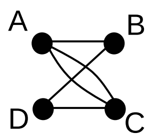
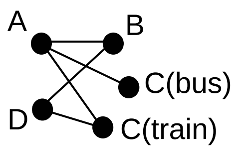
M1
M1
1,A1
1

1
1
1
1</td><td>
For correct method for any two
If each calculated SS is correct

No FT if wrong
No FT if wrong</td><td>12</td></tr>
<tr><td>(vii)</td><td>Independent N (0, σ^2 [constant])</td><td>1
1
1</td><td></td><td>3</td></tr>
</table>

Mark Scheme 4771
June 2007

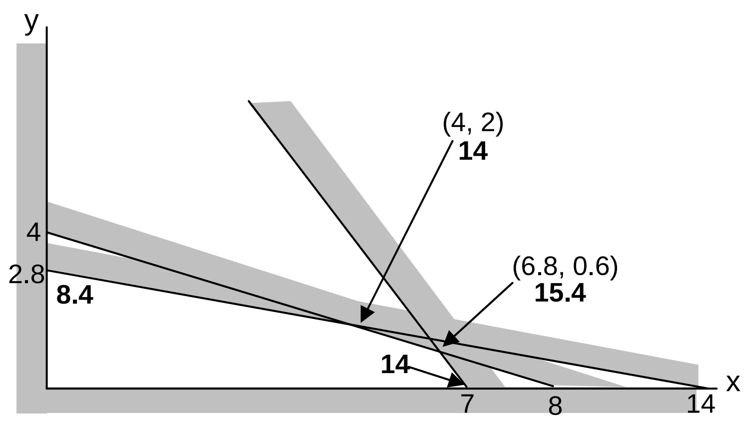
1.

(i) 	M1 4 nodes and 5 arcs A1
(ii) No. Two arcs AC.	M1 A1
(iii) 	M1 5 nodes and 5 arcs A1
(iv) No. ABDC(train)A is a cycle.	M1 A1

2.

(i) Rucksack 1: 14; 6 Rucksack 2: 11; 9 final item will not fit.	M1 6 must be in R1 A1 B1
(ii) Order: 14, 11, 9, 6, 6 Rucksack 1: 14; 11 Rucksack 2: 9; 6; 6	B1 ordering M1 11 in R1 A1
(iii) Rucksack 1: 14; 9 Rucksack 2: 11; 6; 6 e.g. weights.	B1 B1

3.

 Optimum of 15.4 at x = 6.8 and y = 0.6.	B1 axes scaled & used M1 lines A1 B1 shading M1 two intersection A1 points M1 solution A1 (or by using the objective gradient to identify the optimal point)

4.

(i)

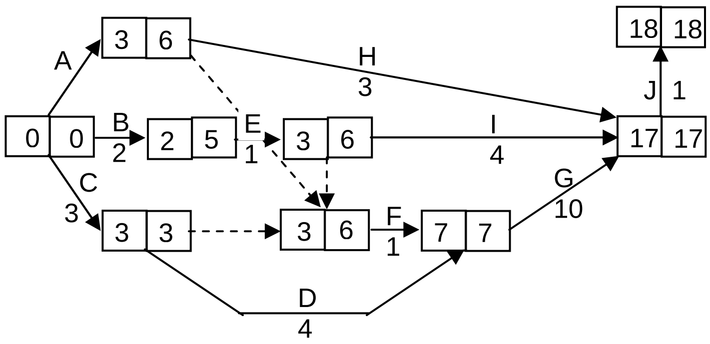
Activity		Duration (minutes)	Immediate predecessors
A	Rig foresail	3	–
B	Lower sprayhood	2	–
C	Start engine	3	–
D	Pump out bilges	4	C
E	Rig mainsail	1	B
F	Cast off mooring ropes	1	A, C, E
G	Motor out of harbour	10	D, F
H	Raise foresail	3	A
I	Raise mainsail	4	E
J	Stop engine and start sailing	1	G, H, I

B1 A, B, C, D, E, H & I

B1 F

B1 G and J

(ii)



M1 A1 forward pass

M1 A1 backward pass

Critical activities: C; D; G; J
Project duration: 18 minutes

B1
B1

(iii) H and I — B1

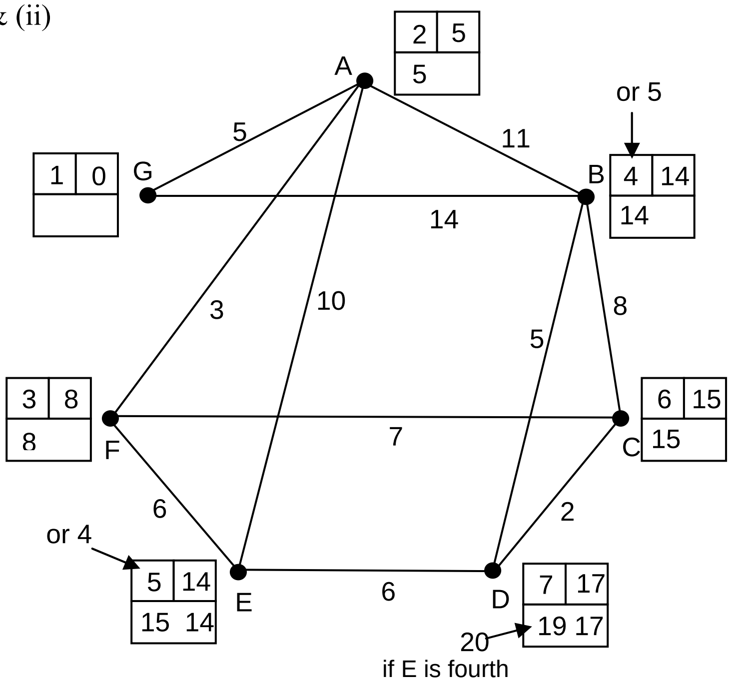
(iv) 25 mins — B1

Must do A, B, E, C, F, D (in appropriate order) then H and I with G, then J. — B1

(v) 18 mins — B1

e.g. Colin does C, D — B1
Crew does A, B, E, F — B1
Thence G et al — B1

5.

(i) & (ii) 	M1 A1 arcs A1 arc weights M1 Dijkstra A1 labels A1 order of labelling A2 working values
Route: G A F C D Weight: 17	B1 B1
(iii) Route: G B C F E D or G B A E D Weight: 6 Any capacitated route application.	B1 B1 B1
(iv) Compute min(label, arc) and update working value if result is larger than current working value. Label unlabelled vertex with largest working value.	B1 B1 B1

6.

(i)(a) e.g. Dry: 00 – 39 Wet: 40 – 69 Snowy: 70 – 99	M1 A1	proportions efficient
(b) e.g. Dry: 00 – 19 Wet: 20 – 69 Snowy: 70 – 99	M1 A1	proportions efficient
(c) e.g. Dry: 00 – 27 Wet: 28 – 55 Snowy: 56 – 97 Reject: 98 & 99	M1 A1 A1	reject some proportions reject 2
(ii) D (today) $\to$ D $\to$ S $\to$ S $\to$ W $\to$ S $\to$ D $\to$ D	M1 A1 A1 A1	applying their rules sometimes dry rules wet rules snowy rules
(iii) 3/7 (or 4/8)	B1	
(iv) a (much) longer simulation run, with a "settling in" period ignored.	B1 B1	
(v) Defining days as dry, wet or snowy is problematical. Assuming that the transition probabilities remain constant. Weather depends on more than just previous day's weather	B1 B1	

Mark Scheme 4772
June 2007

1.

(a)(i) He should salute it. — B1

Since all objects which don't move are painted any unpainted object must move, and anything that moves must be saluted. — M1 A1

(ii) We do not know.

We do not know about painted objects. Some will have been painted because they do not move, but there may be some objects which move which are painted. We do not know whether this object moves or not. — B1, M1 A1

(b)

((m	$\Rightarrow$	s)	$\wedge$	(~	m	$\Rightarrow$	p))	$\wedge$	~	p	$\Rightarrow$	s
1	1	1	**1**	0	1	1	1	0	**0**	1	**1**	1
1	1	1	**1**	0	1	1	0	1	**1**	0	**1**	1
1	0	0	**0**	0	1	1	1	0	**0**	1	**1**	0
1	0	0	**0**	0	1	1	0	0	**1**	0	**1**	0
0	1	1	**1**	1	0	1	1	0	**0**	1	**1**	1
0	1	1	**0**	1	0	0	0	0	**1**	0	**1**	1
0	1	0	**1**	1	0	1	1	0	**0**	1	**1**	0
0	1	0	**0**	1	0	0	0	0	**1**	0	**1**	0

M1 8 rows
A1 $m \Rightarrow s$
A1 $\sim m \Rightarrow p$
A1 first $\wedge$
A1 second $\wedge$
A1 result

(c) $\big((m \Rightarrow s) \wedge (\sim m \Rightarrow p)\big) \wedge \sim p$

$\Leftrightarrow \big(\sim p \wedge (\sim m \Rightarrow p)\big) \wedge (m \Rightarrow s)$

$\Leftrightarrow \big(\sim p \wedge (\sim p \Rightarrow m)\big) \wedge (m \Rightarrow s)$ (contrapositive)

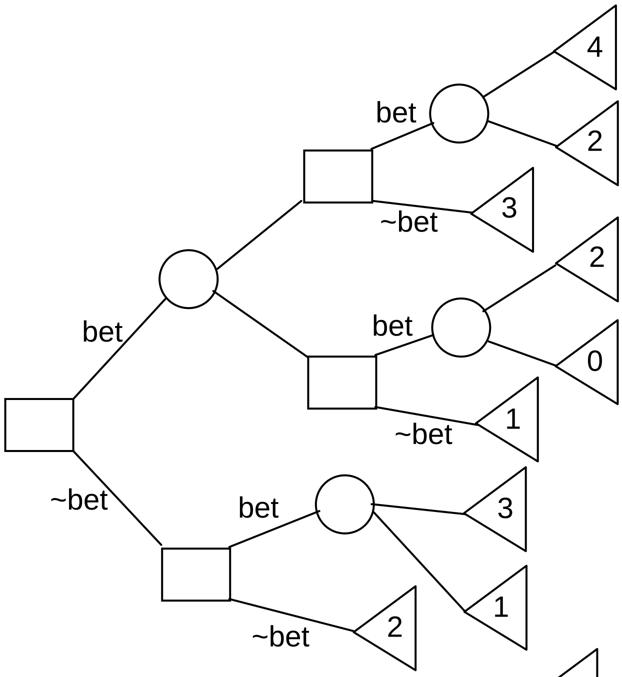
$\Rightarrow m \wedge (m \Rightarrow s)$ (modus ponens)

$\Rightarrow s$ (modus ponens)

M1
A1 reordering
A1 contrapositive
A1 modus ponens

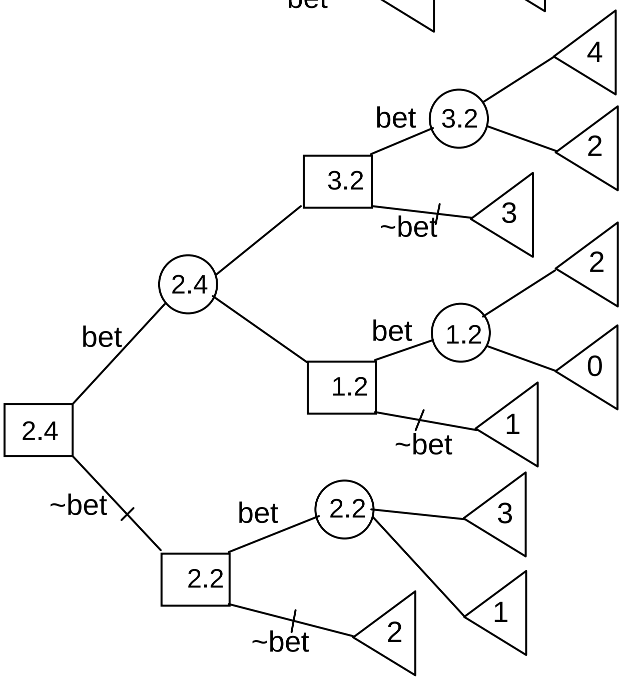
2.

(i)



Mark	Comment
M1	
A1	first D box
A1	D box on ~bet branch
A1	P box on bet branch
A1	D boxes following P box
A1	remaining P boxes
M1	outcomes
A1	

(ii)(A)

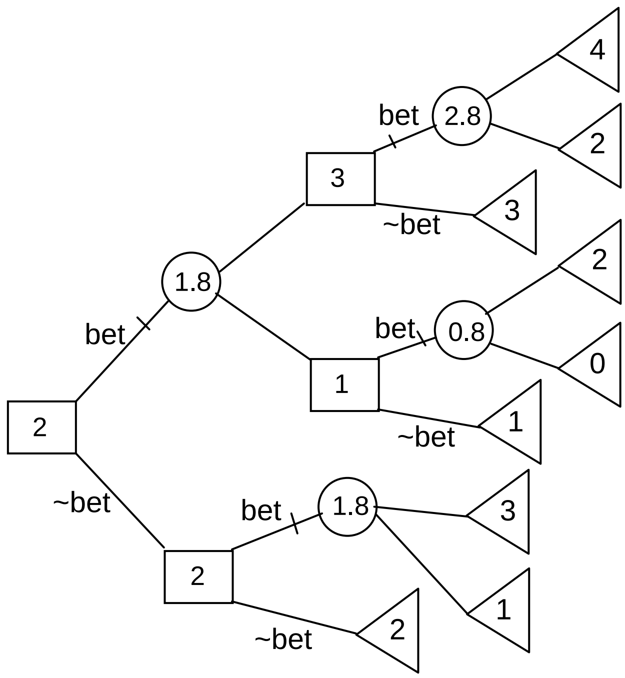


Mark	Comment
M1	
A1	

EMV = 2.4 by betting and betting again

Mark	Comment
B1	course of action

2(cont).

(ii)(B) 	A1
EMV = 2 by not betting	B1 course of action
(iii) $2^{0.5}\times 0.4 = 0.566 < 1$, but $2^{1.5}\times 0.4 = 1.131 > 1$	M1 A1 A1

3.

(i)

	1	**2**	**3**	**4**
1	6	3	6	5
2	3	4	3	2
3	6	3	2	1
4	5	2	1	2

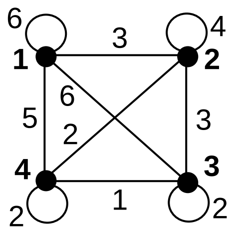
	1	**2**	**3**	**4**
1	2	2	2	2
2	1	4	4	4
3	4	4	4	4
4	2	2	3	3

M1 distances
A2 6 changes
(–1 each error)
M1 a correct update
A1 1 to 3 route (2)
A2 rest
(–1 each error)

(ii) Distance from row 1 col 3 of distance matrix (6)
Route from row 1 col 3 of route matrix (2), then from row 2 col 3 (4), then from row 4 col 3 (3). So 1 2 4 3.

B1 B1
B1
B1

(iii)

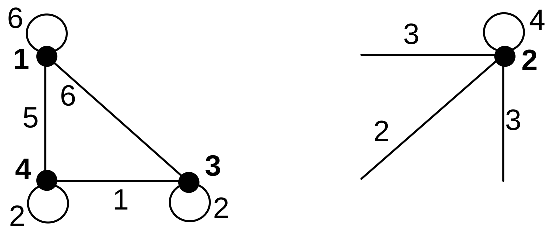


B1 whether or not loops included

(iv) **1 2 4 3 1**
length = 12
1 2 4 3 4 2 1

B1
B1
B1

(v)



MST has length 6, so lower bound = 6 + 2 + 3 = 11

M1
A1 MST
A1 add back

(vi) TSP length is either 11 or 12

B1 11 to 12
B1 either 11 or 12

4.

(i)

P	x	y	s_1	s_2	RHS
1	–1	–1	0	0	0
0	2	1	1	0	1250
0	2	–1	0	1	0
1	1	0	1	0	1250
0	2	1	1	0	1250
0	4	0	1	1	1250

M1 initial tableau
A1

M1 pivot
A2 (–1 each error)

1250 m^2 of paving and no decking — B1 interpretation

(ii) 2-phase

A	P	x	y	s_1	s_2	s_3	a	RHS
1	0	1	0	0	0	–1	0	200
0	1	1	0	1	0	0	0	1250
0	0	2	1	1	0	0	0	1250
0	0	4	0	1	1	0	0	1250
0	0	1	0	0	0	–1	1	200
1	0	0	0	0	0	0	–1	0
0	1	0	0	1	0	1	–1	1050
0	0	0	1	1	0	2	–2	850
0	0	0	0	1	1	4	–4	450
0	0	1	0	0	0	–1	1	200

M1 A1 new objective
B1 surplus
B1 artificial
B1 new constraint

M1
A2

Big-M alternative

P	*x*	*y*	s_1	s_2	s_3	*a*	*RHS*
1	*1–M*	*0*	*1*	*0*	*M*	*0*	*1250–2M*
0	*2*	*1*	*1*	*0*	*0*	*0*	*1250*
0	*4*	*0*	*1*	*1*	*0*	*0*	*1250*
0	*1*	*0*	*0*	*0*	*–1*	*1*	*200*
1	*0*	*0*	*1*	*0*	*1*	*M–1*	*1050*
0	*0*	*1*	*1*	*0*	*2*	*–2*	*850*
0	*0*	*0*	*1*	*1*	*4*	*–4*	*450*
0	*1*	*0*	*0*	*0*	*–1*	*1*	*200*

M1 A1 new objective
B1 surplus
B1 artificial
B1 new constraint

M1
A2

850 m^2 of paving and 200 m^2 of decking. — A1 interpretation

(iii)

C	x	y	s_1	s_2	s_3	s_4	RHS
1	0	0	1.25	0	1.75	0	1212.5
0	0	1	1	0	2	0	850
0	0	0	1	1	4	0	450
0	1	0	0	0	–1	0	200
0	0	0	1	0	1	1	50
1	0	0	–0.5	0	0	–1.75	1125
0	0	1	–1	0	0	–2	750
0	0	0	–3	1	0	–4	250
0	1	0	1	0	0	1	250
0	0	0	1	0	1	1	50

750 m^2 of paving and 250 m^2 of decking at an annual cost of £1125

B1 new objective

B1 new constraint

M1

A1

A1 interpretation

Mark Scheme 4773
June 2007

1.

(i) $u_{n+2} = u_{n+1} + pu_n$	M1 A1
(ii) Auxiliary equation is $\lambda^2 - \lambda - 0.11 = 0$ Solution is $u_n = 22.5(1.1)^n - 2.5(-0.1)^n$	M1 A1 M1 gen homogeneous A1 with 1.1 & –0.1 B1 case 1($u_0 = 20$) +case 2($u_1 = 25$) M1 simultaneous A1 22.5 and −2.5 B1 final answer
(iii)	B1 recurrence relation B1 checking formula B1 discretising

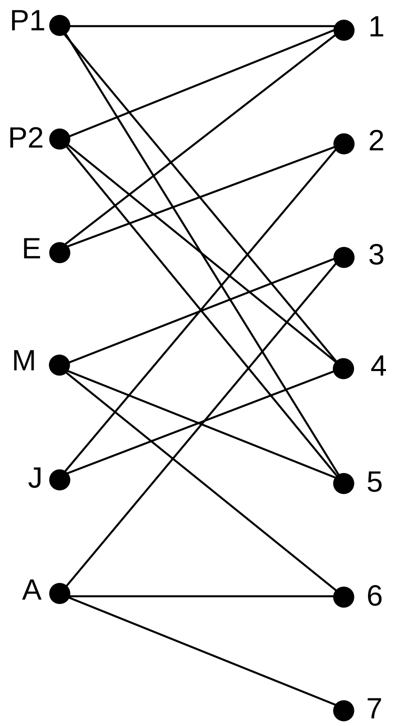
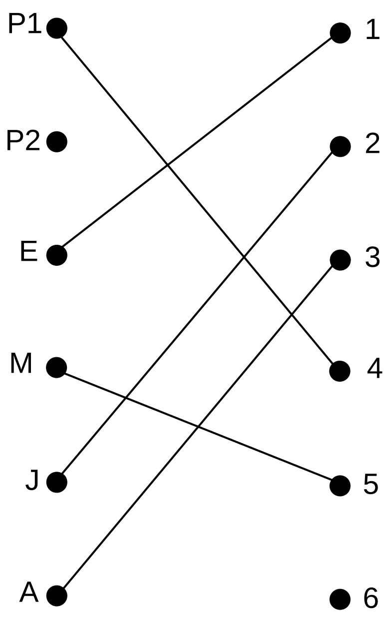
Rec rel		Formula	Int RR
20.0000	0	20.0000	20
25.0000	1	25.0000	25
27.2000	2	27.2000	27
29.9500	3	29.9500	30
32.9420	4	32.9420	33
36.2365	5	36.2365	36
39.8601	6	39.8601	40
43.8461	7	43.8461	44
48.2307	8	48.2307	48
53.0538	9	53.0538	53
58.3592	10	58.3592	58
64.1951	11	64.1951	64
70.6146	12	70.6146	70
77.6761	13	77.6761	77
85.4437	14	85.4437	85
93.9881	15	93.9881	93
103.3869	16	103.3869	102
113.7256	17	113.7256	112
125.0981	18	125.0981	123
137.6080	19	137.6080	135
151.3687	20	151.3687	149

Formula: =INT(H3+B$2*H2+0.5)	B1
(iv) $v_{n+2} = (1-r)v_{n+1} + pv_n$	M1 A1

1. (cont)

(v) Pruning $r =$ 0.026 20 25 27 29 31 33 36 39 42 45 48 52 56 60 65 70 75 81 87 94 101	B1
$r = 0.025$ to 0.027	B1

2.

(i) 	M1 A1
(ii) e.g. locations 3, 6 and 7 for only two trees, so one must be rejected. Therefore other 6 locations needed.	B1 B1
(iii) 	B1
(iv) e.g. P2 – 5 – M – 6	M1 A1
P1 P2 E M J A 4 5 1 6 2 3	A1

2 (cont).

(v) Max P11+P14+P15+P21+P24+P25+E1+E2+M3+M5+M6 +J2+J4+A3+A6+A7 st P11+P14+P15<=1 P21+P24+P25<=1 E1+E2<=1 M3+M5+M6<=1 J2+J4<=1 A3+A6+A7<=1 P11+P21+E1<=1 E2+J2<=1 M3+A3<=1 P14+P24+J4<=1 P15+P25+M5<=1 M6+A6<=1 A7<=1 End	M1 objective A1 M1 tree constraints A2 (–1 each error) M1 location A2 constraints (–1 each error)
LP OPTIMUM FOUND AT STEP 13 OBJECTIVE FUNCTION VALUE 1) 6.000000	B1 running
VARIABLE VALUE REDUCED COST P11 0.000000 0.000000 P14 0.000000 0.000000 P15 1.000000 0.000000 P21 0.000000 0.000000 P24 1.000000 0.000000 P25 0.000000 0.000000 E1 1.000000 0.000000 E2 0.000000 0.000000 M3 0.000000 0.000000 M5 0.000000 1.000000 M6 1.000000 0.000000 J2 1.000000 0.000000 J4 0.000000 0.000000 A3 0.000000 0.000000 A6 0.000000 0.000000 A7 1.000000 0.000000	
P1 P2 E M J A 5 4 1 6 2 7	B1 interpretation

3.

(i) e.g. C2 C3 C5 C7 C9 C11	M1 A1
(ii)	
Min C1+C2+C3+C4+C5+C6+C7+C8+C9+C10+C11+C12	M1 objective
st C1+C2+C3+C4>=1	A1
C4+C5+C6>=1	
C6+C7+C8+C9+C10>=1	M1
C1+C10+C11>=1	A5 constraints
C2>=1	(–1 each error)
C3+C8+C12>=1	
C5+C12>=1	
C11>=1	
C9>=1	
C7>=1	
end	
(iii)	
LP OPTIMUM FOUND AT STEP 7	B1 running
OBJECTIVE FUNCTION VALUE	
1) 6.000000	
VARIABLE VALUE REDUCED COST	
C1 0.000000 0.000000	
C2 1.000000 0.000000	
C3 0.000000 1.000000	
C4 1.000000 0.000000	
C5 0.000000 0.000000	
C6 0.000000 0.000000	
C7 1.000000 0.000000	
C8 0.000000 1.000000	
C9 1.000000 0.000000	
C10 0.000000 0.000000	
C11 1.000000 0.000000	
C12 1.000000 0.000000	
Use locations 2, 4, 7, 9, 11 and 12.	B1
6 cameras needed	B1
(iv) New objective:	M1
5C1+2C2+3C3+5C4+4C5+1.5C6+2C7+2C8+5C9	A1
+3C10+4C11+7C12	
(v) Running	B1
Use locations 2, 5, 7, 8, 9 and 11.	B1
Cost = £19000	B1

4.

(i) e.g.	
1 0 =LOOKUP(RAND(),B1:B3,A1:A3) 2 0.1 3 0.4	B1 rand B1 probs B1 outcomes
(ii) =LOOKUP(RAND(),$B$3:$B$5,$A$3:$A$5) + accumulation e.g.	M1 formula A1 repeats B1 accumulation
2 2 3 5 2 7 3 10 2 12 3 15 3 18 3 21 3 24 3 27 2 29 2 31 3 34 2 36 2 38 3 41	
(iii) e.g.	
day 14 day 15 day16 no. of replacements 0 0 1 5 1 0 0 6 0 0 1 5 0 0 1 5 1 0 0 6 0 1 0 5 0 1 0 6 1 0 1 5 0 1 0 5 1 0 1 6	M1 first run A1 B1 repetitions
0.4 0.3 0.5 5.4	B1 probabilities B1 expected no. of replacements

Q4 (cont)

(iv) e.g.

				Replacements day 1	Replacements day 2
1	0	1	1	2	6
2	0.1	1	2	0	7
		2	4	0	7
		2	6	1	6
		2	8	1	6
		2	10	0	7
		2	12	1	6
		2	14	0	7
		1	15	0	7
		2	17	1	6
		2	19		
		2	21	0.6	6.5

B1 changed probabilities

B1 repetitions

B1 results

B1 averages

5.4*(50+25) = 405 versus B1

0.6*(50+25) + 6.5*(30+25) = 402.5 B1

(v) More repetitions. B1

Mark Scheme 4776
June 2007

1	x	f(x)					
	1	2.414214	< 3				
	1.4	3.509193	> 3	change of sign hence root in (1, 1.4)			**[M1A1]**
	1.2	2.92324	< 3	root in (1.2, 1.4)	est 1.3	mpe 0.1	
	1.3	3.206575	> 3	root in (1.2, 1.3)	est 1.25	mpe 0.05	
	1.25	3.0625	> 3	root in (1.2, 1.25)	**est 1.225**	mpe 0.025	**[M1A1A1]**
						mpe	**[A1]**

mpe reduces by a factor of 2, 4, 8, …
Better than a factor of 5 after 3 more iterations **[M1A1]**

[TOTAL 8]

2	x	1/(1+x^4)		values:	**[A1]**
	0	1	M = 0.498054		**[A1]**
	0.25	0.996109	T = 0.485294		**[A1]**
	0.5	0.941176	S = (2M + T) / 3 = 0.493801		**[M1]**

h	S	ΔS		
0.5	0.493801			
0.25	0.493952	0.000151	/ one term enough	**[M1]**

Extrapolating: 0.493952 + 0.000151 $(1/16 + 1/16^2 + ...)$ = 0.493962 **[M1A1]**
0.49396 appears reliable. (Accept 0.493962) **[A1]**

[TOTAL 8]

3	Cosine rule:	5.204972	**[M1A1]**
	Approx formula:	5.205228	**[A1]**
	Absolute error:	0.000255	**[B1]**
	Relative error:	0.000049	**[B1]**

[TOTAL 5]

4(i) r represents the relative error in X **[E1]**

(ii) $X^n = x^n(1 + r)^n \approx x^n(1 + nr)$ for small r **[A1E1]**
hence relative error is nr

(iii)

pi =	3.141593	(abs error:	0.001264)	**[M1]**
22/7 =	3.142857	rel error:	0.000402	**[A1]**

approx relative error in π^2 (multiply by 2): 0.000805 (0.0008)
approx relative error in sqrt(π) (multiply by 0.5): 0.000201 (0.0002) **[M1M1A1]**

[TOTAL 8]

5	x	f(x)	
	-1	3	
	0	2	
	4	9	

f(x) = 3 x (x-4) / (-1)(-5) + **[M1A1]**
2 (x+1)(x-4) / (1)(-4) + **[A1]**
9 (x+1) x / (5)(4) **[A1]**

f(x) = $0.55 x^2 - 0.45 x + 2$ **[A1]**

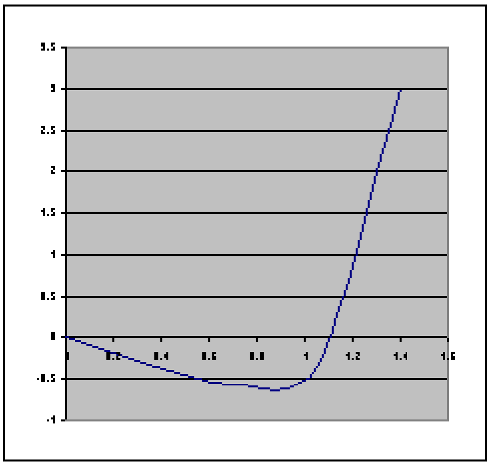
f '(x) = 1.1 x - 0.45 **[B1]**
Hence minimum at x = 0.45 / 1.1 = 0.41 **[A1]**

[TOTAL 7]

6(i) Sketch showing curve, root, initial estimate, tangent, intersection of tangent with x-axis as improved estimate **[E1E1E1]**

[subtotal 3]

(ii)



Sketch showing root, α **[G2]**

E.g. starting values just to the left of the root can produce an x1 that is the wrong side of the asymptote **[M1]** **[E1]**

E.g. starting values further left can converge to zero. **[M1]** **[E1]**

[subtotal 6]

(iii) Convincing algebra to obtain the N-R formula **[M1A1]**

r	0	1	2	3	4
xr	1.2	1.169346	1.165609	1.165561	1.165561

[M1A1A1]

root is 1.1656 to 4 dp **[A1]**

differences from root	-0.03065	-0.00374	-4.8E-05	Accept diffs of successive terms
ratio of differences		0.1219	0.012877	**[M1A1]**

ratio of differences is decreasing (by a large factor), so faster than first order **[E1]**

[subtotal 9]

[TOTAL 18]

7

(i)

x	g(x)	Δg	Δ^2g
1	2.87		
2	4.73	1.86	
3	6.23	1.50	-0.36
4	7.36	1.13	-0.37
5	8.05	0.69	-0.44

[M1A1A1]

Not quadratic **[E1]**

Because second differences not constant **[E1]**

[subtotal 5]

(ii)

x	g(x)	Δg	Δ^2g
1	2.87		
3	6.23	3.36	
5	8.05	1.82	-1.54

[B1]

Q(x) = 2.87 + 3.36 (x - 1)/2 - 1.54 (x - 1)(x - 3)/8 **[M1A1A1A1]**

= 0.6125 + 2.45 x - 0.1925 x^2 **[A1A1A1]**

[subtotal 8]

(iii)

x	Q(x)	g(x)	error	rel error
2	4.7425	4.73	0.0125	0.002643
4	7.3325	7.36	-0.0275	-0.00374

Q: **[A1A1]**

errors: **[A1]**

rel errors: **[M1A1]**

[subtotal 5]

[TOTAL 18]

Mark Scheme 4777
June 2007

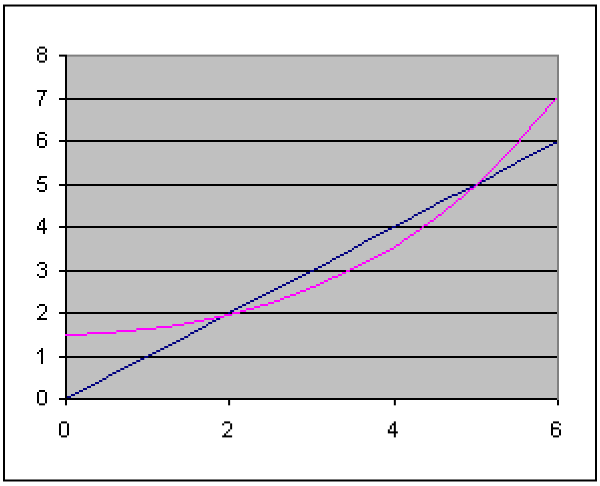
1(i) Convincing algebra to $k = (x_2 - x_1)/(x_1 - x_0)$ **[M1A1]**

Convincing algebra to $\alpha = (x_2 - k\,x_1)/(1 - k)$ or equivalent **[M1A1A1]**

[subtotal 5]

(ii)

x	y=x	y=f(x)
0	0	1.5
0.5	0.5	1.527842
1	1	1.612144
1.5	1.5	1.755252
2	2	1.961151
2.5	2.5	2.235574
3	3	2.586161
3.5	3.5	3.022674
4	4	3.557265
4.5	4.5	4.204819
5	5	4.983366
5.5	5.5	5.914581
6	6	7.024391



[G2]

converges	2	*diverges*	4.5	5	5.5	*set up*	
slowly	1.961151	*from*	4.204819	4.983366	5.914581	*iteration*	**[M1A1]**
to	1.942783	*root*	3.807921	4.95514	6.820878		
root	1.934241	*near*	3.339412	4.90763	9.3175	*near 2*	**[A1]**
near	1.9303	*5*	2.872419	4.828739	21.8726		
2	1.928489		2.488967	4.70068	1466.344	*near 5*	**[A1A1]**
	1.927657		2.228729	4.500432	1.9E+212	(theoretical arguments	
	1.927276		2.07777	4.205432	#NUM!	involving f 'acceptable)	

[subtotal 7]

(iii)

x0	x1	x2	k	new x0			
2	1.961151	1.942783	0.472807	1.92631		k	**[M1A1]**
1.92631	1.926659	1.926818	0.458143	1.926953		est of root	**[M1A1]**
1.926953	1.926953	1.926953	0.45827	**1.926953**	**=alpha**	use as x0	**[M1]**
						iterate	**[M1A1]**

x0	x1	x2	k	new x0			
5	4.983366	4.95514	1.696813	5.023872		alpha	**[A1]**
5.023872	5.024167	5.024673	1.71656	5.023461			
5.023461	5.023461	5.023461	1.716217	**5.023461**	**= beta**	beta	**[A1]**

x0	x1	x2	k	new x0		
4.6	4.349412	3.996895	1.406756	5.216066		
5.216066	5.365628	5.647933	1.887551	5.047555	**range**	
5.047555	5.064991	5.095267	1.73646	5.02388	**4.6 to 5.7**	**[M1A1A1]**
5.02388	5.024181	5.024697	1.716567	5.023461		
5.023461	5.023461	5.023461	1.716217	**5.023461**		

[subtotal 12]

[TOTAL 24]

2 (i) Substitute $f(x) = 1, x^2, x^4, x^6$ into the integration fomula **[M1M1M1M1]**

Obtain $a + b = h$ **[A1]**

$a\alpha^2 + b\beta^2 = h^3/3$ **[A1]**

$a\alpha^4 + b\beta^4 = h^5/5$ **[A1]**

$(a\alpha^6 + b\beta^6 = h^7/7)$

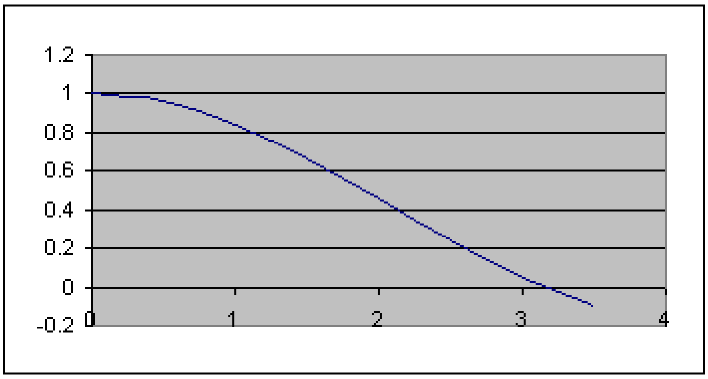
[subtotal 7]

(ii) E.g.

x	0.1	0.01	0.001	
sin(x) / x	0.998334	0.999983	1	**[B1]**

(ii)

x	sin(x) / x
0	1
0.5	0.958851
1	0.841471
1.5	0.664997
2	0.454649
2.5	0.239389
3	0.04704
3.5	-0.10022



[G2]

Single application of Gaussian 4-pt rule

m=	1.570796	h=	1.570796			
α, β	x	f(x)	a, b			
-0.86114	0.218127	0.992089	0.347855	0.345103		**set up**
-0.33998	1.036755	0.830241	0.652145	0.541438		**[M4]**
0.339981	2.104837	0.408942	0.652145	0.26669		
0.861136	2.923466	0.074022	0.347855	0.025749		
			sum:	1.17898		
			integral:	**1.851937**		**[A1]**

Subdividing the interval

m=	0.785398	h=	0.785398			
			gives	1.370762		**[M1A1]**
m=	2.356194	h=	0.785398			
			gives	0.481175		**[M1A1]**
			sum	**1.851937**	(= 6dp)	**[A1]**

[subtotal 13]

(iii) By trial and error

m=	0.53242	h=	0.53242		
α, β	x	f(x)	a, b		
-0.86114	0.073934	0.999089	0.347855	0.347538	**trial**
-0.33998	0.351407	0.979546	0.652145	0.638806	**and**
0.339981	0.713433	0.917302	0.652145	0.598214	**error**
0.861136	0.990906	0.8442	0.347855	0.293659	**[M1A1]**
				1	
		Hence t = 2m =	**1.065**	(1.06484)	**[M1A1]**

[subtotal 4]

[TOTAL 24]

3	**(i)** Euler	h	x	y	y '	new y	
		0.2	0	0	0.1	0.02	
		0.2	0.2	0.02	0.30202	0.080404	**setup**
		0.2	0.4	0.080404	0.508372	0.182079	**[M2]**
		0.2	0.6	0.182079	0.719971	0.326073	
		0.2	0.8	0.326073	0.938552	0.513783	**estimates**
		0.2	1	0.513783			**[A1A1]**

h	y(1)	diffs	ratio of diffs		
0.2	0.513783				**differences**
0.1	0.569802	0.056019			**[M1A1]**
0.05	0.598337	0.028535	0.509387		
0.025	0.612748	0.014411	0.505038	approx 0.5, so first order	**[E1]**

[subtotal 7]

(ii)	Modified	h	x	y	k1	k2	new y	
	Euler	0.2	0	0	0.02	0.060404	0.040202	
		0.2	0.2	0.040202	0.06082	0.102126	0.121675	**setup**
		0.2	0.4	0.121675	0.102588	0.145028	0.245483	**[M2]**
		0.2	0.6	0.245483	0.145565	0.189571	0.413051	
		0.2	0.8	0.413051	0.190228	0.236562	0.626446	**estimates**
		0.2	1	0.626446				**[A1A1]**

h	y(1)	diffs	ratio of diffs		
0.2	0.626446				**differences**
0.1	0.627065	0.000619			**[A1]**
0.05	0.627213	0.000147	0.238113		
0.025	0.627249	3.58E-05	0.242993	approx 0.25, so second order	**[E1]**

[subtotal 6]

(iii) predictor-corrector

h	x	y	y '	pred	corr1	corr2	corr3	
0.2	0	0	0.1	0.02	0.040202	0.04041	0.040412	
0.2	0.2	0.040412	0.304124	0.101237	0.12189	0.122121	0.122124	**setup**
0.2	0.4	0.122124	0.512989	0.224722	0.245942	0.246211	0.246214	**[M3]**
0.2	0.6	0.246214	0.727917	0.391798	0.413802	0.414132	0.414137	
0.2	0.8	0.414137	0.951306	0.604398	0.627569	0.627998	0.628006	
0.2	1	0.628006						**estimates**
								[A1A1]

h	y(1)				
0.2	0.628006				**differences**
0.1	0.627447	-0.00056			**[A1]**
0.05	0.627307	-0.00014	0.250462	Still second order. Differences very	
0.025	0.627272	-3.5E-05	0.250113	similar in magnitude to modified Euler.	**[E1E1]**

[subtotal 8]

(iv)	mod Euler	pre-corr	average	diffs	ratio of diffs		
							values
	0.626446	0.628006	0.627226				**[A1]**
	0.627065	0.627447	0.627256	3.04E-05			**differences**
	0.627213	0.627307	0.62726	3.78E-06	0.124555	approx 0.125	**[A1]**
	0.627249	0.627272	0.627261	4.21E-07	0.111332	so third order	**[E1]**

[subtotal 3]

[TOTAL 24]

4 (i)

0	1	2	3	1			
3	0	1	2	2	x1 =	0.666667	**elimin'n**
2	3	0	1	3			**[M1M1M1]**
1	2	3	0	4			**[A1A1]**
	1	2	3	1			
	3	-0.66667	-0.33333	1.666667	x2 =	0.666667	**back sub**
	2	2.666667	-0.66667	3.333333			**[M1]**
		2.222222	3.111111	0.444444			**solutions**
		3.111111	-0.44444	2.222222	x3 =	0.666667	**[A1A1A1A1]**
			3.428571	-1.14286	x4 =	-0.33333	

pivot (shaded) is element **[M1]**
of largest magnitude in column **[E1]**
Demonstrate check by substituting values back into equations. **[B1]**

[subtotal 13]

(ii)

Apply to	1	0	0	0		**at least one v**
v =	0	1	0	0		**[M1]**
	0	0	1	0	*NB: clear*	**other three**
	0	0	0	1	*evidence*	**[M1]**
					required	
To get	-0.20833	0.29167	0.04167	0.04167	*that own*	
M^{-1} =	0.04167	-0.20833	0.29167	0.04167	*routine*	
	0.04167	0.04167	-0.20833	0.29167	*is used*	**columns**
	0.29167	0.04167	0.04167	-0.20833		**[A1A1A1A1]**

[subtotal 6]

(iii) The product of the pivots is 96 **[M1A1]**
In each of the first three cases, the pivot is in the second row
of the reduced matrix. This is equivalent to three row
interchanges. Hence multiply by $(-1)^3$. **[M1E1]**
i.e. determinant is -96 **[A1]**

[subtotal 5]

[TOTAL 24]

7895-8, 3895-8 AS and A2 MEI Mathematics
June 2007 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	54	46	38	31	24	0
4752	Raw	72	54	47	40	33	26	0
4753	Raw	72	60	52	45	38	30	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	65	57	49	41	34	0
4755	Raw	72	59	51	44	37	30	0
4756	Raw	72	52	45	38	32	26	0
4757	Raw	72	53	46	39	32	25	0
4758	Raw	72	55	47	40	33	25	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	59	51	43	36	29	0
4762	Raw	72	59	52	45	38	31	0
4763	Raw	72	61	53	45	37	30	0
4764	Raw	72	62	54	46	38	31	0
4766	Raw	72	55	48	41	35	29	0
4767	Raw	72	58	51	44	37	30	0
4768	Raw	72	62	53	45	37	29	0
4769	Raw	72	54	47	40	33	27	0
4771	Raw	72	59	53	47	41	35	0
4772	Raw	72	52	45	39	33	27	0
4773	Raw	72	59	51	43	36	29	0
4776	Raw	72	53	46	40	33	26	0
4776/02	Raw	18	13	11	9	8	7	0
4777	Raw	72	55	47	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	43.5	64.3	80.2	90.9	97.5	100	9403
7896	57.9	78.6	90.1	96.2	98.6	100	1301
7897	88.2	97.1	100	100	100	100	34
7898	100	100	100	100	100	100	2
3895	27.4	42.6	57.3	70.9	82.9	100	12342
3896	55.4	73.4	85.1	92.1	97.1	100	1351
3897	75.2	87.2	97.3	99.1	100	100	109
3898	71.4	82.1	82.1	96.4	96.4	100	28

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

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