RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Report on the Units

January 2007

3895-8/7895-8/MS/R/07J

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REPORTS FOR THE UNITS

4751 - Introduction to Advanced Mathematics (C1)

General Comments

As usual, candidates for this paper gained the full range of marks. This time, much of the straightforward work was in section A, with some of the work in section B proving stretching for the most able.

The trend of some centres entering all their C1 candidates in January has continued, so that the mark distribution includes very low marks from some very weak candidates who will need to improve dramatically if they are to succeed at AS Mathematics – such candidates demonstrated little idea of how to proceed on any of the straightforward questions in this paper where E grade candidates would be expected to score. There were also many excellent scripts from strong candidates, although fewer than usual gained full marks, mainly due to performance on question 11(i)(*B*).

The removal of graph paper from the list of additional materials on the front of the paper resulted in a reduction in the number of candidates drawing graphs when requested to sketch them, although some centres still issued graph paper to all the candidates.

In general, time was not an issue. Some candidates petered out towards the end of question 13, but there were few for whom this appeared to be because they had run out of time. As ever, a long method used unnecessarily in any question takes valuable time from other questions.

Comments on Individual Questions

Section A

1 This was generally well-answered, although a few candidates confused parallel and perpendicular lines.

2 Most candidates knew the shape of an inverted quadratic graph and labelled the intersection of the graph with the y-axis. Omitting to label the intersections with the x-axis was a common error.

3 The correct rearrangement of this formula was common, but weaker candidates often failed to realise the need to collect the terms in a and then to factorise. A few candidates did not simplify $7c - 5c$.

4 Those who used the remainder theorem by starting with f(2) = 3 were usually successful. However, some attempted long division, which was difficult in this case and was rarely done successfully.

5 Many successfully calculated the coefficient, although some could not cope with the arithmetic, particularly if starting from ${}^6C_4$, whilst others found the 15 from Pascal's triangle but omitted the 5^2 factor.

6 The first part was well done, with the successful using the strategy $\left(\sqrt{25}\right)^3$. As expected, those who used $\sqrt{25^3}$ rarely got further, whilst poorer candidates interpreted $25^{\frac{3}{2}}$ as $3\sqrt{25}$ or $\sqrt[3]{25^2}$. In part (ii). most candidates knew that the reciprocal was involved, but many could not proceed beyond $\frac{1}{\left(\frac{49}{9}\right)}$ or made errors in squaring one or both terms, such as giving 3/49 as their answer.

7 There were more problems with the fractions than with the surds, with some candidates taking half a page to add the fractions, and the denominator frequently being wrong when the fractions were multiplied. Some used the given answer to discover their errors, but some happily wrongly cancelled their wrong answers to 'obtain' the required result. However, there were also many correct answers, efficiently obtained and showing clearly what the candidates had done.

8 The less able often used trial and error to try to solve the equation. Better candidates used $b^2 - 4ac$ but fewer used clearly the fact than this should be negative. Some of the most able candidates gave $k < 4$ as the solution to $k^2 < 16$, omitting the requirement $k > -4$. Some candidates only worked with $b^2 - 4ac = 0$ and were unable to convert correctly to the desired inequality.

9 The first part was found straightforward by many, but some invented their own algebras instead of using the laws of indices. In the second part, there were many correct solutions, but some 'cancelled' $\frac{x+2}{x-3}$ and gave their final answer as $-\frac{2}{3}$. Some very weak candidates had no idea how to factorise the quadratic expressions.

10 Missing brackets caused the main error – those who expanded separately and then subtracted tended to do better. Those who successfully found $4m^2$ sometimes ignored the 'Hence' and started again, or could not cope with finding the square root of $4m^2$. Final answers such as $p = 4\,m^2$ or $4m$ were common. Some weak candidates who worked carefully picked up several marks here, whilst some stronger but careless candidates lost the accuracy marks.

Section B

11 (i) This part was answered particularly poorly with a high proportion of candidates ignoring the instructions to use the insert and to use the graph to solve the equations. Analytical methods were not accepted here. In part (*A*), many candidates did partly solve the equation $x + \frac{1}{x} = 4$ by reading off at $y = 4$ as expected, but often gave only one of the two roots, not realising that the line intersects the curve twice.

The examiners accept that wording such as 'By drawing a straight line on the graph of $y = x + \frac{1}{x}$, solve the equation…' might have assisted more candidates to know what was required in part (*B*) – it was pleasing to see some good solutions using the line $y = 4 - x$, but these were rare. A few candidates correctly used the graph to plot appropriate values of the curve $y = 2x + \frac{1}{x}$ and read off at $y = 4$, and this was accepted. Many candidates omitted part (*B*).

(ii) This part was done well by many candidates, although giving only the positive solution to $y^2 = 3$ was common. Some found the intersections with the *x*-axis instead of the *y*-axis.

(iii) Most candidates obtained a mark for correctly giving the radius as 2 in this part and somewhat fewer gave (1, 0) as the centre of the circle, with (−1, 0) the minority view. The explaining proved a challenge to many, to the extent that a significant proportion did not attempt it. There were some excellent explanations also, and drawing the circle on the insert was a good way of supporting the case (and deemed sufficient to earn the 2 marks).

12 Most candidates scored more marks on this question than in the other two questions in section B, and a mark in the range 6 to 9 was common.

(i) The vast majority were successful in determining the midpoint of AB. Where errors were made it was either in the arithmetic involving negative numbers, giving an answer of (4, 6), or in using an incorrect 'formula' to give (4,2). Candidates usually made a successful attempt at the rest of part (i). Most found the gradient of AB, then of the line perpendicular to it and then used the coordinates of M to obtain the given equation of the line. A significant number of candidates determined the gradients of AB and the given line, and then showed them to be perpendicular; however, they did not always show that the given line did pass through the midpoint. A few candidates failed to make it clear that they understood that the gradient of the perpendicular bisector was −2, and a few also did not make the use of the product of the gradients explicit. When the answer is given in the question, candidates need to make their methods very clear. However, many candidates gained full marks on this part.

(ii) Many candidates successfully found the intersections of AB and the perpendicular bisector with the *y*-axis. However, only a minority of the candidates used the straightforward method of determining the length of the 'base' of the triangle (along the *y*-axis) and multiplying by the 'height' i.e. the *x*-coordinate of the midpoint – this method easily gave the correct answer of 45/4 square units. Most chose to determine the lengths of the other two sides of the triangle. This method gave answers involving surds and many left them like this, hence losing the final accuracy mark for failing to give the answer in a simple form. Some candidates were confused by the reference to the line AM in the question, or by the fact that the point A appeared to be close to the *y*-axis: instead of using the point of intersection of AB with the *y*-axis, they used either the point A (−1,4) or the point (0,4) to calculate one side of the triangle. However, the mark scheme made allowance for these potential misreads, allowing candidates to pick up 4 or 3 of the 6 marks in this part.

13 (i) Weak candidates often floundered around at the start, attempting trials using the factor theorem, not taking the hint from the wording of the question that the other roots would not be integers. Most candidates appreciated that $x - 2$ was a factor, with the majority attempting long division rather than inspection or coefficient methods. Errors in division leading to an answer such as $x^2 + 2x + 1$ meant that the subsequent method mark for attempting the quadratic formula or completing the square was lost, since in factorising the difficulty was not the same. Again, in spite of the wording, many candidates assumed that the quadratic expression found would factorise. Those who used the quadratic formula often gained one accuracy mark for correctly finding the roots in the form $\frac{-2 \pm \sqrt{8}}{2}$, but only about half of them were able to simplify to $-1 \pm \sqrt{2}$,

whereas those who used completing the square arrived at this result more easily.

(ii) Those candidates who gained full marks in this part usually did so by substituting $x - 3$ for x in the expression for f(x). Some strong candidates successfully completed the task using the product of the factors as their first step, having done part (iii) first, but with the difficulty of the surds, errors tended to creep in during this process. Some candidates did not show sufficient evidence of how they had obtained the given expression for f($x - 3$) from their starting point. Since there was some working backwards (sensibly done when candidates found their errors and were able to correct them), in such situations the examiners require steps to be shown. Some candidates did not realise what was going on and attempted to find f(3) or to divide the given expression by ($x - 3$).

(iii) Some candidates had given up on this question by now, and some attempted trials. Some realised that the graph was a translation of the graph of y = f(x) and were able simply to state the roots as intended. A few realised that 5 was a root and started again using the methods of part (i) to find the other roots, and had time to do so.

4752 - Concepts for Advanced Mathematics (C2)

General Comments

The paper was well received with a full range of achievement. There were many excellent scripts as usual and there were fewer scripts scoring under ten marks. There was some evidence that a few candidates were short of time as the last question was incomplete or missing on some scripts. This was possibly due to inefficient working or inefficient use of the calculator in questions 5(i), 7, 8, 11 and 12. Three pieces of advice, which would help to enhance candidates' scores are as follows.

- When a question asks for an exact answer, keep off the calculator; in question 3, 2.828 usually led to a score of zero.
- When using the cosine rule, the examiner wants to see $189^2 = 118^2 + 82^2 - 2\times118 \times 82\cos C$, or equivalent, to check the method is right, but then all that is needed is the correct answer; there is no need to show all the numbers in the intermediate steps.
- When a question gives the answer, e.g. "show that the angle AOB is 1.63 radians to 3 s.f." it is clear that the examiner will not award the final mark for 1.63, it will be earned for 1.628, the number which rounds to 1.63.

Comments on Individual Questions

Section A

1 This was very well done; just a few did not know the basic rule for differentiation; just a few integrated.

2 Again, this was very well done. Very few candidates misquoted the formula as $S = a/(r - 1)$. Weaker candidates made errors in the algebra.

3 Too many put 1/3 in their calculators to find θ, then took their calculator answer for $\tan\theta$ as 2.828…; this received no credit. The majority put 1/3 onto a right angled triangle, used the theorem of Pythagoras (usually correctly) and wrote down that $\tan\theta = \sqrt{8}$. Those who used $(1/3)^2 + \sin^2\theta = 1$ were less successful as $\sqrt{(1 - 1/9)}$ sometimes defeated them.

4 All three parts were very well done by the majority of candidates. Some candidates did not understand the words periodic or convergent; some could not cope with the expression ar^{n-1}.

5 (i) The main problem was in finding the y value at x = 2.1. It is 0.9070 and at least 0.907 is needed to achieve the correct answer to two decimal places. Many took it to be 0.91 or even 0.9, thus losing the accuracy mark. The method for this part was usually good, a few lost the minus sign and a few used run/rise for the gradient.

(ii) Not all candidates understood the situation here; the response 2 did not score, nor did 1.9.

(iii) This was very well done, the one frequent error being $dy/dx = -8x^{-1}$

6 Almost all candidates produced a sine curve, and the majority convinced the examiner that they knew the period and amplitude for the second mark. The majority found one, usually both, of the correct angles satisfying the equation.

7 This was not well done. The majority of candidates had not met the phrase “increasing function” and many of those that had met it failed to use the simple condition y’ > 0. A small minority coped perfectly to obtain x < 0 and x > 6. A few of these presented the answer as 6 < x < 0; this received full marks, but the examiner’s disapproval must be noted. Many tried to solve $x^2 – 6x = 0$, but lost the zero root. Many tried to work with y or y’’, but usually without any progress. Many worked out a series of y values and things like x ≥ 7 appeared, not elegant and not correct.

8 Some had no idea how to tackle this question. Some treated it as a geometric progression and quoted appropriate formulae. Some quoted the correct A.P. formulae, but could not substitute the given data properly. Some quoted the expression $\frac{n}{2}$ (a + l), which is not wrong, but the third letter muddied the waters somewhat and they extricated themselves with great difficulty, or more usually, did not. Many did everything correctly and scored full marks, some very neatly, some less so. There were attempts of varying ingenuity that did not involve the standard formulae. Some guessed that the common difference was 2 and produced a list of numbers which could be checked for a sum of 30. A delightful method was seen: 5 pairs add up to 30, 6 each, 4th and 7th is a pair, hence 4th is 0, 5th and 6th must be 2 and 4.

9 A few calculated y’ at x = 1 and said the curve was y = 14x + c. Similarly a few got involved with $y = (6x^2 + 8x)x + c$. The majority knew what had to be done and duly scored full marks. A common slip was $6x^3/3 = 3x^3$.

10 (i) There was some good work done here, but it did not always lead to full marks. There was a mark for 4logx or –logx or $\log x^3$, but for both marks the multiple of logx was needed.

(ii) logbc = 3 did not score any marks. The candidates had to deal with the 3 to earn a mark and they found that remarkably difficult. $bc = 10^3$ scored 1 mark and $b = 10^3/c$ scored the other.. A common attempt, which only scored 1, was logb = 3 – logc, hence $b = 10^{(3 - \log c)}$, correct, but not nice. Some suggested b = 10 and c = 100. This is not wrong, but it is not good enough. A sizeable minority suggested $bc = e^3$

Section B

11 (i) Most candidates recognised the need for the cosine rule and most applied it correctly to find C, B or A. Many did not appreciate that there are three forms, depending on which angle one needs. A few, calculating C, lost the minus sign to obtain 38.8, thus reading rather a lot into “not to scale” on the diagram. Some did excellent work but threw away a mark by giving the bearing as 038.84.

(ii) The formula (absinC)/2 was well known and many earned both marks for this part; some by using the appropriate numbers having found A or B in part (i)

(iii) Part A was very well done by various methods and most arrived correctly at 1.63. Unfortunately many lost a mark for failing to show 1.628.
In part B, most good candidates scored full marks; they knew how to find the areas of the sector and the triangle and correctly did both and subtracted. Just a few could not organise their areas and calculated sector minus triangle ABC.

12 (i) Most candidates scored full marks for this part, some taking two lines, some taking two pages. They solved the quadratic by formula and then laboriously substituted both roots to confirm y = 0.

(ii) Many scored full marks, doing the six or seven operations perfectly correctly. Some created difficulties by taking the gradient as – 1/ (4x – 11) and substituting this into a line formula. Some recovered, some did not. A few did not convert 5 to – 1/5; it was left as 5 or converted to 1/5. The area of the triangle was well done using (base × height)/2 or integration from 0 to 4.

(iii) The method for this part was well known and most candidates scored 2 marks, 3 if they coped with the arithmetic – many did. A complication here was the interpretation of "bounded by the curve and the x-axis". Some deemed the area between x = 0 and x = 1.5 to be "between the curve and the x-axis" and so included it in their calculations.

13 (i) All candidates knew that logarithms had to be taken, but a very common error was $\log(k10^{ax}) = \log k \times \log 10^{ax}$. Many did convert correctly to ax + logk and this, with any mention of a and c, earned full marks.

(ii) This part was very well done. For the table mark we required e.g. 3.28 or 3.3. Drawing the line of best fit freehand cost a mark.

(iii) Candidates had to find the gradient of the line and the logy intercept (or an equivalent calculation using a point on the line) and put these into Y = mx + c, where Y = y or logy. To score the third mark they had to produce $y = 316 \times 10^{0.2x}$ approximately.

(iv) There was a method mark for substituting 75 000 into any x/y equation and a second method mark if the x/y equation was of the correct form i.e. logy = 0.2x + 2.5 or $y = 316 \times 10^{0.2x}$ Covering all previous small inaccuracies, 11,12 or 13 scored the third mark. Full marks were awarded if the correct answer came from a convincing attempt at extrapolating 48 000 to 75 000 with appropriate increments.

(v) Anything suggesting that profits are not impervious to any one of hundreds of outside influences scored. A few hit on the serious point that indefinite exponential growth is never realistic.

4753 - Concepts for Advanced Mathematics (C3)

General Comments

The paper proved to be accessible to all but a few candidates, and there were many excellent scripts scoring over 60 marks out of 72, and few scripts scoring less than 20. The standard of presentation was often pleasing, though we did penalise inaccurate notation by withholding E marks for errors in notation, and for lack of brackets – see questions 5(i) and 7(ii) and (iv). Virtually all the candidates appeared to have enough time to complete the paper.

Topics which continue to cause problems to candidates are proof and disproof – the lack of familiarity of question 4, which would not appear to be particularly difficult, seemed to put some candidates off – and the modulus function, which seems to create misunderstandings. The calculus techniques are usually sound, but some aspects of function notation and language, such as inverting, domain and range, are less generally assured.

It should be pointed out that although graph paper is available, and may be requested by candidates, it is almost invariably unnecessary, and can lose time for candidates who try to plot graphs accurately when sketches are requested.

Comments on Individual Questions

Section A

1 Part (i) proved to be an easy 'write down' for most candidates. However, candidates' success with part (ii) was mixed. The easiest approach (which the word 'verify' suggests) is to argue that on $y = |x|$, $y = 1½ \Rightarrow x = 1½$, and then substituting this value into $y = |x-2| +1$ to verify that $y = 1½$. Students using the direct approach of solving $|x| = |x-2| +1$ often came unstuck by failing to see that for the point of intersection $y = 2 - x + 1$. Other tactics, such as squaring both sides, are not very satisfactory. Many candidates showed weaknesses in handling moduli.

2 This was found to be one of the harder questions, with a modal score of 3. While there were plenty of completely correct solutions, a substantial minority tried to use $u = x^2$ and $dv/dx = \ln x$, falsely giving $v = 1/x$.
Another source of error was failing to simplify $\int \frac{1}{3}x^3.\frac{1}{x}dx$.

3 This question was very well answered, with the majority of candidates scoring full marks. Virtually all found $A = 10000$, and in general candidates solved the equations by taking lns effectively. There were occasional errors caused by 'fudging' the negative signs.

4 Although only worth 3 marks, this proved to be the least well done question on the paper. A surprising number of candidates ignored the question altogether, perhaps being put off by the 'method of exhaustion' mentioned in the question. Some candidates also interpreted 2-digit perfect squares to mean squares of two-digit numbers, and tested up to 99^2! We wanted to see 0^2 to 9^2 evaluated, together with a comment that none end in 2, 3, 7 or 8. However, we condoned the lack of 0^2, which was quite common, and any other superfluous squares evaluated. Many candidates, for the generalisation, commented on the pattern rather than generalising the given statement to all integers or whole numbers.

5 This proved to be high scoring and straightforward. The given result helped candidates to keep on track. We withheld the 'E' mark if brackets were missing in the numerator of the quotient rule. Weaker candidates missed the $x = 0$ solution in part (ii), or set numerator equal to denominator. We deducted a mark (when one was earned) for equating the denominator to zero.

6 Many candidates made a meal of part (i), often finding AQ = $3 - y$ but failing to conclude that AP = $3 + y$. Part (ii) was quite well done – many successfully expanded the brackets first, and some deduced the result from $y = \sqrt{(x^2 + 9)} - 3$. Part (iii) was also generally well done, provided it was initiated by a correct chain rule.

Section B

7 The given results made many of the marks in this question accessible, and very few candidates failed to score half marks for this question – the modal mark was 14.

(i) Verifying the coordinates of P was a straightforward mark, but the domain $x \geq -1$ was less well known.

(ii) The product rule was well done, but the algebra required to show the given result proved to be beyond weaker candidates.

(iii) The turning point coordinates were correctly derived by the majority of candidates; the range was sometimes omitted, and y was occasionally approximated.

(iv) We required to see $\mathrm{d}u/\mathrm{d}x = 1$, or $\mathrm{d}u = \mathrm{d}x$, limits consistent with $\mathrm{d}x$ and $\mathrm{d}u$, and brackets in $(u - 1)\sqrt{u}$, to secure the E mark. The integration was done well – negative answers were accepted.

8 Parts of this question were more challenging for candidates than the rest of the paper. In particular, the last part required geometrical insight which was often lacking, and maximum marks were rare. The modal mark was 11 out of 18.

(i) Most candidates differentiated correctly, either by expanding the brackets or using the chain rule. Many went on to evaluate the gradients at $x = 0$ and $x = \ln 2$ correctly. Errors in the derivative leading to a fortuitous zero gradient at the origin were not allowed follow through.

(ii) Although there was some confusion between $f^{-1}(x)$ and $f'(x)$, $f(-x)$ or $1/f(x)$, most candidates attempted to invert $y = (e^x - 1)^2$ and many completed this work successfully. Other approaches were to show that $fg(x)$ or $gf(x)$ equals x, or use of a flow diagram, which was accepted provided the order of operations was clear. We did not expect particularly accurate graphs, provided the $g(x)$ appeared to be a reasonable reflection in $y = x$; however, we wanted to see the curve touching the y-axis, and graphs which dipped substantially below the horizontal were penalised. The gradient at $(1, \ln 2)$ was often calculated by differentiating. A fairly common error was $-\frac{1}{4}$.

(iii) Even with the help of the result given, many candidates failed to expand brackets before integrating term by term, and tried substitution or parts, usually without success. This sometimes wasted a lot of time. The final result often suffered from losing negatives after substituting the limits.

(iv) This proved to be a good discriminator for A grade candidates. Some tried to evaluate the integral directly, which is difficult though not impossible. Others guessed that the integral equalled the result in (iii) or its reciprocal. Only the best candidates were able to sort out the geometry successfully. Unsupported answers, presumably obtained by graphics calculator, gained no credit.

4754 - Applications of Advanced Mathematics (C4)

General Comments

This paper was of a similar standard to that of last January. Candidates found it much more straightforward than the June 2006 paper. There was a wide range of responses but all questions were answered well by some candidates. There were some excellent scripts.
Candidates should be advised to read questions carefully. There were instances, particularly in the Comprehension, where instructions were not followed.
There was also some use of inefficient methods. Those that were competent at algebra and surds and were familiar with manipulating trigonometric formulae generally achieved good results. Some of the arithmetic in the trapezium rule and the integration of the polynomial was disappointing.
There was some evidence of shortage of time as a small proportion of candidates failed to complete question 8.

Comments on Individual Questions

Paper A

Section A

1 The algebraic fraction equation was almost always answered correctly.

2 (i) There seemed to be a lack of familiarity with the trapezium rule formula. Common errors were use of A=0.5h(y_0+y_4) +2(y_1+y_2+y_3) but omission of the other brackets. Or alternatively omitting y_0 and using A=0.5h((y_1+y_4)+2 (y_2+y_3)). Most obtained at least one ordinate correctly but there were many errors in the calculation of the answer.

(ii) Those without correct, or almost correct, answers in the first part could not make a valid comment about which of Chris or Dave was correct in their calculations. There were some poor explanations given, such as 'the trapezium rule always overestimates results'.

3 Most candidates correctly used the compound angle formula as the first stage. Those that used *sin/cos* 45° as $\sqrt{2}/2$ rather than $1/\sqrt{2}$ could not always deal with cancelling $(\sqrt{6}+\sqrt{2})/4$. The sine rule was usually correct.

4 There were some efficient solutions but weaker candidates found it difficult to see ahead to what was needed. In some cases poor knowledge of trigonometric identities and their rearrangement was the problem. Some tried to work on both sides simultaneously - some more clearly than others.
There were some confused starts using incorrect identities in the second part but many did obtain the first solution. The solution θ=150° was often lost - in some cases due to missing the negative square root.

5 This was well answered. The improvement seen in the binomial expansion was pleasing although this was possibly due to the first number in the bracket being a 1. There were still some candidates who used *x* rather than *3x* throughout the calculation and many could not deal successfully with the range for the validity.

6 The partial fractions were almost always correct.
The second part was less successful. Some separated the variables to *ydy*=…. Many integrated 2/ (2*x*+1) as 2ln (2*x*+1) and there were many instances of the omission of the constant. Poor use of the laws of logarithms meant that c was often not found correctly. For example, lny=ln(2x+1) – ln(x+1) +c leading to y= (2x+1)/(x+1) +c was common. Those that found c before combining their logs were more successful. 2ln(2x+1) – ln(x+1) = ln 2(2x+1)/(x+1) was also a common error.

Section B

7 (i) This was usually correctly answered although some candidates used long methods to show that $\theta=0$ at A and others gave the value of θ at B in degrees.

(ii) There were many errors in *dy/dθ*- usually the coefficient of *cos 2θ* being incorrect - and there were also sign errors. Most knew that they had to equate *dy/dx* to zero but made errors in their simplification to the given equation.

(iii) Some omitted this or tried to factorise and then abandoned the attempt. Of those that did use the formula, a common mistake was to solve the quadratic equation for *cos θ* but then to use this as *θ* in the expression for *y*.

(iv) This was disappointing. The first part was usually correct but a significant number failed to integrate the polynomial. Of those that did integrate, many surprisingly made numerical errors when substituting the limits.

8 (i) Most candidates correctly found the distance AB.

(ii) Many failed to find the required angle ABC.

(iii) This proved to be very successful for many. Those that gave the required vector equation in terms of *a* and *b*, however, could rarely make progress. A few found *a* and *b* successfully without explicitly writing down the equation of the line.

(iv) Once again too many candidates failed to realise that in order to prove that a vector is perpendicular to a plane it is necessary to show it is perpendicular to **two** vectors in the plane. Others did not evaluate their dot product, merely stating it was zero. Most used the Cartesian form of the equation with success. There were still some candidates who approached this from the vector equation of the plane and they were more likely to make errors.

Paper B

Comprehension

1 The tables in (i) and (ii) were usually correct but there were occasional slips. In (iii) candidates often failed to calculate using Benford's Law. It was unclear what their methods were in (iii) but they may have been trying to use Fig.9.

2 This was often successful but it was not always clear which tables the candidates were referring to.

3 Some failed to explain about the multiplication of leading digits. For those that did, the multiplication factor quoted did not always work for the complete range. Multiplying 3,4 and 5 by 3.5 or 4 was commonly seen.

4 Usually correct although log (n+1) –log n=log(n+1)/log n was seen.

5 The approach encouraged by the question was not always used. There were some very long and often confused solutions involving changing all 'L' expressions to strings of 'p' equations and eliminating.

6 Candidates often seemed not to have read this question carefully. There were many good solutions, but too often the proportions were calculated rather than using the frequencies in the table.

4755 - Further Concepts for Advanced Mathematics (FP1)

General Comments

This paper was well answered with many high scores and few really low ones. However, the mistakes that some quite good candidates made left the impression that they may not have been quite ready to take the examination.

The paper was of an appropriate standard and length, with the high marks reflecting the talented candidature it attracts.

The entry continues to grow, indicating that centres are encouraging more candidates to study Further Mathematics.

Comments on Individual Questions

1 **Proof**

Almost all candidates got this question right.

2 **Roots of a quadratic**

While almost all candidates knew what was expected in this question, many lost marks through errors in simplifying $\frac{4 \pm \sqrt{-12}}{2}$.

The second part of the question asked for the points to be plotted on an Argand diagram and this was well answered.

3 **Matrix transformation**

In the first part of this question candidates applied a matrix to a triangle and plotted its image. This was well done, though several failed to label the image points clearly. Candidates were then asked to describe the transformation, which was a stretch with different scale factors in the *x*- and *y*- directions; many candidates said it was an enlargement and many more chose a combination of reflections and rotations, failing to take account of what had happened to the individual points.

4 **Series summation using standard results**

This question was well answered, though there was much evidence of careless notation. The most common mistake was failing to factorise the expression and some candidates multiplied everything out before attempting to factorise, which introduces many opportunities for error; another common mistake was to use the wrong standard results, typically $\sum r^2$ instead of $\sum r$.

5 **Manipulating the roots of an equation**

Although there were many correct answers to this question, it was also the case that many candidates who knew what they were doing lost marks through careless mistakes. A common mistake was to present the final answer as an expression rather than an equation. The question asked candidates to find a new equation with roots related to those of the given equation; the method of working from the sum and products of the roots of the given equation was more popular than that of substituting for *x* and was on the whole carried out slightly more accurately.

6 **Proof by induction**

This question was often well answered and full marks were fairly common. A few candidates failed to present the correct structure. Others were not explicit about the assumption that the result is true for $n = k$; statements like "Let $n = k$" were not uncommon. It was also not uncommon for candidates to skimp on the final few statements necessary to complete the proof and so lose marks.

Many candidates got into a muddle with notation, for example by writing $\sum(k+1)^2$ when they meant $\sum_{r=1}^{k+1} r^2$.

7 **Graph**

This question was well answered and even the weakest candidates were able to get some marks on it.

(i) This asked for the intercept with the *y*-axis. It was almost universally answered correctly.

(ii) This asked for the equations of the asymptotes. Almost all got the vertical asymptotes right, but a few failed to give the correct horizontal asymptote.

(iii) Most candidates sketched the graph correctly and labelled the asymptotes, but many lost a mark by failing to mark in the intercept with the *y*-axis.

(iv) Candidates were asked to solve a related inequality and this was less well answered; many gave only one of the three regions in which it held. They could have avoided this mistake had they used their graph sketch to help them.

8 **Complex numbers**

This question proved the most taxing for many candidates.

(i) Candidates were asked to find the reciprocal of the complex number $-4+2\mathrm{j}$. While most candidates got this right, many knew to multiply the numerator and denominator by the conjugate but then made careless mistakes; a few displayed complete ignorance of the topic, writing things like $\frac{1}{-4+2\mathrm{j}} = \frac{-1}{4} + \frac{1}{2}\mathrm{j}$.

(ii) Candidates were asked to write the complex number in modulus argument form; most could calculate the modulus, but had problems with the argument; many gave the supplement of the argument, a mistake that they would not have made if they had drawn a sketch Argand diagram to help them. Many calculated the modulus and argument but lost a mark by not actually expressing the complex number in modulus argument form, or by doing so incorrectly.

(iii) Part (iii) involved two related loci and many candidates lost some marks here, either by starting their half-lines at $-m$ rather than m, or by failing to draw the lines in part (iii) (*B*) broken to show they were excluded. A few candidates shaded the unwanted region but did not state that this was what they were doing. A few of the weaker candidates thought that the loci were circles.

Several candidates might have earned more marks had they ensured their diagrams were clearly labelled.

9 **Matrices**

Parts (i) and (ii) were generally done well. Part (iii) was more taxing but the standard of attempts was pleasing and many got full marks.

(i)(ii) In parts (i) and (ii) candidates were asked to multiply matrices and to find their inverses. This was well done and many candidates got full marks for these parts. Loss of marks was usually due to careless mistakes but some candidates were unaware that matrix multiplication is not commutative.

(iii) In part (iii) candidates were asked to prove a general result. Almost all candidates knew what was expected of them and many got it fully right; the commonest mistake was to fail to write the matrices on the left hand side in the correct order after post-multiplying by $\mathbf{P}^{-1}$.

The best solutions were annotated to set out very clearly the steps and properties involved.

4756 - Further Methods for Advanced Mathematics (FP2)

General Comments

There was a wide range of performance on this paper, with about a quarter of the candidates scoring 60 marks or more (out of 72), and about a quarter scoring less than 30 marks. The standard integrals involving inverse trigonometric and hyperbolic functions were handled particularly well, but marks were often lost through carelessness when differentiating trigonometric functions. Many candidates used very long methods to find eigenvalues, and some were unable to complete the paper as a result.
Each of the three questions in Section A had an average mark of about 11 or 12 (out of 18). In Section B, Q.4 (on hyperbolic functions) was chosen by almost all the candidates, and the average mark was about 10.

Comments on Individual Questions

1 **Polar curve and Maclaurin series**

In part (a)(i) the curve was usually sketched correctly, although it sometimes spiralled in the wrong direction and often went through the origin. In part (a)(ii) most candidates used $\int \frac{1}{2} r^2 \, d\theta$ with the correct limits, although a substantial minority forgot to square *r* even though they had written it down. The work was very often completed correctly, but the integration of $e^{-2k\theta}$ proved to be surprisingly challenging with factors of *k* going astray and answers such as $-\dfrac{1}{2k\theta} e^{-2k\theta}$ or even $\dfrac{1}{-2k\theta+1} e^{-2k\theta+1}$ were quite common.
The integral in part (b) was usually found confidently and correctly; the only difficulty was with the factor $\dfrac{1}{2\sqrt{3}}$ in front of the integral.
In part (c), the method for finding the Maclaurin series was well understood, but the triple differentiation of $\tan x$ very often went wrong. The first derivative was sometimes written as $\dfrac{1}{\cos^2 x}$ or even $\dfrac{2}{1+\cos 2x}$ before proceeding, making the work much more difficult than is necessary. Some strong candidates observed that $f''(x) = 2f(x)f'(x)$, and obtained $f'''(0) = 2$ very efficiently. When the Maclaurin series had been found correctly, the final part (ii) was usually also completed correctly.

2 **Complex numbers**

Part (a) was well answered; most candidates were able to work with modulus and argument correctly, although solutions were quite often spoilt by careless errors (such as an incorrect argument for *z*).
The identity in part (ii) was usually handled successfully.
In part (iii), almost all candidates realised that they should consider $C + jS$, and there were very many fully correct solutions. However, a fair proportion of candidates failed to recognise the resulting series as binomial, and were determined to use the formulae for a geometric series, thereby losing most of the marks for this part.

3 **Matrices**

In part (i) almost all candidates knew a method for finding the inverse matrix, and the process was very often completed accurately.

In part (ii), it was expected that, for each of the given vectors **e**, the candidates would evaluate **Me** and see that this is a multiple of **e**. Many did this, but a large number of candidates found the characteristic equation, then the eigenvalues, and finally the eigenvectors. This did sometimes yield the correct results, but it must have been very time-consuming. Another common method was to write $(\mathbf{M}-\lambda\mathbf{I})\mathbf{e}=\mathbf{0}$ and use one component to find λ; however, this does not establish that **e** is an eigenvector unless the other two components are checked, and this was rarely done.
Many candidates knew how to answer part (iii) by forming the product $\mathbf{S}\mathbf{D}^n\,\mathbf{S}^{-1}$, although the order of the product was often wrong, and inaccuracies in evaluation frequently prevented the emergence of the given answer.

4 **Hyperbolic functions**

Most candidates knew how to tackle the standard proof in part (i), although few gave the correct reason (arcosh $x \geq 0$) for discarding the other root.

The integral in part (ii) was very often found correctly, although the factor $\frac{1}{2}$ was quite frequently omitted.

In part (iii) the differentiation was usually done correctly. Setting the gradient equal to $\frac{1}{9}$ gives a quadratic in sinh*x* which was often solved correctly; then the logarithmic form of arsinh was usually correctly employed to obtain the values of *x*. Many wrote the gradient in terms of e^x and e^{-x}, and rarely made any further progress; and a common error was to equate the gradient to zero instead of $\frac{1}{9}$. Some made heavy weather of finding the values of *y*; from $\sinh x = 1$ and $x = \ln(1+\sqrt{2})$ they evaluated $\cosh x$ as $\cosh(\ln(1+\sqrt{2}))$ instead of using $\sqrt{1+\sinh^2 x}$.

5 **Investigation of curves**

There were only a few attempts at this question, and all of these scored less than half marks.

4758 - Differential Equations

General Comments

The standard of work was generally very good, demonstrating a clear understanding of the techniques required. Almost all candidates answered questions 1 and 4; questions 2 and 3 were equally popular as a third choice. Candidates often produced accurate work when solving second order differential equations. However, when solving first order equations, it was common to see errors in integration, omission of the constant of integration, or not dealing properly with the constant.

With regard to graph sketching, I would like to emphasise the following advice given in last summer's report. Candidates should note that generally the expectation of sketches in this unit is that any known information (such as initial conditions or relevant results found earlier in the question) should be indicated on the sketch. Also any obvious shape and features of the graph (e.g. oscillating, increasing, decreasing, unbounded, asymptotes), should be shown. Detailed calculations are not required, unless specifically requested.

Comments on Individual Questions

1 **Second order differential equations**

(i) This was often completely correct.

(ii) Many candidates correctly found the particular solution, but a few candidates were unable to use the asymptotic condition. Only a minority of candidates were able to demonstrate that the solution is zero *only* when $t = 0$. Commonly candidates simply showed that $t = 0$ implies $y = 0$, or gave a vague argument why it is the only root. Sketch graphs were often good, except they often showed y positive rather than negative.

(iii) The solution was often correct, but some did not realise the correct form of the particular integral and some made slips with their arbitrary constants.

2 **First order differential equations**

(i) This derivative (using the chain rule) was intended to help with the integral later in the question, but unfortunately many candidates misunderstood the request and approached it as an integral or a differential equation.

(ii) This was often done well, but slips in integration were common, in particular not dealing with the $2x$ properly and omitting the constant. Another common error was not dealing with the constant properly when rearranging.

(iii) Most candidates attempted to use an integrating factor, but slips were common. A common error was $I = \exp\left(\int \cot 2x \, dx\right) = \exp\left(\frac{1}{2}\ln\sin 2x\right) = \frac{1}{2}\sin 2x$, which gave a valid integrating factor but contained two errors, i.e. omitting the 2 in the integral and then wrongly dealing with the $\frac{1}{2}$.

(iv) Many candidates successfully found the particular solution, but some made slips in the calculation. The sketch graphs varied widely in quality. Some candidates simply drew a sketch of cot*x*. Many sketches did not show the given condition, indeed some wrongly had a break in the curve around the point $\left(\frac{1}{4}\pi, 0\right)$, presumably due to entering *y* = 1/tan2*x* into a graphical calculator. If using a graphical calculator, candidates must use it intelligently and not just copy the screen without thinking about what the graph should look like.

(v) Very few candidates successfully completed this part. Many tried to solve the differential equation again, which was rarely successful, rather than use their general solution.

3 **Modelling the motion of a ball-bearing falling through a liquid**

(i) Candidates usually formulated the differential equation correctly from Newton's second law, and many solved it correctly. However, errors did occur, in particular with the integral. Also, some candidates omitted the constant or made mistakes with the constant when rearranging their solution.

(ii) Most candidates integrated the previous solution, but many omitted the constant or did not calculate it.

(iii) This part was often done well, but candidates were not as successful as in part (i). Common errors with formulating the equation were: not starting from Newton's second law or not recalling the alternative form of acceleration. Common errors when solving the differential equation were: not integrating correctly, omitting the constant, or including the constant but wrongly stating that it was the initial displacement and hence zero.

(iv) This was usually correct, although as usual with an Euler's method calculation, a few candidates produced unrecognisable figures with no method shown. Some candidates mistakenly gave the value of *v* at *t* = 0.3 by tabulating $t, v_r, \dot{v}_r$ and v_{r+1} in each row and then stating the final value of *v* in their final row.

(v) Virtually all attempts at this calculation were correct.

4 **Simultaneous differential equations**

(i) This calculation was usually correct, although some made slips.

(ii) The elimination of *y* was often done well, although a few differentiated the first equation with respect to *x* rather than *t*.

(iii) The solution for *x* was often done very well. When finding *y*, many candidates correctly used the first equation. As usually occurs, a few tried instead to set up and solve a differential equation for *y*. Such attempts were very lengthy and time-consuming and never attempted to relate the arbitrary constants in the two solutions, which is a vital feature of the solutions.

(iv) The particular solutions were often done well.

(v) The sketch was often correct, but some candidates seemed to ignore the request to make clear the initial and the long-term behaviour of the solution on the sketch.

4761 - Mechanics 1

General Comments

The majority of the candidates seemed well prepared for this paper and were able to finish or, at least, make substantial progress on every question; there were many beautifully presented and clearly argued solutions. Many of the candidates showed good algebraic and arithmetic skills.

Very few candidates seemed unfamiliar with all of the principles required for the unit but rather more struggled to make much progress. It was notable that this latter group tended not just to have poor general mathematical skills (as well as a lack of specific knowledge about mechanics) but handicapped themselves further by poor presentation. There were many examples of such candidates overlooking some parts of questions and in extreme cases mixing up their attempts to a whole question by not specifying which part was being answered. Candidates should know that if an answer cannot be reasonably associated with a specific question then no credit may be the consequence.

Many candidates did not know how to do Q6 (ii) and not many were able to assemble complete arguments to satisfy the requests in Q8 (iv) & (v). A lot of candidates did not appreciate the help offered through the structure of Q4 but those who saw what was required found it easy to answer. Apart from these questions, most candidates knew what was expected of them at each step and many did much of it very well, especially in Q7 where full marks were common.

Comments on Individual Questions

Section A

1 **Using a velocity – time graph**

This was generally done fairly efficiently for full marks but a few candidates could not get started. Relatively few candidates used the area of a trapezium but instead considered two triangles and a rectangle. Surprisingly, quite a few used an argument based on the fraction of the whole area represented by each of the three sections. The most common error was to attempt to apply a constant acceleration formula once only for the entire 100 second interval.

2 **A kinematics problem involving calculus**

Most candidates realised that they should differentiate to find the velocity, did so accurately and correctly equated their expression to zero. Many forgot the negative root or discarded it despite the time interval clearly indicating the inclusion of negative values. Many candidates forgot that the final answer was to be the displacement not the time.

3 **The static equilibrium of a box on a rough horizontal floor with a force applied at an angle to the floor.**

(i) Most candidates obtained the correct normal reaction.

(ii) Many candidates omitted the frictional force or (fewer) the normal reaction. A common error from generally weaker candidates was to label the normal reaction with the value found in part (i).

(iii) There were pleasingly many correct solutions, the most common error being to transpose sine and cosine. There were also some sign errors, often from candidates who when considering the vertical equilibrium tried to write down an expression with the normal reaction as subject.

4 **The resultant of two forces and the magnitude and direction of the acceleration when they are applied to a particle**

This question was structured to lead candidates through a common method of solution. However, a number struggled to do much at all or used the cosine and sine rules instead.

(i) This part presented unexpected problems of comprehension and many candidates gave the complete resultant vector instead of the required component.

(ii) Perhaps a majority of the candidates used the cosine rule; for these a common error was to take the angle between the forces as 60° instead of 120°. Many of those who used Pythagoras' Theorem (as expected) had a sign error in the calculation of the component in the direction of the 20 N force or failed to use components or used incomplete resolutions.

(iii) Many candidates were able to follow through from part (ii) to obtain the acceleration. Others started from first principles, often making different mistakes with components to those made in part (ii); some included a weight of $2g$ in the vertical component. With wrong components the direction found was wrong and there was limited follow through allowed from earlier parts. The few candidates who used the sine rule did well.

5 **Motion of a block on a horizontal connected by a string over a pulley to a hanging object**

This standard problem was recognised as such by many candidates who then went on to obtain full marks. A few clearly did not know what to do and came up with wrong equations not obtained from any clear method.

(i) A common error from those who broadly knew what to do was to produce two equations with inconsistent signs. Other errors were to include the weight of the block in its equation of motion and the resistance on the block in the equation of motion of the sphere.

6 **The kinematics of particle in a plane with constant acceleration. The direction of motion and of displacement**

Many candidates answered part (i) quite well but very few understood what was required for part (ii), (the worst answered part on the paper). In part (i) they were given a velocity vector in terms of t and they then correctly used its direction when $t = 2.5$ as the direction of the motion at that time (perhaps without thinking). In part (ii) they had to realize that the direction required was that of a displacement and most failed to do so.

(i) This was usually quite well done with the direction established acceptably by many. Quite a few candidates failed to give the speed – possibly they forgot it was required.

(ii) Most candidates either left the part out or considered **v**(0) or Δ **v**. Quite a few who realized they must find a displacement used integration instead of a constant acceleration result in vector form; these were candidates who had shown themselves generally strong and they usually obtained the correct answer.

7 **Newton's second law applied to motion on a horizontal plane and down a slope**

This question was answered well by all but the weakest candidates and even most of those made some progress. Almost all of the stronger candidates scored high or even full marks. It was particularly pleasing to see so many applying Newton's second law correctly to a block on an inclined plane.

(i) Usually done correctly.

(ii) This was done well by most but there were lapses from some weak candidates who used distance = speed $\times$ time as if the acceleration were zero. There were also a surprisingly large number of errors when substituting (for example a formula was written down correctly but the value of *t* was substituted where t^2 was required).

(iii) This was done pleasingly well by many candidates. The most common errors were to use cosine instead of sine when resolving or to resolve the 15N or even to include a component of the velocity.

(iv) Most candidates managed to find the time and the distance using the value of their acceleration but some found only one of these – the other was, perhaps, forgotten.

(v) Again, the response to this question was very pleasing. Most candidates knew that they must find a new acceleration and did so accurately, the most common error from these being with signs.

8 **Two objects with the same initial horizontal and vertical speeds but different initial heights projected towards one another**

Parts (i) to (iii) seemed to be found straightforward by most candidates but fewer could manage the much more sophisticated arguments required in parts (iv) and (v).

(i) Surprisingly many candidates could not establish this result clearly, leaving the reason for the negative sign ambiguous.

(ii) Most candidates did this well. The most common method was to equate the vertical height expression to zero but some considered twice the time to the highest point and others first found the cartesian equation of the trajectory. A common omission was to show that A landed between the initial positions of A and B. It seemed that many candidates thought this so obvious that they simply repeated the words in the question without saying *why* it was obvious.

(iii) Most candidates also did this part well. The most popular method used was to equate the vertical height expression to – 15; the most common error was to equate to 15 instead. Candidates who used this method mostly solved the quadratic equation accurately. Some candidates split the motion into two parts but these often made sign errors somewhere in this more complicated approach. Some of the weaker candidates produced some very poor attempts where they equated the vertical and horizontal displacements or added distances to speeds etc.

(iv) This part was found difficult by many candidates and was not done particularly well mainly because the candidates did not communicate their arguments sufficiently clearly. When trying to show that A and B do not collide, many argued that the two equations for the vertical motion were inconsistent for the same time and others tried to show that the heights were different when the horizontal positions were the same. Fewer candidates made a good attempt to show that the paths intersect; most who were successful considered the sum of the ranges they had found in the two previous parts. There were a number of well thought out valid arguments presented by strong candidates.

(v) This part was also done in a variety of ways. The most popular was to demonstrate that the vertical displacements were equal at the two times; however, a large number of these candidates omitted to check that the horizontal positions were also the same. A number of candidates attempted to derive one of the times. This was usually unsuccessful due to (t + 2) being used instead of (t - 2) in the motion for A. However, the problem was well within the capabilities of the stronger candidates and a pleasing number of them scored full marks for this part using, as with part (iv), a variety of interesting methods.

4762 - Mechanics 2

General Comments

Many excellent scripts were seen in response to this paper with the majority of candidates able to make some progress worthy of credit on every question. The majority of candidates seemed to understand the principles required. However, diagrams in many cases were poor and not as helpful to the candidate as they could have been and some candidates did not clearly identify the principle or process being used. As has happened in previous sessions, those parts of the questions that were least well done were those that required an explanation or interpretation of results or that required the candidate to *show* a given answer. In the latter case some candidates failed to include all of the relevant steps in the working.

Comments on Individual Questions

1 Many candidates gained significant credit on this question. Those that drew a diagram were usually more successful than those who did not. Many candidates did not indicate which direction was to be positive and this did lead to some errors in signs or inconsistencies between equations.

(i) The majority of candidates were able to gain some credit for this part of the question. Sign errors occurred in a few cases in the use of Newton's experimental law and many candidates forgot to indicate the direction in which the ball was travelling after the impact.

(ii) (A) Almost all the candidates could gain full credit for this part of the question.

(B) Very few candidates could obtain any credit for this part. Many did not appreciate the vector nature of the problem and merely stated (incorrectly) that the sledge would speed up because the mass had decreased and momentum had to be conserved. A small number of candidates appreciated that there would be no change in the velocity of the sledge but could not give a valid reason for this. Few mentioned that there was no force on the sledge in the direction of motion.

(iii) Many candidates were able to gain full credit for this part of the question. Of those who did not, a significant minority drew an inadequately labelled diagram or made errors with the masses. A small number of candidates did not understand the significance of the velocity of the snowball being relative to the sledge and assumed that the snowball had a speed of 10 m s^{-1}.

2 Some excellent answers to this question were seen, with many candidates gaining almost full credit. It was pleasing to see that there were fewer mistakes made with inconsistent equations than has been the case in previous sessions.

(i) This part was well done by almost all the candidates.

(ii) Most candidates did well on this part of the question. Some arithmetic errors were seen and, in a few cases, some 'creative' algebra to try to establish the given results.

(iii) In most cases the standard of the diagrams was satisfactory and worth some credit but a significant minority of candidates did not label the internal forces and/or omitted one or more of the external forces. A few candidates obviously changed the diagram in response to their answers as they worked through the following parts of the question and this then led to mistakes.

(iv) This part of the question caused few problems.

(v) Many candidates scored well on this part of the question. Those who did not were usually those who drew an inadequate diagram or who ignored their diagram; these made mistakes with signs or produced equations that were inconsistent with each other and with the diagram drawn in part (iii).

(vi) This part of the question was not as well done as previous parts. Arguments based purely on symmetry at B were common but few appreciated that the vertical equilibrium had to be considered. Those candidates who attempted to look at the vertical equilibrium often forgot to resolve the forces in BD and BE. A common answer was to simply write down $T_{BD} = \pm T_{BE}$ without any supporting argument or interpretation of the result.

3 Only the last two parts of this question caused any problems to the vast majority of candidates. The principles behind the calculation of centres of mass appeared to be well understood and candidates who adopted column vector notation made fewer mistakes than those who calculated the co-ordinates separately.

(i) Most obtained full credit for this part.

(ii) Few candidates had difficulty with this part.

(iii) Most candidates were able to obtain full credit for this part. A minority of them wrongly assigned a mass of 100 units to the lid and it was common to see the z co-ordinate of the centre of mass of the lid assumed to be 2.5 cm. However, many who made this error realised the mistake and clearly corrected it. Unfortunately, there were some candidates who made both of the above errors, completed the working and still stated 2.1 as the z component of the centre of mass.

(iv) Few candidates made much progress here. The most common mistake was to ignore one of the components of the weight. Trigonometric errors were also common.

(v) Very few correct responses to this part were seen. Many of those candidates who appreciated that the moment of P had to be equated to the clockwise moment of the weight from the previous part had inconsistent units for the distance of P from the pivot.

4 It was very pleasing to see some good answers to this question with a significant number of completely correct responses.

(i) Most candidates were able to obtain significant credit for this part of the question but many failed to explain fully their interpretation of *their* calculations.

(ii) (A) Almost all candidates gained credit for this part.

(B) Few candidates had difficulty with this part.

(C) A small number of candidates forgot to raise 10 tiles, others forgot to include the work done in raising the tiles 6 m from the ground and only calculated the power required to move the tiles up the roof.

(iii) Many candidates found this part difficult. Most candidates obtained the change in kinetic energy and made an attempt at the change in potential energy but then forgot the work done against friction; others obtained the correct terms but made mistakes with signs. It was pleasing to see that the vast majority of candidates attempted to use an energy method as specified and not Newton's second law and the constant acceleration formulae.

4763: Mechanics 3

General Comments

This paper was found to be somewhat more difficult than the 2006 papers, but the majority of candidates showed that they had a good understanding of most of the topics examined. About 30% of the candidates scored 60 marks or more (out of 72), and only about 15% scored less than half marks.

Comments on Individual Questions

1 **Dimensional Analysis**

This question was very well answered, with the majority of candidates scoring either 13 or 14 marks (out of 16). All parts were usually answered correctly, except for the change of units in part (iii); the conversion factor was very often the reciprocal of the correct one, and the given value (6.67×10^{-11}) was often omitted from the calculation.

2 **Circular Motion**

There was a much greater spread of marks on this question, and the average mark was about 13 (out of 20).
The vertical circle problem in part (a) was usually answered correctly, although the weight was often omitted from the calculation in part (ii), and a few assumed that the speed remained constant.

Part (b) was a horizontal circle problem, and most candidates were able to obtain the given result in part (i). The force diagram in part (ii) was usually correct, although the frictional force was often in the wrong direction, and sometimes omitted. The final part (iii) was found to be quite challenging. There were many excellent solutions; those who wrote down the vertical and horizontal equations of motion usually made substantial progress and often obtained the correct answer. However, a substantial number tried resolving in inappropriate directions and were not able to score any marks.

3 **Elasticity and Simple Harmonic Motion**

The average mark on this question was about 13 (out of 18).

Part (i) was usually answered correctly, although some candidates confused stiffness with the modulus of elasticity.

Part (ii) was generally well done (but some measured x from the equilibrium position and so could not obtain the required quadratic equation), and most candidates were able to obtain the given result in part (iii).

Part (iv) was found quite difficult. Those who recognised that the equation in part (iii) implied simple harmonic motion usually used $x = A\sin\omega t$ but the values of A and/or x were often incorrect; few seemed to realise that what was required was just one quarter of the period. A surprising number of candidates tried to apply constant acceleration formulae in this part.

In part (v), the acceleration was usually found correctly, and many candidates expressed concern for Ben's safety.

4 **Centres of Mass**

The principles involved in finding centres of mass by integration were very well understood, but the work was often spoilt by careless errors such as dropped minus signs, and powers of *a* going astray; the average mark was about 12 (out of 18). More serious errors, such as integrating $\frac{1}{x^4}$ to obtain $\frac{-1}{5x^5}$, or even $\ln(x^4)$, were also quite common.

In part (ii) the angle was often calculated as $\tan^{-1}\left(\frac{\bar{x}}{1-\bar{y}}\right)$ instead of $\tan^{-1}\left(\frac{\bar{x}-1}{1-\bar{y}}\right)$. In part (iii), showing that $\bar{x} < 1.5$ proved to be quite challenging, even for those who had correctly obtained $\bar{x} = \frac{3(a^3 - a)}{2(a^3 - 1)}$. It was only necessary to observe that $a > 1$; but many candidates just substituted in a single value of *a* (usually $a = 2$), and some stated that $\bar{x} \to 1.5$ as $a \to \infty$ without saying that $\bar{x}$ is an increasing function of *a*.

4766 - Statistics 1

General Comments

The paper attracted a fairly wide range of responses, although there were relatively few scripts with either exceptionally high or exceptionally low scores. However candidates did seem to perform at a somewhat lower level than in recent sessions. There was no evidence to suggest that candidates had insufficient time to attempt all questions, apart from those who chose very long winded methods in more than one question. Answers were often well presented but a good number of candidates do not appreciate the implications of using rounded answers in subsequent calculations.

Most candidates gave good answers to and were able to earn substantial marks from Questions 1i, 2, 3, 4, and parts of 6 and 7. Question 5 was not well answered; as was reported last summer, candidates were again evidently unclear about how to manipulate probabilities in a Venn diagram and many scored at best 3 out of 8 marks. The performance on question 6 (i) – (iii) was also variable, with many candidates making errors in accurately reading the graph scales and in calculating outliers. Several parts of Question 7 were not well answered. Many candidates are still not meeting the requirement to define *p* in words in the context of questions on hypothesis testing and many candidates are also using point probabilities rather than tail probabilities.

Comments on Individual Questions

Section A

1 **Carbon dioxide emissions; mean, median, midrange and comments.**

(i) The mean and median were almost always correct but the midrange was confused with the range or IQR. Many candidates calculated (max-min)/2 = 8.3 instead of (max + min)/2 = 14.5.

(ii) Despite the question saying 'for these data', most comments did not relate specifically to the data, but were general in nature. Examiners were looking for two aspects here: the suitability of each measure of central tendency (i.e. good, poor, useful, etc) and how each measure was or was not influenced by the outlier of 22.8. Many candidates thought that the midrange was a measure of spread in the data. Others felt that the mean and/or midrange were good measures 'because they detected outliers'. Relatively few convincing responses were seen.

2 **Absentees; vertical line chart, mean and root mean squared deviation, calculation of new mean and rmsd.**

(i) The vertical line chart was almost without exception correctly drawn, with only a tiny minority failing to label the axes.

(ii) The mean and rmsd were generally tackled well; the main errors seen were failure to take the square root: using an (n – 1) divisor in the rmsd instead of n, dividing by 30 (the number of pupils in the class) or dividing by 21 (from 0+1+2+3+4+5+6) instead of dividing by 50 (the number of days the data were collected over).

(iii) All that was required here was to be aware that 'new mean = 30 – original mean' and 'new rmsd = original rmsd'. Unfortunately very few candidates recognised the transformation $x \to 30 - x$, and instead most produced inordinately long solutions by re-calculating, often going wrong in the process. The fact that only 2 marks were available should have alerted candidates that this did not warrant a further 2 pages of calculations.

3 **Travel times; histogram and skewness.**

(i) There were many very good responses to this question with full marks often achieved for the whole question. Some candidates who favoured the frequency per 5 minutes or frequency per 10 minutes approach failed to label the vertical axis of the histogram as such but instead simply used a label of 'frequency density', thus losing a mark. This label only gains credit when the candidate is using frequency per unit *x* value. Some of the weaker candidates used non-linear scales on the horizontal axis or labelled the axis with a series of inequalities ($0 \leq t < 5$, $5 \leq t < 10$, etc) rather than the correct linear scale. Only a small minority drew a frequency diagram.

(ii) Almost all candidates recognised the positive skewness for the shape of the distribution.

4 **Dice; evaluating *k*; calculation of E(*X*), probability.**

(i) Most candidates correctly found k = $^1/_{36}$ although there were a few sightings of $^1/_{35}$ or $^1/_{37}$.

(ii) The expectation E(*X*) was almost always found correctly but there were many candidates who then went on to calculate E(*X*) / n, for some value of *n,* usually with *n* = 6 or *n* = 21.

(iii) Many candidates used the previous probabilities instead of 1/6, 1/6, 1/6. A significant minority could not identify the 6 ways of getting a total of 16, often only coming up with 2 ways or even 12 ways. Numbers that did not add up to 16 were sometimes seen, especially (4, 4, 4).

5 **Wearing a tie or jacket, conditional probability, Venn diagram, probability calculations.**

(i) Many candidates were unable to correctly deal with the conditional probability aspect of this question and instead of the required $0.4 \times 0.3 = 0.12$ in part (i), answers of 0.08 or 0.06 from 0.4×0.2 or 0.3×0.2 respectively were often seen.

(ii) Disappointingly, the vast majority of candidates produced an incorrect Venn diagram with the 0.4 placed inside the 'jacket circle' and the 0.2 placed inside the 'tie circle' instead of 0.28 and 0.08 respectively. In many cases the sum of the probabilities in the diagram was not one and probabilities were sometimes omitted. This is a relatively straightforward concept, but centres would be advised to deal very carefully with it when preparing candidates as they seem to have a great deal of difficulty with it. Such errors in the Venn diagram usually make it impossible to allow follow through marks in the next part of the question and so candidates lose a significant number of marks. There have now been 3 questions on Venn diagrams and probability in recent examinations but this remains an area where centres need to improve candidates' understanding of the concepts and their labelling of diagrams.

(iii) All sorts of errors were seen in here, but in (*B*) a common misconception was 1 – answer (*A*).

Section B

6) **Birth weights; Cumulative frequency, median, IQR, outliers, percentiles, binomial distribution, expected value.**

(i) Many correct answers were seen although a substantial number of candidates were unable to read the scales accurately in order to find the median or quartiles. The most common error was the belief that 1 small square on the vertical scale was 10 instead of 20, thus leading to half the correct number of outliers in (ii). In view of the fact that these topics are examined at Intermediate Tier GCSE, significant penalties were imposed on candidates who did not read the scales correctly.

(ii) The definition of an outlier still remains unclear for a large number of candidates, with many thinking it is defined as median $\pm$ 1.5 IQR, or UQ + 2IQR, LQ - 2IQR or UQ + IQR, LQ – IQR or median $\pm$ 2 IQR instead of the correct LQ -1.5 IQR, UQ + 1.5 IQR.

(iii) The comments about outliers were often vague. The fact that in such a large data set a considerable number of genuine data values were likely to lie outside the limits was rarely mentioned. Equally only a few candidates made a reference to either premature or overweight babies or mentioned the relevance of the purpose for which the data was being used (eg health care provision).

(iv) The 10^{th} percentile was very often correct although occasionally 2,500 or 550 were seen instead of 2600.

(v) This was usually very well answered with only a few candidates attempting incorrectly to use binomial tables by approximating 0.12 as 0.1. In fact most candidates seemed happier with this AS work than with the GCSE work earlier in the question. In part (*B*) the most common errors were 1 – [P(0) + P(1)] or 1 – [P(1) + P(2)] instead of the correct 1 – [P(0) + P(1) + P(2)].

(vi) Nearly all candidates found 100 $\times$ answer (v) (*B)* but occasionally the 100 was replaced by 17.

7) **Germination and growth of onion seeds; binomial distribution, independence, calculation of E(*X*) and Var(*X*), expected frequency, hypothesis test on the binomial distribution.**

(i) This was often correct but failure to multiply by 2 in (*B*) sometimes resulted in an answer of $^2/_9$.

(ii) Many candidates gave good explanations here, although some failed to mention that lack of independence would mean that the **probability** of one event would be altered by the occurrence of the other event. It was very pleasing to see some candidates state that independence is a required condition for the use of the binomial distribution.

(iii) Most candidates calculated both expectation and variance correctly, although some inaccuracy was seen when candidates used decimal probabilities.

(iv) There was a great deal of confusion between ‘the number of seeds’ and ‘the number of **pairs** of seeds’ with many halving when they should not have. Some

only considered the pairs where both germinate, with 113.33 being a common wrong answer, for which some credit was awarded.

(v) It is pleasing to note that many candidates stated their hypotheses in symbolic form. However, as in previous papers, very few candidates defined the parameter '*p*'. Previous reports have referred to the importance of this matter. There are three marks available for the correct statement of hypotheses, including the definition of the *p*. Many candidates correctly evaluated $P(X \leq 14) = 0.0982$ as the tail probability but some then made an erroneous statement along the lines of '0.0982 is not in the critical region at the 5% significance level'. It is important for centres to stress to candidates that the critical region contains only *x*-values and NOT *p* –values. However a pleasing number of candidates did explicitly show a correct comparison of 0.0982 with 0.05. Overall the work on hypothesis testing is slowly improving year by year but there are still too many candidates who base their arguments on point probabilities instead of tail probabilities.

4767 - Statistics 2

General Comments

Most candidates were well prepared for this examination, demonstrating a good command of the necessary calculation techniques, and were able to complete all questions within the allowed time. None of the questions stood out as being either noticeably difficult or easy. Few candidates scored all of the available marks for explanation.

Comments on Individual Questions

Section A

1 (i) Very well answered, mostly producing full marks. Most lost marks occurred in the calculation of the gradient of the regression line, usually through the use of an incorrect method. Some candidates obtained the correct gradient but did not use the centroid of the data to find the t-intercept. Some candidates relying on calculators gave the incorrect equation t = 12.6v + 167.

(ii)A & B Most candidates scored both marks for the predictions. Many candidates gave suitable comments regarding the reliability of the predictions. Comments which failed to provide reasons for reliability/unreliability of the predictions scored no marks.

(iii) Few candidates scored marks on this part of the question. Many simply pointed out that the coefficient was the gradient of the line. Some managed to explain that it gave an indication of the rate of change of time taken for the kettle to boil with respect to the volume of water in the kettle. Very few mentioned units of time &/or volume.

(iv) Many scored full marks. Most knew to find the difference between the predicted and observed values but were not always sure of the signs of the residuals.

(v) Many candidates scored full marks. Marks were lost for failing to explain that the *distance* that needed to be measured was *vertical*. Some candidates did not realise that the question was asking how to measure residuals from a diagram and simply explained how to find residuals from an equation. Most provided an acceptable explanation of how to obtain the sign of the residual.

2 (a)(i) This question was well answered with many scoring full marks. Common errors included use of variance instead of standard deviation, and unnecessary continuity corrections.

(ii) Well answered with many candidates working to a suitable degree of accuracy and gaining full marks.

(iii) Well answered. A few candidates lost marks through using -1.645 instead of +1.645 in their equation although candidates who used -1.645 × 4 = (*k* - 33) were given the benefit of the doubt. A small number of candidates failed to use 33, with 28 and 24 seen in its place on several occasions.

(b)(i) Most candidates provided correct hypotheses. Few candidates identified μ as the population mean.

(ii) Well answered with full marks awarded to a reasonable proportion of candidates. Many lost marks through failing to use an appropriate test statistic despite help being available in the formula booklet. Omissions of square root signs were common. A small number failed to recognise that the value, 4.77 kg, was a *total weight* when calculating their test statistic – those preferring to work with total weight throughout could still obtain full marks. Most candidates now appreciate the requirement to provide conclusions in context.

3 (i) Nearly all candidates provided an acceptable justification of the given answer.

(ii) Many candidates lost marks on this question through failing to calculate the variance. Many gave the incorrect reason "the mean is approximately equal to the standard deviation" to support the Poisson model. A small number gained no marks for stating "events occur randomly and independently with a uniform mean rate", and/or "n is large and p is small"

(iii) Nearly all candidates obtained the correct value for $P(X = 1)$ and most then went on to make a suitable comparison to receive full marks. A few lost marks for not providing enough detail – e.g. finding the expected number of 1s as 22.6 but not specifying the value in the table with which it was being compared.

(iv) Most candidates scored full marks. A small number mistakenly thought that $P(X \geq 12)$ was the same as $1 - P(X \leq 12)$. A similar number used the Poisson p.d.f. to find $P(X = 12)$ and used $1 - P(X = 12)$, which gained no credit.

(v) Most candidates picked up two of the three available marks – usually for noticing that the previous answer was small, and for explaining that in the laundrette the machines will be used more often than in the home. The mark for appreciating that a "tail" probability was used tended to be the mark dropped.

(vi)*A* Well answered, with most scoring full marks. Some candidates lost marks for failing to use the correct mean. Those who failed to combine the means and use a single Poisson distribution, preferring to work with separate distributions, often lost marks – usually for failing to identify all four combinations – though some scored full marks with this method.

(vi)*B* Not so well answered, with many adding rather than multiplying their probabilities. Most managed to obtain P(Drier needs 1 repair).

4 (i) Well answered. Most managed to provide correct hypotheses. In the calculation of X^2, some lost marks through excessive rounding of their expected frequencies. Candidates should be encouraged to work to at least 2dp when finding expected frequencies. Nearly all candidates used 1 degree of freedom as required, and found the correct critical value. Some lost marks for making the wrong conclusion. As ever, those failing to provide context in their conclusion were penalised. Simply stating "there is no evidence of association (between the two/variables)" did not earn the mark.

(ii)*A* Well answered

(ii)*B* Well answered, with those who scored the last four marks in part (i) usually gaining full marks.

(ii)*C* Poorly answered. One requirement in such questions is for candidates to identify the large contributions, indicating strong association (which most candidates can do), and to distinguish between positive and negative association (which tends to be neglected). Another requirement is to identify the small contributions, which show little association between the categories. Candidates commonly fail to refer to the contributions at all. Many referred only to “strong ambition” for those living in the country, without distinguishing between the two categories of ambition.

(iii) Poorly answered.

4768 - Statistics 3

General Comments

Once again the overall standard of the scripts seen was pleasing: many candidates appeared well prepared for the paper. However, as in the past, the quality of their comments, interpretations and explanations was consistently below that of the rest of the work.

It was noticeable that candidates' use of correct mathematical notation was often poor. For example: integrals written without the terminator "dx" and interchanging the symbols "=" and "⇒". Also many candidates showed a lack of appreciation of the level of detail of arithmetic required to convince the examiner that an answer printed in the question has been obtained genuinely.

Invariably all four questions were attempted, and attempted well on the whole. Questions 1 and 3 were found to be particularly high scoring. There was no evidence to suggest that candidates found themselves short of time at the end.

Comments on Individual Questions

1 **Continuous random variables; no context.**

(i) Although the accuracy of notation left much to be desired (as noted above) virtually all candidates were able to establish the value of k satisfactorily.
Also, most candidates sketched the graph of f(x) with little difficulty. The only note of disappointment was the number of candidates who neglected to draw a sketch at all.

(ii) The value of E(X) was almost always obtained with no difficulty. Similarly Var(X) was found correctly. Candidates need to be aware, however, that a little more effort is appropriate when establishing the exact value printed in the question.

(iii) As in the past, candidates did not acquit themselves at all well when attempting to find the cumulative distribution function (c.d.f.). They must be encouraged to realise that a definite integral (with suitable limits) is expected.
It was then interesting to see that an appreciable proportion of candidates seemed not to know how to use and/or interpret their c.d.f. Instead, in both this part and the next, they set up and evaluated integrals that were completely unnecessary. Having said that there were very many who did eventually find the correct probability of X greater than the mean.

(iv) Perhaps fewer than half of the candidates used their c.d.f. and substitution to verify that the given value was the median. The majority (including those who first integrated) obtained and solved a quadratic equation for m, and this left them needing to remember to distinguish between the two roots.

(v) Most candidates were able to write down the correct distribution here, based on the Central Limit Theorem.

2 **The *t* distribution: hypothesis test for the population mean; confidence interval for a population mean; heights of saplings.**

(i) The null hypothesis was usually correct, although some used "≥" instead of "=", but there were fewer correct alternative hypotheses. Furthermore it remains the case that too many candidates neglect to define in words the symbol "μ".
At this level it is expected that candidates are going to use the built-in statistical functions of their calculators for the mean and sample variance. There were an appreciable number of scripts where this did not seem to be the case, and so the accuracy of their results suffered a little from premature approximation. Nonetheless the test statistic was usually worked out correctly. Similarly the test was carried out and concluded correctly, the most common problem being the use of the wrong critical value (-2.201 instead of -1.796). When the test is one-tailed, requiring the lower tail critical value and involving a negative test statistic, candidates are often less than clear and careful about the negative signs. Centres are advised that the "special case", shown in past mark schemes to allow for a particular form of misreading the tables, will not be applied from June 2007 onwards.

(ii) Most candidates showed that they were familiar with how to construct a confidence interval, and did so successfully. Unsurprisingly, there were a number who seemed to forget that they should still be using the *t* distribution. Clear and accurate descriptions of the meaning of a confidence interval were disappointingly rare.

(iii) There were many correct responses to this part. However it was also quite common to see answers that were only partially correct, for example by identifying the Wilcoxon single sample rank sum test but then suggesting a null hypothesis that was inconsistent with it.

3 **Combinations of Normal distributions; times of gardening tasks.**

This question was very well answered with very many scoring full marks. Candidates seemed well prepared for it and understood what was expected. In many cases their answers were concise and to the point. Those who take the trouble to provide simple sketch graphs of the standard Normal distribution do much to enhance the quality of their responses. There was evidence from some quarters of effective use of the built in functions on graphics calculators.

(i) This part was almost always correct.

(ii) This part, too, was almost always correct.

(iii) Again, correct answers were often seen here too, but this time weaker candidates experienced difficulty with the formulation of the requirement.

(iv) Usually the mean total time was correct, but often the variance was not. Typically the error came about through a lack of proper understanding of the difference between $\text{Var}(2X)$ ($= 2^2\text{Var}(X)$) and $\text{Var}(X_1 + X_2)$ ($=\text{Var}(X_1) + \text{Var}(X_2)$). Here the former was used when it should have been the latter.

(v) There were many good answers to this part, and they were evenly split between those who adapted their answers to part (iv) to obtain the probability distribution of the monthly charges and hence the probability of the charge not exceeding £40, and those who found the time (in minutes) corresponding to a total charge of £40 and then the probability of not exceeding that time.

4 **Chi-squared test of goodness of fit; Wilcoxon paired sample test for a difference in population medians; air pressures.**

(a) Although there were plenty of good attempts at this part of the question many broke down in one or more of the following ways. Some, but not very many, candidates neglected to merge the first two classes. Quite a few used the wrong number of degrees of freedom, usually because they forgot to allow for the two estimated parameters, and hence their critical value was inappropriate. Following the conclusion of the test, many simply neglected to comment on their findings. For this last point it is expected that candidates will undertake a brief discussion of what can be deduced by looking at the data in order to explain the outcome of the test.

(b) There were very many good answers to this part and most of these scored full or nearly full marks for it. It was a rare script indeed where the candidate did not know to take differences and then rank the absolute values. An occasional slip with the arithmetic was seen here. The vast majority of candidates found and used the correct test statistic and compared it with the correct critical value, which led to a correct conclusion.

4771 - Decision Mathematics 1

General Comments

This was the first session in which candidates were provided with a printed answer book. This seemed to work well. There were examples in which candidates did not have sufficient space, and had to use supplementary sheets, or where second attempts were needed. But such inconveniencies were greatly outweighed by the positive benefits for most students.

It is hoped that candidates will only need the printed answer book to write on, which is provided inside the question paper. For the summer session, 4-page answer booklets and graph paper will be available should the candidate request them.

Candidates were generally well prepared.

Comments on Individual Questions

1 **Graphs**

This was a very straightforward first question, and most candidates did very well on it.

2 **Algorithms**

(i) Examiners often found it difficult to see whether or not candidates had fully followed the algorithm, including the last two iterations on which no swaps were made.

(ii) Many candidates were able to give the correct ordering, and the majority of those were able to count 10 swaps.

(iii) Only about 25% of candidates were able to do the quadratic computation.

3 **Simulation**

Most candidates coped very well with this question. Quite a number insisted on giving their mean in part (iii) to the nearest integer, and were penalised when they did so. A minority erroneously thought that the accuracy could be improved by using 2-digit random numbers.

4 **CPA**

(i) & (ii) These parts were well done. Again, it was pleasing to see an aspect of modelling being tackled so well. (This copied from last summer's report.) Having said that, there was an unfortunate resurgence of "activity-on-node" from some centres. This gains no credit.
One recurrent minor error was having activities D and E share the same "i" and "j" events – a dummy was needed.

(iii) Fewer than 50% of candidates were able to demonstrate knowledge about both total and independent float. A few candidates who did have that knowledge proceeded to incorrect answers as a consequence of unnecessary dummy activities.

(iv) Able candidates found this very easy. Less able did not. It was a good discriminator.

5 **LP**

(i) The June 2006 report on this question started "How **do** we persuade candidates properly to define their variables?" Some improvement was seen this session, although far too many candidates stated "Let x = lawn …" etc. Nearly all candidates who failed in all or part of the subsequent formulation, and there were many, had failed properly to define their variables.

(ii) Graphs were often better than might have been expected from formulations. In particular the graph of y=2x (or equivalent) was seen more often than was the expression y=2x.

(iii) Surprisingly few candidates scored the marks here. They needed to be evaluating at vertices and comparing, or to be clearly applying an objective gradient.

(iv) Only a few succeeded with this, as had been expected.

6 **Networks**

(i) Most candidates were able to apply Kruskal successfully.

(ii) Candidates needed to convince the examiner that they were in fact applying the tabular form of Prim – not all did so.

(iii) This was the least satisfactory part of the paper. The point of the question was that a greedy approach – choose the minimum connector followed by the minimum connector of the remainder – does not produce the best answer. Allowing a suboptimal "first" connector allows, in this case, for the second connector more than to compensate.
The question, the last on the paper, was deliberately left very open-ended, and the outcomes were very poor. Students interpreted the invitation to give possible reasons as an excuse to let their imaginations run riot. Nearly all of the answers offered involved suppositions, with no basis to support them.
In such questions candidates should restrict themselves to that which is known, and that will almost always be what is given. (There are few cases in which knowledge of a real world situation can be assumed across the candidature.) In this case the knowns were pipe lengths, and it was there that candidates should have been focusing in constructing their answers.

4776 - Numerical Methods

General Comments

There was, as usual, considerable variation in the level of preparation shown by candidates, but there seemed to be fewer than usual who were completely out of their depth. Many candidates are good at applying routine techniques accurately, though very often work is not presented concisely and logically. It is very difficult (for the candidate and for the examiner) to see what is and is not correct in a jumble of numbers. Setting down the numerical work systematically helps towards getting it right. The analysis and interpretation of results still presents challenges to some of those who can cope easily with the numerical work. It should be remembered that the numbers themselves, without analysis and interpretation, are almost meaningless.

Comments on Individual Questions

1 **Error analysis**

This was a routine exercise in finding absolute and relative error and it was frequently well done. The second part asked why a calculator displaying 8 digits might work to 11 digits internally. Most answered that the extra digits gave greater accuracy: true, but a rather poor answer. The aim is (as far as possible) to make the displayed answer correct to 8 significant figures.

2 **Approximation and errors**

Most candidates had no difficulty with this question. The second part, finding the constant *k*, sometimes produced sign errors.

3 **Solution of equation; extrapolation**

This question was frequently well answered, but there were two common mistakes. Firstly, the ratio of differences was calculated as the reciprocal of the common form: this is not a problem if it is handled correctly subsequently. Secondly, there was a tendency to suppose that the ratio of differences should be a ‘neat’ number: in this case ⅓ was popular. Though this has little effect on the final answer, it is faulty reasoning. The rate of convergence of a first order process might be any number at all between –1 and 1.

4 **Solution of equation: secant method**

The graphs to show that there is only one root were of variable quality. Inevitably the least convincing were those taken unthinkingly from a graphical calculator with its domain set inappropriately. Almost all could locate the root by means of a sign change and the secant method was well done by many. (In some cases there was a numerical error in the secant process but it was followed by a recovery. This attracted most of the credit.)

5 **Numerical differentiation**

The numerical values of the various estimates were generally found correctly, The comments on the forward difference method were usually appropriately cautious and the majority were aware that the central difference formula is more accurate than the forward difference method. Some candidates tried to make something of the fact that one of the numerical answers in part (i) is identical to the numerical answer in part (ii).

6 **Interpolation: Lagrange's method**

In part (i), the equation of the straight line and the estimate of the root, α, were generally found accurately. (A few, however, were thrown by the use of Lagrange's method for a straight line.) Finding the range of values of α caused problems, however: the x values pair off as –0.085 with 0.155 and –0.095 with 0.145. Many had these pairings wrong.

7 **Numerical integration**

This question was the least well done. This is both surprising and disappointing as the topic is straightforward and (one would have thought) familiar. The function, x^{-x}, was unusual but with a careful explanation and with some given values that did not seem to be the problem. Rather, it seemed that candidates were just not familiar with the basics of Simpson's rule.

The first request was to 'use the values in the table to find the Simpson's rule estimate … with $h = 0.5$'. The range of integration was from 1 to 2, and the values in the table were $x = 1, 1.5, 2$. The standard form of Simpson's rule to integrate from a to b has $h = (b - a)/2$. It was therefore deeply puzzling to find candidates calculating f(1.25) and f(1.75) and then working with $h = 0.25$. These candidates then went on to find the Simpson's rule estimate with $h = 0.125$ when asked to use $h = 0.25$, and were surprised (or not) to find that their answer coincided (or didn't) with the given answer for $h = 0.125$. As much credit as possible was given for the ability to calculate a Simpson's rule estimate, but inevitably these candidates lost marks.

Principal Moderator's Report

Centres are reminded that the deadline for coursework marks (and scripts if there are 10 or fewer from the centre) is December 10 for this specification. Moderators were pleased to receive the MS1 and the sample of work from the vast majority of centres before the Christmas holidays, but there were a few centres where inconvenience was caused by the late arrival of the work.

Assessors are reminded that marks for each criterion should be awarded rather than a domain mark. It is often not possible to see where the assessor has withheld a mark if a domain mark of 2 out of 3 is awarded. Assessors are also encouraged to write comments in the spaces given on the cover sheet and also to annotate the work. Moderators also appreciate a note of what work has been checked; conversely we do not expect assessors to tick work if it has not been checked. There have been a number of cases where incorrect work is ticked and given credit.

The majority of centres also included the Centre Authentication Form, CCS160, but again there was a degree of inconvenience caused by a few centres where this form had to be requested.

This report, in common with all previous reports, outlines the reasons why moderators recommend adjustment to the marks awarded by the Centre. As a result, teachers will find that most of what is stated below has been said before. We feel we need to repeat what we have said before because we continue to experience the same difficulties with marking.
These documents are therefore crucial to centres who are engaged in the process of assessment and we would encourage Heads of Departments and Examination Officers to ensure that all those involved in the assessment have a copy of the report to inform them for future sessions.
We wish to stress that the vast amount of work we have seen displays a high level of commitment by candidates and assessors with appropriate marks being awarded. In a few cases, however, this is unfortunately not the case.

Methods for Advanced Mathematics (C3); Numerical solution of equations (4753)

A small number of centres assessed the work using an incorrect cover sheet. This incorrect sheet was originally published with the specification but was amended within weeks of publication. Subsequently, centres have been sent the correct sheets and asked to destroy the old versions. Some centres even used both the correct and incorrect ones within their assessment and even within a single group.
This will have been noted on individual reports to centres, and if you receive such a comment please will you ensure that all incorrect sheets are destroyed.

Change of sign
As in other domains, candidates sometimes do some theoretical work which is not worthy of credit. Candidates should be illustrating the method working on their equation, and this means more than simply drawing the curve of the function. Such a curve either needs to be annotated, or redrawn more than once with a smaller range of *x* to illustrate the way the iterates are converging on a root.

The failure of the method is often carried out inappropriately; for example:

- The equation has no roots.
- The equation has no roots, yet the candidate asserts that it has.
- The search of a sign change actually locates the root or locates the discontinuity. Equations such as $\frac{1}{x-1}=0$ or $(x-2)^2=0$ are therefore deemed as trivial and should not be used.
- The search for a sign change for equations such as $(x-2)^2=0$ using values of *x* = 1.8, 1.9, 2.1, 2.2,… and declaring that there is no sign change.

Newton-Raphson method

It is unnecessary for candidates to derive the formula as there is no credit for doing so. However, it is necessary to describe by graphical illustration how the method works, and, as with the last domain, this illustration should be using the equation being solved.
The second mark in this domain is for finding all remaining roots, the first mark being for the first root. If there is only one root then this mark should not be awarded. The requirements of the task include the need for the equation to be used in this domain to have at least two roots (see specification book, page 62). A candidate who uses an equation with only one root should therefore not be awarded this second mark.
Error bounds need to be established. It is not enough to note that successive iterates agree to *n* decimal places and that therefore the root is found to *n* decimal places. Error bounds are typically established by a change of sign calculation.
The need to illustrate failure of the method to converge to an expected root is "despite a starting value close to it". Merely choosing an unrealistic value for x_0 does not satisfy this criterion. Typically, a candidate will embark on this method by doing a sign change search using integer values, thus locating the integer interval for each root. It is therefore reasonable to take one of the integer end points as a starting value.
Particularly in this domain, candidates who use computer resources to do the work for them should give some indication that they understand the method by doing some of the work themselves, either using a spreadsheet or calculator.

Rearrangement method

The main problem in this domain was the description of why convergence or not was achieved. It is expected that candidates will make some reference to the fact that the gradient of the line $y = x$ is 1 and that convergence will therefore only be achieved if $|g'(x)| < 1$. Merely stating that $g'(x) < 1$ with no explanation does not fulfil the criterion.
The moderators spotted some work given credit in this domain, but the rearrangement was incorrect, thus yielding a value for a root that was incorrect.

Comparison

It is expected that candidates will find a particular root already found by one of the methods by the other two methods, using the same starting value. Without this condition, any comparison of speed of convergence is a little limited.
Candidates are also expected to comment on the resources (hardware and software) they have used. Candidates who have used spreadsheets and/or "Autograph" may well come to an entirely different conclusion from candidates who have only had a scientific calculator at their disposal.

Written communication

Some candidates find it difficult to write equations and functions well enough for the reader to understand what is being said, and to relate what they are writing to the curves they are drawing. Such candidates should be penalised in this domain.
Rather more particularly, many candidates confuse equations with functions and even expressions.
A candidate who writes "I am going to solve the equation $y = x^3 + x - 7$" or even "I am going to solve the equation $x^3 + x - 7$" should not be credited with having written correct notation and terminology. The moderators found that a large number of candidates were awarded this mark with a positive comment given, yet the work was full of the confusions described above.

Oral communication

As with the investigations in the other two units, it is a requirement that the assessor fulfils this criterion and writes a brief report on how it was done and the results. Assessors are reminded that it is not permissible to give credit for any of the other criteria as a result of this oral communication.

Differential Equations (4758)

Fewer changes were made to the submitted marks this season.
The usual tasks were submitted along with one or two original tasks.
Teachers should note the following points which contributed to the generosity of marking.

In domain 3, the source of the data often needs to be explicit, together with some discussion of accuracy. (This is particularly so in the task "Aeroplanes".)
A comparison of predicted and measured values, in the form of a table, should be included wherever possible, while it is accepted that this is not always possible or sensible.

Students should be encouraged to label graphs clearly; it is not always evident to the reader whether the values shown are measured or predicted.

A crucial part of the task is to demonstrate the modelling cycle. Care must therefore be taken to ensure that the second model is a revision of the first rather than a new model for a new situation. New and original investigations are encouraged, but again, care must be taken to ensure that the modelling cycle of comparison and revision of the original model is possible.

Numerical Methods (4776)

Most candidates tackled suitable problems, mainly on numerical integration, but a few were disadvantaged by the choice of topic which meant that they were unable to address the criteria adequately.
Rather more marks were adjusted this session than is usual. In many cases this was because incorrect work was being given credit.
An example is in numerical integration where, for a well-behaved curve, there is a theoretical value to which the ratio of differences of the area found by a standard procedure (midpoint, trapezium or Simpson's rules) will converge. Candidates were being given full credit for using this theoretical value even when there was no justification from their work to do so. In addition, if the curve is not well-behaved, then the value is actually wrong and the use of such a value will lead to errors in the solution of the problem. If such work is given full credit then there is a generosity of marks of at least 2 in the error analysis domain and at least 2 in the interpretation domain.
Other problems in assessment experienced by the moderators are outlined below.

Domain 1.

If all the candidates from a centre do much the same task using the same template then it is difficult to justify the first mark which is for a candidate to identify a suitable problem. Additionally it is expected that a brief explanation of why it is an appropriate problem is expected. (In the case of numerical integration it is **not** expected that candidates will attempt to justify this by "trying" standard methods of integration known to him/her.)
It is worth noting that at this level and in this task, creating artificial contexts does little to enhance the task - our experience is that more usually it causes confusion to the candidate.

Domain 2.

It is not necessary to describe the methods or reproduce mere bookwork to explain how the methods work. In this domain it is sufficient to explain why the procedure was used. Many candidates doing numerical integration chose all three procedures without any justification or explanation and were given full credit.

Domain 3.

In a few cases, full credit was given to candidates whose work could not be classified as "substantial". In the case of numerical integration and Simpson's rule, for instance, we would expect to see S_2, S_4, S_8, S_{16}, S_{32}, S_{64} and possibly also S_{128} in order to justify convergence of *r*. Obtaining only up to S_{32}, or perhaps a single application to find S_{128} would not be enough to satisfy the other criteria.
While it is not expected that assessors will check all the work of all their candidates, there was evidence that incorrect work had been ticked, and this was particularly so in numerical integration.

Domain 4.

The wording of the criteria should make it clear that it is expected that spreadsheets be used. There are still a few centres where candidates have completed their work on calculators and in these cases the criteria in this domain are difficult to obtain.
Where a spreadsheet is used, a printout of formulae used needs to be annotated to explain what is being done.

Domain 5.

Many candidates working on numerical integration were given credit for work that was incomplete or incorrect.
Candidates who obtained a ratio of differences that looked as though *r* was converging to 0.35, but then used a theoretical value of 0.0625 to extrapolate has not got the application fully correct and the subsequent analysis will be wrong. Likewise, candidates who do not work through the ratio of differences but uses $r = 0.0625$ has not got the application fully correct and may get the analysis wrong.
There are some integrals which may be theoretically outside the experience of the candidate at this particular point of their career and there may be some integrals which can now be found using an algebraic manipulator (such as "Derive") and as a result, candidates may have an alternative means of obtaining an accurate value for their integral. In some cases candidates have taken this value and used it in their error analysis. For this piece of coursework this is inappropriate; error analysis should be worked from within their calculations. Thus, working out differences and then ratio of differences for numerical integration is appropriate, while error from some other (assumed correct) value is not.

Domain 6.

Where there is incorrect work as outlined above, marks for accuracy of solution are also difficult to justify.

7895-8, 3895-8 AS and A2 MEI Mathematics

January 2007 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	50	43	36	29	23	0
4752	Raw	72	52	45	38	31	25	0
4753	Raw	72	61	54	47	39	31	0
4753/02	Raw	18	14	12	10	9	8	0
4754	Raw	90	68	60	52	44	37	0
4755	Raw	72	59	51	43	35	27	0
4756	Raw	72	53	46	39	32	25	0
4758	Raw	72	58	50	42	33	24	0
4758/02	Raw	18	14	12	10	9	8	0
4761	Raw	72	56	48	40	33	26	0
4762	Raw	72	58	50	43	36	29	0
4763	Raw	72	53	46	39	32	25	0
4766	Raw	72	51	44	38	32	26	0
4767	Raw	72	59	52	45	38	31	0
4768	Raw	72	59	51	43	35	28	0
4771	Raw	72	55	47	40	33	26	0
4776	Raw	72	52	46	40	33	27	0
4776/02	Raw	18	13	11	9	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	28.9	59.8	83.5	95.9	96.9	100	97
7896	30.8	69.2	100	100	100	100	13
7897	100	100	100	100	100	100	1
7898							0
3895	18.0	39.1	61.6	78.4	94.4	100	445
3896	33.3	66.7	83.3	100	100	100	6
3897	100	100	100	100	100	100	2
3898	84.6	92.3	92.3	100	100	100	13

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

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