ADVANCED GCE UNIT

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper A

TUESDAY 23 JANUARY 2007

4754(A)/01

Afternoon

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **6** printed pages and **2** blank pages.

HN/7

Section A (36 marks)

1 Solve the equation $\dfrac{1}{x} + \dfrac{x}{x+2} = 1$. [4]

2 Fig. 2 shows part of the curve $y = \sqrt{1 + x^3}$.

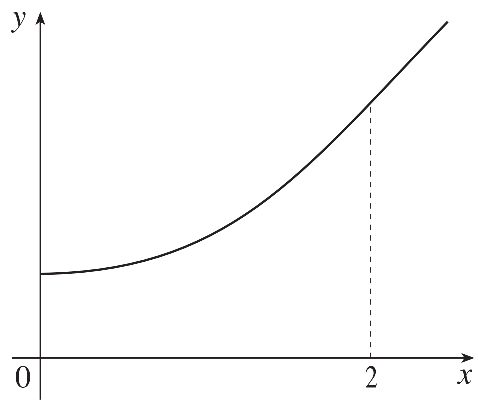

Fig. 2

(i) Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{1 + x^3}\,\mathrm{d}x$, giving your answer correct to 3 significant figures. [3]

(ii) Chris and Dave each estimate the value of this integral using the trapezium rule with 8 strips. Chris gets a result of 3.25, and Dave gets 3.30. One of these results is correct. Without performing the calculation, state with a reason which is correct. [2]

3 **(i)** Use the formula for $\sin(\theta + \phi)$, with $\theta = 45°$ and $\phi = 60°$, to show that $\sin 105° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$. [4]

(ii) In triangle ABC, angle BAC = 45°, angle ACB = 30° and AB = 1 unit (see Fig. 3).

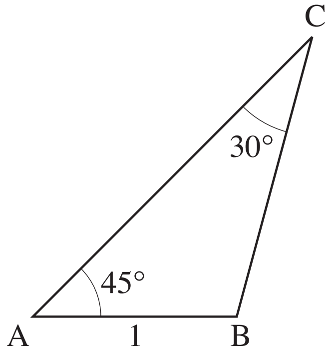

Fig. 3

Using the sine rule, together with the result in part (**i**), show that $\text{AC} = \dfrac{\sqrt{3}+1}{\sqrt{2}}$. [3]

4 Show that $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \sec 2\theta$.

Hence, or otherwise, solve the equation $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = 2$, for $0° \leqslant \theta \leqslant 180°$. [7]

5 Find the first four terms in the binomial expansion of $(1+3x)^{\frac{1}{3}}$.

State the range of values of x for which the expansion is valid. [5]

6 **(i)** Express $\dfrac{1}{(2x+1)(x+1)}$ in partial fractions. [3]

(ii) A curve passes through the point $(0, 2)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{(2x+1)(x+1)}.$$

Show by integration that $y = \dfrac{4x+2}{x+1}$. [5]

Section B (36 marks)

7 Fig. 7 shows the curve with parametric equations

$$x = \cos\theta,\ y = \sin\theta - \tfrac{1}{8}\sin 2\theta,\ 0 \leqslant \theta < 2\pi.$$

The curve crosses the x-axis at points $A(1, 0)$ and $B(-1, 0)$, and the positive y-axis at C. D is the maximum point of the curve, and E is the minimum point.

The solid of revolution formed when this curve is rotated through 360° about the x-axis is used to model the shape of an egg.

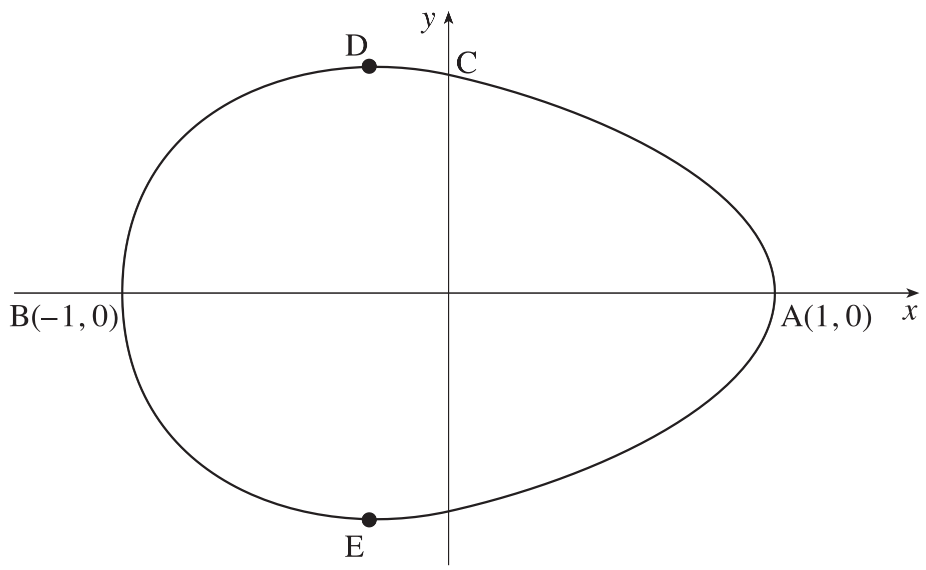

Fig. 7

(i) Show that, at the point A, $\theta = 0$. Write down the value of θ at the point B, and find the coordinates of C. [4]

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

Hence show that, at the point D,

$$2\cos^2\theta - 4\cos\theta - 1 = 0.$$ [5]

(iii) Solve this equation, and hence find the y-coordinate of D, giving your answer correct to 2 decimal places. [5]

The cartesian equation of the curve (for $0 \leqslant \theta \leqslant \pi$) is

$$y = \tfrac{1}{4}(4 - x)\sqrt{1 - x^2}.$$

(iv) Show that the volume of the solid of revolution of this curve about the x-axis is given by

$$\tfrac{1}{16}\pi\int_{-1}^{1}\left(16 - 8x - 15x^2 + 8x^3 - x^4\right)\mathrm{d}x.$$

Evaluate this integral. [6]

8 A pipeline is to be drilled under a river (see Fig. 8). With respect to axes Oxyz, with the x-axis pointing East, the y-axis North and the z-axis vertical, the pipeline is to consist of a straight section AB from the point $\mathrm{A}(0, -40, 0)$ to the point $\mathrm{B}(40, 0, -20)$ directly under the river, and another straight section BC. All lengths are in metres.

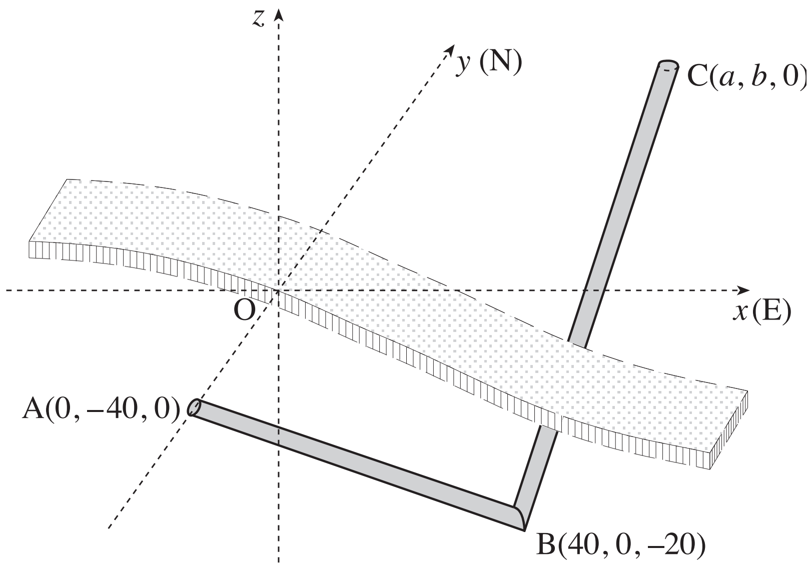

Fig. 8

(i) Calculate the distance AB. [2]

The section BC is to be drilled in the direction of the vector $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.

(ii) Find the angle ABC between the sections AB and BC. [4]

The section BC reaches ground level at the point $\mathrm{C}(a, b, 0)$.

(iii) Write down a vector equation of the line BC. Hence find a and b. [5]

(iv) Show that the vector $6\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ is perpendicular to the plane ABC. Hence find the cartesian equation of this plane. [5]

BLANK PAGE

BLANK PAGE

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4754(B)/01**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper B: Comprehension

INSERT

TUESDAY 23 JANUARY 2007

Afternoon

Time: Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions.

This document consists of **10** printed pages and **2** blank pages.

[Turn over

Benford's Law

Leading digits

This article is concerned with a surprising property of the leading digits of numbers in various sets. The leading digit of a number is the first digit you read. In the number 193 000 the leading digit is 1. When a number is written in standard form, such as 1.93×10^5 or 2.78×10^{-7}, the leading digit is the digit before the decimal point, in these examples 1 and 2 respectively.

Mathematical sequences

Table 1 shows the integer powers of 2, from 2^1 to 2^{50}. In this table, which digits occur more frequently as the leading digit? You might expect approximately one ninth of the numbers to have a leading digit of 1, one ninth of the numbers to have a leading digit of 2, and so on. In fact, this is far from the truth.

2	2 048	2 097 152	2 147 483 648	2 199 023 255 552
4	4 096	4 194 304	4 294 967 296	4 398 046 511 104
8	8 192	8 388 608	8 589 934 592	8 796 093 022 208
16	16 384	16 777 216	17 179 869 184	17 592 186 044 416
32	32 768	33 554 432	34 359 738 368	35 184 372 088 832
64	65 536	67 108 864	68 719 476 736	70 368 744 177 664
128	131 072	134 217 728	137 438 953 472	140 737 488 355 328
256	262 144	268 435 456	274 877 906 944	281 474 976 710 656
512	524 288	536 870 912	549 755 813 888	562 949 953 421 312
1 024	1 048 576	1 073 741 824	1 099 511 627 776	1 125 899 906 842 624

Table 1

The frequencies of the different leading digits in these powers of 2 are shown in Table 2.

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	15	10	5	5	5	4	1	5	0

Table 2

You can see that, for these data, 1 and 2 appear more frequently than any other numbers as the leading digit. Is this just a peculiarity of the first fifty powers of 2, or is a general pattern emerging?

Here is another example. Imagine you invest £100 in an account that pays compound interest at a rate of 20% per year. Table 3 shows the total amount (in £), after interest is added, at the end of each of the following 50 years.

120.00	743.01	4 600.51	28 485.16	176 372.59
144.00	891.61	5 520.61	34 182.19	211 647.11
172.80	1 069.93	6 624.74	41 018.63	253 976.53
207.36	1 283.92	7 949.68	49 222.35	304 771.83
248.83	1 540.70	9 539.62	59 066.82	365 726.20
298.60	1 848.84	11 447.55	70 880.19	438 871.44
358.32	2 218.61	13 737.06	85 056.22	526 645.73
429.98	2 662.33	16 484.47	102 067.47	631 974.87
515.98	3 194.80	19 781.36	122 480.96	758 369.85
619.17	3 833.76	23 737.63	146 977.16	910 043.82

Table 3

The frequencies of leading digits for these data are shown in Table 4.

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	15	9	6	5	4	3	4	2	2

Table 4

The pattern that emerges is much the same as that in Table 2, with 1 appearing as the leading digit in 30% of cases and 2 appearing in 18% of cases.

Now imagine that a second person invests £200 rather than £100. Each amount in this person's table (Table 5) is double the corresponding amount in Table 3.

240.00	1 486.02	9 201.02	56 970.32	352 745.18
288.00	1 783.22	11 041.23	68 364.38	423 294.21
345.60	2 139.86	13 249.47	82 037.25	507 953.05
414.72	2 567.84	15 899.37	98 444.70	609 543.66
497.66	3 081.40	19 079.24	118 133.65	731 452.40
597.20	3 697.69	22 895.09	141 760.37	877 742.88
716.64	4 437.22	27 474.11	170 112.45	1 053 291.45
859.96	5 324.67	32 968.93	204 134.94	1 263 949.74
1 031.96	6 389.60	39 562.72	244 961.93	1 516 739.69
1 238.35	7 667.52	47 475.26	293 954.31	1 820 087.63

Table 5

The frequencies of leading digits for these data are shown in Table 6.

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	15	9	6	5	4	3	3	3	2

Table 6

A remarkable result now emerges. The frequencies in Table 6 are almost the same as those in Table 4.

In Table 3 there are 15 numbers with a leading digit of 1. Each of these numbers, when doubled, has a leading digit of either 2 or 3, as can be seen in Table 5. Similarly, the numbers in Table 3 with leading digit 5, 6, 7, 8 or 9 give numbers in Table 5 with leading digit 1. These outcomes are reflected in the frequencies in Table 4 and Table 6.

Physical phenomena

The numbers in Tables 1, 3 and 5 were all generated mathematically. Now look at something less mathematical in origin.

The frequencies of the leading digits of the areas of the world's 100 largest countries, measured in square kilometres, are given in Table 7. (For interest the data are given in Appendix A.)

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	33	20	13	8	6	6	4	4	6

Table 7

You will notice that these data, even though they have a non-mathematical origin, show essentially the same pattern of frequencies. The populations of cities and countries also show this general pattern. As further examples, if you take the heights of the world's tallest mountains, the lengths of Europe's longest rivers, the numbers of votes cast for each political party in every constituency in a general election or the values of a wide range of scientific constants, you will find a similar pattern in many cases. The remainder of this article looks at such physical data, rather than mathematically generated data, and answers the following question.

Why does this pattern in leading digits occur, and how can it be modelled mathematically?

Benford's Law

This phenomenon was noted in 1881 by Simon Newcombe, an American mathematician and astronomer, and then rediscovered by the physicist Frank Benford in 1938. Benford analysed 20 229 sets of data, including information about rivers, baseball statistics and all the numbers in an issue of *Reader's Digest*. He was rewarded for his efforts by having the law named after him.

Benford's Law gives a formula for the proportions of leading digits in data sets like these. This formula will be derived over the next few pages.

The proportions given by Benford's Law are illustrated in Fig. 8.

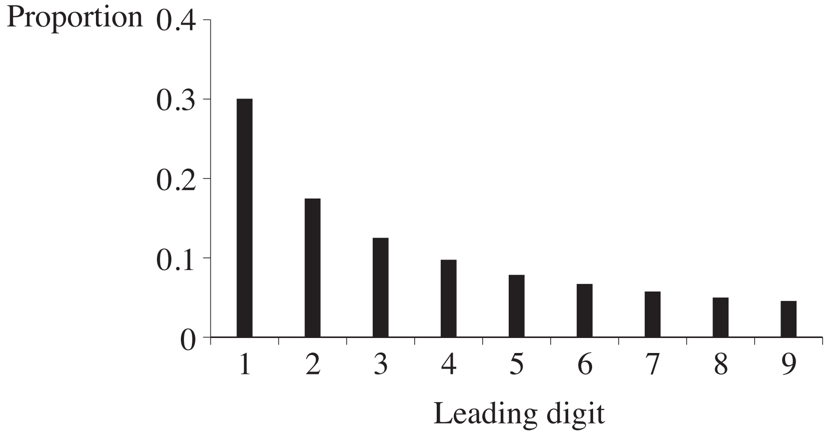

Fig. 8

This shows that, in a typical large data set, approximately 30% of the data values have leading digit 1 but fewer than 5% have leading digit 9. For small data sets, you cannot expect the leading digits to follow Benford's Law closely; the larger the data set, the better the fit is likely to be.

Fig. 9 shows the proportions for the leading digits of the areas of the world's largest countries, derived from Table 7, together with the proportions given by Benford's Law. You will see that there is a very good match.

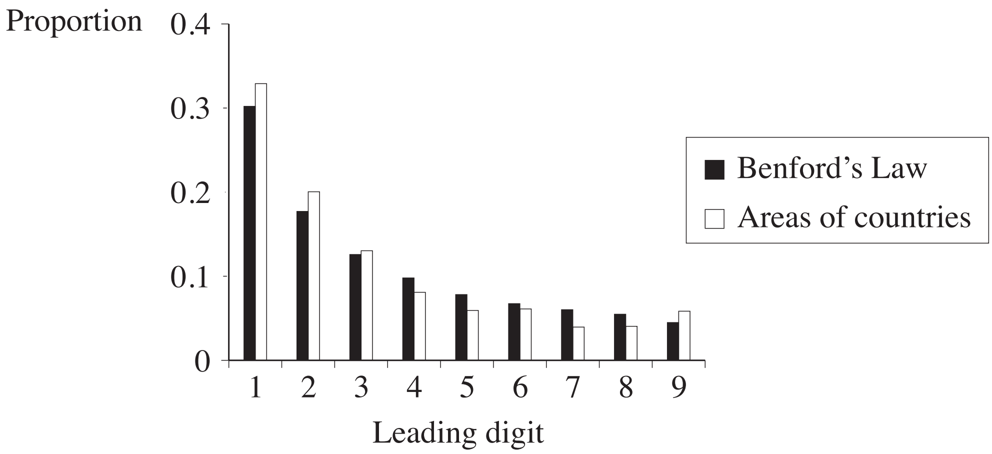

Fig. 9

Benford's Law does not apply to all situations, even when there is a large data set. There is still debate about the conditions under which it applies. The rest of this article relates to situations in which it does apply.

Scale invariance

If the areas of the countries in Appendix A are measured in square miles, rather than square kilometres, it turns out that the leading digits still follow the same pattern. This is a feature of all data sets which follow Benford's Law; it does not matter what units are used when measuring the data. The next example illustrates this.

Table 10 shows the frequencies of the leading digits of the share prices of the 100 largest UK companies on 28 February 2006, in pounds sterling, US dollars and euros.

Leading digit	1	2	3	4	5	6	7	8	9
Frequency (£)	33	14	7	4	14	11	3	8	6
Frequency ($)	41	18	13	6	6	2	3	7	4
Frequency (€)	36	18	10	8	2	4	8	8	6

Table 10

These results are illustrated in Fig. 11 along with the proportions given by Benford's Law. Despite there being only 100 items of data, two features are evident.

- There is a reasonable agreement between the proportions for the three currencies.
- Benford's Law gives a reasonable approximation in each case.

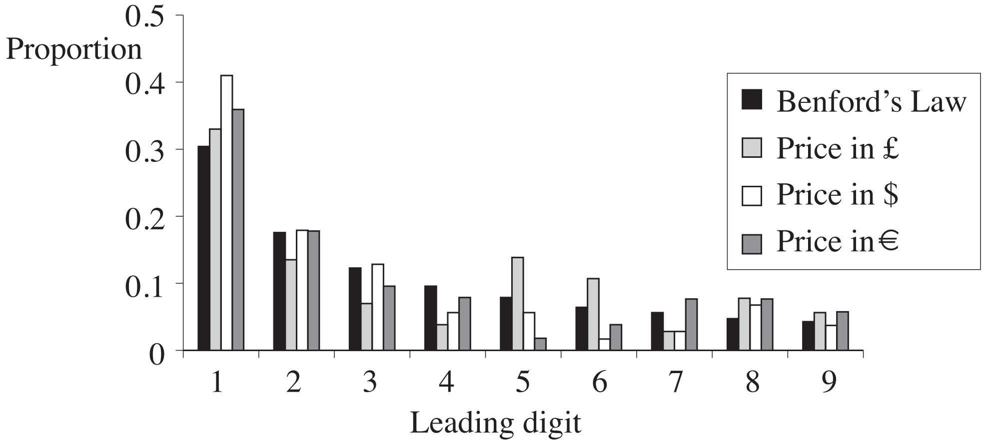

Fig. 11

This property, that it does not matter what units are used when measuring the data, is called *scale invariance*.

The frequencies of leading digits

The idea of scale invariance is important. If scale invariance applies, what does this tell us about the frequencies of leading digits?

In order to answer this question it is helpful to use the following notation.

- p_n represents the proportion of data values with leading digit n.

Thus p_1 represents the proportion of data values with leading digit 1, p_2 represents the proportion of data values with leading digit 2, and so on. Clearly $\sum_{n=1}^{9} p_n = 1$.

The proportions $p_1, p_2, \ldots, p_9$ can be represented by the areas of the rectangles on a diagram such as Fig. 12. The total area is 1.

Fig. 12

Many things can be deduced about the values of $p_1, p_2, \ldots, p_9$ by thinking about a large data set in which scale invariance holds exactly. Here are some of them.

- If every number in the data set is multiplied by 2, then all the numbers with leading digit 1, and no others, are mapped to numbers with leading digit either 2 or 3. Since this does not change the distribution of leading digits, it follows that

$$p_1 = p_2 + p_3.$$

- Similarly, when multiplying by 2, all numbers with leading digit 5, 6, 7, 8 or 9, and no others, are mapped to numbers with leading digit 1. Therefore

$$p_5 + p_6 + p_7 + p_8 + p_9 = p_1.$$

As a consequence of these two results,

$$p_1 + (p_2 + p_3) + p_4 + (p_5 + p_6 + p_7 + p_8 + p_9) = 3p_1 + p_4,$$

from which it follows that

$$3p_1 + p_4 = 1$$

and so $p_1 < \frac{1}{3}$.

- By using a multiplier of 4, instead of 2, it follows that

$$p_1 = p_4 + p_5 + p_6 + p_7.$$

This shows that $p_1 > p_4$. Using the fact that $3p_1 + p_4 = 1$, it follows that $p_1 > \frac{1}{4}$. Therefore $\frac{1}{4} < p_1 < \frac{1}{3}$. This is consistent with the value of about 0.3 observed in several of the data sets considered earlier in the article.

In a similar way, other relationships connecting values of p_n, such as $p_1 = p_3 + p_4 + p_5$, $p_6 + p_7 = p_3$ and $p_2 = p_6 + p_7 + p_8$, can be derived.

Deriving Benford's Law

It is helpful now to introduce the quantities L(1), L(2), …, L(10), defined as follows.

- $\mathrm{L}(1) = 0$
- $\mathrm{L}(2) = p_1$
- $\mathrm{L}(3) = p_1 + p_2$
- $\mathrm{L}(4) = p_1 + p_2 + p_3$

  …
- $\mathrm{L}(10) = p_1 + p_2 + \ldots + p_9 = 1$

The quantities L(1), L(2), …, L(10) are the cumulative proportions. They are illustrated in Fig. 13.

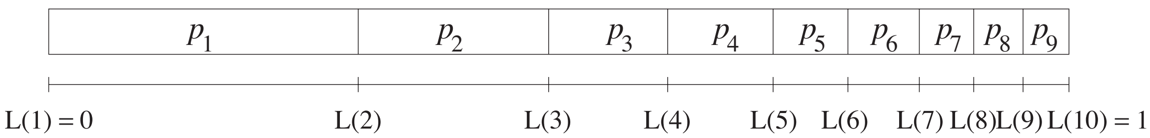

Fig. 13

What can you say about the quantities L(1), L(2), …, L(10)?

- You know $p_1 = p_2 + p_3$.

This corresponds to $\mathrm{L}(2) - \mathrm{L}(1) = \mathrm{L}(4) - \mathrm{L}(2)$ which simplifies to $\mathrm{L}(4) = 2 \times \mathrm{L}(2)$.

- Similarly $p_1 = p_3 + p_4 + p_5$.

This corresponds to $\mathrm{L}(2) - \mathrm{L}(1) = \mathrm{L}(6) - \mathrm{L}(3)$ which simplifies to $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$.

- Also $p_6 + p_7 = p_3$.

  This corresponds to $\mathrm{L}(8) - \mathrm{L}(6) = \mathrm{L}(4) - \mathrm{L}(3)$. Combining this with the last two results gives $\mathrm{L}(8) = 3 \times \mathrm{L}(2)$.

These results, and others like them, suggest that $\mathrm{L}(n)$ is a logarithmic function. The fact that $\mathrm{L}(10) = 1$ shows that the base of the logarithms is 10, and so $\mathrm{L}(n) = \log_{10} n$.

It follows that $p_n = \mathrm{L}(n+1) - \mathrm{L}(n) = \log_{10}(n+1) - \log_{10} n$. That is, the proportion of data values with leading digit n (where $1 \leqslant n \leqslant 9$) is $\log_{10}(n+1) - \log_{10} n$. This is Benford's Law.

Uses of Benford's Law

Since the 1980s Benford's Law has, on several occasions, been used successfully to convict people accused of fraud. When concocting figures to include in fictitious company accounts, it is natural to try to make the amounts look 'random' or 'average'. This might, for example, be done by including a high proportion of 'average' amounts beginning with 'middle digits' such as 4, 5 or 6, or including amounts just under £1000, £10 000 and £100 000, in an attempt to avoid closer analysis. In this way fraudsters are generating data which do not follow Benford's Law and thereby attracting the kind of scrutiny they were trying to avoid.

Appendix A Areas of countries (thousands of km^2)

Russia	17 075
Canada	9976
USA	9629
China	9597
Brazil	8512
Australia	7687
India	3288
Argentina	2767
Kazakhstan	2717
Sudan	2506
Algeria	2382
Congo (Dem. Rep.)	2345
Greenland	2166
Mexico	1973
Saudi Arabia	1961
Indonesia	1919
Libya	1760
Iran	1648
Mongolia	1565
Peru	1285
Chad	1284
Niger	1267
Angola	1247
Mali	1240
South Africa	1220
Colombia	1139
Ethiopia	1127
Bolivia	1099
Mauritania	1031
Egypt	1001
Tanzania	945
Nigeria	924
Venezuela	912
Namibia	825
Pakistan	804
Mozambique	802
Turkey	781
Chile	757
Zambia	753
Myanmar	679
Afghanistan	648
Somalia	638
C. African Republic	623
Ukraine	604
Botswana	600
Madagascar	587
Kenya	583
France	547
Yemen	528
Thailand	514
Spain	505
Turkmenistan	488
Cameroon	475
Papua New Guinea	463
Sweden	450
Uzbekistan	447
Morocco	447
Iraq	437
Paraguay	407
Zimbabwe	391
Japan	378
Germany	357
Congo (Rep.)	342
Finland	337
Malaysia	330
Vietnam	330
Norway	324
Cote d'Ivoire	322
Poland	313
Italy	301
Philippines	300
Ecuador	284
Burkina Faso	274
New Zealand	269
Gabon	268
Western Sahara	266
Guinea	246
Great Britain (and N Ireland)	245
Ghana	239
Romania	238
Laos	237
Uganda	236
Guyana	215
Oman	212
Belarus	208
Kyrgyzstan	199
Senegal	196
Syria	185
Cambodia	181
Uruguay	176
Tunisia	164
Suriname	163
Bangladesh	144
Tajikistan	143
Nepal	141
Greece	132
Nicaragua	129
Eritrea	121
Korea (North)	121
Malawi	118

BLANK PAGE

BLANK PAGE

OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper B: Comprehension

TUESDAY 23 JANUARY 2007

4754(B)/01

Afternoon
Time: Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF2)

Candidate Name

Centre Number

Candidate Number

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 18.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with the question paper.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

For Examiner's Use	
Qu.	Mark
1	
2	
3	
4	
5	
6	
Total	

This document consists of **4** printed pages and an insert.

1 In a certain country, twenty cars are on display in a car showroom. The costs of the cars in the local currency, the zen, are shown below.

10 255	23 250	48 500	25 950	12 340
34 750	5 690	13 580	7 450	9 475
18 890	14 675	6 295	21 225	37 850
51 200	43 340	16 575	8 380	28 880

(i) Complete the table giving the frequencies of the leading digits. [1]

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	6	4	2						

The country joins the European Union and so the costs of the cars are converted to euros. The exchange rate is 1 zen = 3 euros.

(ii) Give the costs of the cars in euros in the space below and then complete the table giving the frequencies of the leading digits in euros. [2]

....

....

....

....

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	7								0

(iii) In the table below, give the frequencies predicted by Benford's Law, in each case correct to **one decimal place**. [2]

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	6.0								

(iv) Compare the results in the three tables. [1]

....

....

2 On lines 28 and 29 it says 'Similarly, the numbers in Table 3 with leading digit 5, 6, 7, 8 or 9 give numbers in Table 5 with leading digit 1'. Explain how this is reflected in the frequencies in Table 4 and Table 6. [1]

....

....

....

3 Line 104 refers to the relationship $p_1 = p_3 + p_4 + p_5$. Explain how this relationship is obtained. [2]

....

....

....

....

4 Benford's Law is quoted on lines 126 to 127. Show that this is equivalent to

$$p_n = \log_{10}\left(1 + \frac{1}{n}\right).$$

[2]

....

....

....

....

5 Using the results $\mathrm{L}(4) = 2 \times \mathrm{L}(2)$ and $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$, and the relationship $p_6 + p_7 = p_3$, derive the result $\mathrm{L}(8) = 3 \times \mathrm{L}(2)$ stated on line 123. [3]

....

....

....

....

....

....

For Examiner's Use

6 The distribution of leading digits in the **daily** wages, in pounds sterling, of the employees of a firm is given in the table below.

Leading digit	1	2	3	4	5	6	7	8	9
Frequency (**daily** wages)	29	16	12	10	8	7	6	5	4

The employees all work a 5-day week. Using the values for the **daily** wages above, find the entries marked a and b for the **weekly** wages in the table below. Explain your reasoning.[4]

Leading digit	1	2	3	4	5	6	7	8	9
Frequency (**weekly** wages)	a			b					

....

....

....

....

....

....

....

....

Mark Scheme 4754
January 2007

Paper A – Section A

1 $\frac{1}{x}+\frac{x}{x+2}=1$ $\Rightarrow$ $x+2+x^2=x(x+2)$ $=x^2+2x$ $\Rightarrow$ $x=2$	M1 A1 DM1 A1 [4]	Clearing fractions solving cao
2(i) x: 0, 0.5, 1, 1.5, 2; y: 1, 1.0607, 1.4142, 2.0917, 3 $A \approx 0.5[(1+3)/2 + 1.0607 + 1.4142 + 2.0917]$ $= 3.28$ (3 s.f.)	B1 M1 A1 [3]	At least one value calculated correctly or 13.13…or 6.566… seen
(ii) 3.25 (or Chris) area should decrease with the number of strips used.	B1 B1 [2]	ft (i) or area should decrease as concave upwards
3(i) sin 60 = √3/2, cos 60 = 1/2 , sin 45 = 1/√2, cos 45 = 1/√2 sin(105°) = sin(60°+45°) = sin60°cos45° + cos60°sin45° $=\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}+\frac{1}{2}.\frac{1}{\sqrt{2}}$ $=\frac{\sqrt{3}+1}{2\sqrt{2}}$ *	M1 M1 A1 E1 [4]	 splitting into 60° and 45°, and using the compound angle formulae
(ii) Angle B = 105° By the sine rule: $\frac{AC}{\sin B}=\frac{1}{\sin 30}$ $\Rightarrow$ AC = $\frac{\sin 105}{\sin 30}=\frac{\sqrt{3}+1}{2\sqrt{2}}.2$ $=\frac{\sqrt{3}+1}{\sqrt{2}}$ *	 M1 A1 E1 [3]	 Sine rule with exact values www
4 $\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ $=\frac{1}{\cos 2\theta}$ $=\sec 2\theta$ $\sec 2\theta = 2 \Rightarrow \cos 2\theta = ½$ $\Rightarrow$ $2\theta = 60°, 300°$ $\Rightarrow$ $\theta = 30°, 150°$	M1 M1 M1 E1 M1 B1 B1 [7]	$\tan\theta=\frac{\sin\theta}{\cos\theta}$ or $1+\tan^2\theta=\sec^2\theta$ used simplifying to a simple fraction in terms of $\sin\theta$ and/or $\cos\theta$ only $\cos^2\theta-\sin^2\theta=\cos 2\theta$ oe used or $1+\tan^2\theta=2(1-\tan^2\theta) \Rightarrow \tan\theta=\pm1/\sqrt{3}$ oe 30° 150° and no others in range

5 $(1+3x)^{\frac{1}{3}} =$ $= 1+\frac{1}{3}(3x)+\frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(3x)^3+...$ $=1+x-x^2+\frac{5}{3}x^3+...$ Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	M1 B1 A2,1,0 B1 [5]	binomial expansion (at least 3 terms) correct binomial coefficients (all) x, $-x^2$, $5x^3/3$
6(i) $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$ $\Rightarrow$ $1 = A(x + 1) + B(2x + 1)$ $x = -1$: $1 = -B \Rightarrow B = -1$ $x = -½$: $1 = ½ A \Rightarrow A = 2$	M1 A1 A1 [3]	or cover up rule for either value
(ii) $\frac{dy}{dx}=\frac{y}{(2x+1)(x+1)}$ $\Rightarrow$ $\int\frac{1}{y}dy=\int\frac{1}{(2x+1)(x+1)}dx$ $=\int(\frac{2}{2x+1}-\frac{1}{x+1})dx$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + c$ When $x = 0$, $y = 2$ $\Rightarrow$ $\ln 2 = \ln 1 - \ln 1 + c \Rightarrow c = \ln 2$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + \ln 2$ $=\ln\frac{2(2x+1)}{x+1}$ $\Rightarrow$ $y=\frac{4x+2}{x+1}$ *	M1 A1 B1ft M1 E1 [5]	separating variables correctly condone omission of c. ft A,B from (i) calculating c , no incorrect log rules combining lns www

Section B

7(i) At A, cos $\theta = 1 \Rightarrow \theta = 0$ At B, cos $\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow$ cos $\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ dy/dx = 0 when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding $dy/d\theta$ and $dx/d\theta$ correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ (1 + ½ √6 > 1 so no solution) $\Rightarrow \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ = 1.35π = 4.24	 M1 M1 E1 B1 M1 A1cao [6]	 correct integral and limits expanding brackets correctly integrated substituting limits

8 (i) $\sqrt{(40-0)^2+(0+40)^2+(-20-0)^2}$ $= 60$ m	M1 A1 [2]	
(ii) $\overrightarrow{BA} = \begin{pmatrix}-40\\-40\\20\end{pmatrix} = 20\begin{pmatrix}-2\\-2\\1\end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix}-2\\-2\\1\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}}{\sqrt{9}\sqrt{26}} = -\dfrac{13}{3\sqrt{26}}$ $\Rightarrow \quad \theta = 148°$	M1 A1 A1 A1 [4]	or $\overrightarrow{AB}$ −13 oe eg -260 √9√26 oe eg 60√26 cao (or radians)
(iii) $\mathbf{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ At C, $z = 0 \Rightarrow \lambda = 20$ $\Rightarrow \quad a = 40 + 3 \times 20 = 100$ $b = 0 + 4 \times 20 = 80$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}40\\0\\-20\end{pmatrix} + \ldots$ $\ldots + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ or….$+\lambda\begin{pmatrix}a-40\\b\\20\end{pmatrix}$ 100 80
(iv) $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}-2\\-2\\1\end{pmatrix} = -12+10+2=0$ $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix} = 18-20+2=0$ $\Rightarrow \begin{pmatrix}6\\-5\\2\end{pmatrix}$ is perpendicular to plane. Equation of plane is $6x - 5y + 2z = c$ At B (say) $6 \times 40 - 5 \times 0 + 2 \times -20 = c$ $\Rightarrow c = 200$ so $6x - 5y + 2z = 200$	B1 B1 M1 M1 A1 [5]	(alt. method finding vector equation of plane M1 eliminating both parameters DM1 correct equation A1 stating Normal hence perpendicular B2)

Paper B Comprehension

<table>
<tr><td>1(i)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>6</td><td>4</td><td>2</td><td>2</td><td>2</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
</table>
</td><td>B1 Table</td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>7</td><td>3</td><td>2</td><td>3</td><td>1</td><td>2</td><td>1</td><td>1</td><td>0</td></tr>
</table>
</td><td>M1 A1 Table</td></tr>
<tr><td>(iii)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>6.0</td><td>3.5</td><td>2.5</td><td>1.9</td><td>1.6</td><td>1.3</td><td>1.2</td><td>1.0</td><td>0.9</td></tr>
</table>
</td><td>B1any 4 correct
B1 other 4 correct</td></tr>
<tr><td>(iv)</td><td>Any sensible comment such as:
• The general pattern of the frequencies/results is the same for all three tables.
• Due to the small number of data items we cannot expect the pattern to follow Benford's Law very closely.</td><td>E1</td></tr>
<tr><td>2</td><td>Evidence of 4+3+4+2+2 from Table 4 frequencies is the same as 15 in Table 6</td><td>B1</td></tr>
<tr><td>3</td><td>$p_1 = p_3 + p_4 + p_5$: on multiplication by 3, numbers with a leading digit of 1 will be mapped to numbers with a leading digit of 3, 4 or 5 and no other numbers have this property.</td><td>B1 Multiplication
B1… by 3</td></tr>
<tr><td>4</td><td>$\log_{10}(n+1) - \log_{10} n = \log_{10}\left(\frac{n+1}{n}\right) = \log_{10}\left(\frac{n}{n} + \frac{1}{n}\right) = \log_{10}\left(1 + \frac{1}{n}\right)$</td><td>M1
E1</td></tr>
<tr><td>5</td><td>Substitute $\mathrm{L}(4) = 2 \times \mathrm{L}(2)$ and $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$ in
$\mathrm{L}(8) - \mathrm{L}(6) = \mathrm{L}(4) - \mathrm{L}(3)$:
this gives $\mathrm{L}(8) = \mathrm{L}(6) - \mathrm{L}(3) + \mathrm{L}(4) = \mathrm{L}(2) + 2 \times \mathrm{L}(2) = 3 \times \mathrm{L}(2)$</td><td>M1
M1 subst
E1
(or alt
M1 for 2 or more Ls used
M1 use of at least 2 given results oe
E1)</td></tr>
<tr><td>6</td><td>$a = 28$. All entries with leading digit 2 or 3 will, on multiplying by 5, have leading digit 1. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td>$b = 9$. Similarly, all entries with leading digit 8 or 9 will, on multiplying by 5, have leading digit 4. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td></td><td>**Total 18**</td></tr>
</table>

4754 - Applications of Advanced Mathematics (C4)

General Comments

This paper was of a similar standard to that of last January. Candidates found it much more straightforward than the June 2006 paper. There was a wide range of responses but all questions were answered well by some candidates. There were some excellent scripts.
Candidates should be advised to read questions carefully. There were instances, particularly in the Comprehension, where instructions were not followed.
There was also some use of inefficient methods. Those that were competent at algebra and surds and were familiar with manipulating trigonometric formulae generally achieved good results. Some of the arithmetic in the trapezium rule and the integration of the polynomial was disappointing.
There was some evidence of shortage of time as a small proportion of candidates failed to complete question 8.

Comments on Individual Questions

Paper A

Section A

1 The algebraic fraction equation was almost always answered correctly.

2 (i) There seemed to be a lack of familiarity with the trapezium rule formula. Common errors were use of A=0.5h(y_0+y_4) +2(y_1+y_2 +y_3) but omission of the other brackets. Or alternatively omitting y_0 and using A=0.5h((y_1+y_4)+2 (y_2 +y_3)). Most obtained at least one ordinate correctly but there were many errors in the calculation of the answer.

(ii) Those without correct, or almost correct, answers in the first part could not make a valid comment about which of Chris or Dave was correct in their calculations. There were some poor explanations given, such as 'the trapezium rule always overestimates results'.

3 Most candidates correctly used the compound angle formula as the first stage. Those that used *sin/cos* 45° as $\sqrt{2}/2$ rather than $1/\sqrt{2}$ could not always deal with cancelling $(\sqrt{6}+\sqrt{2})/4$. The sine rule was usually correct.

4 There were some efficient solutions but weaker candidates found it difficult to see ahead to what was needed. In some cases poor knowledge of trigonometric identities and their rearrangement was the problem. Some tried to work on both sides simultaneously - some more clearly than others.
There were some confused starts using incorrect identities in the second part but many did obtain the first solution. The solution θ=150° was often lost - in some cases due to missing the negative square root.

5 This was well answered. The improvement seen in the binomial expansion was pleasing although this was possibly due to the first number in the bracket being a 1. There were still some candidates who used *x* rather than *3x* throughout the calculation and many could not deal successfully with the range for the validity.

6 The partial fractions were almost always correct.
The second part was less successful. Some separated the variables to *ydy*=….
Many integrated 2/ (2*x*+1) as 2ln (2*x*+1) and there were many instances of the omission of the constant. Poor use of the laws of logarithms meant that c was often not found correctly. For example, lny=ln(2x+1) – ln(x+1) +c leading to y= (2x+1)/(x+1) +c was common. Those that found c before combining their logs were more successful. 2ln(2x+1) – ln(x+1) = ln 2(2x+1)/(x+1) was also a common error.

Section B

7 (i) This was usually correctly answered although some candidates used long methods to show that $\theta=0$ at A and others gave the value of θ at B in degrees.

(ii) There were many errors in *dy/dθ*- usually the coefficient of *cos 2θ* being incorrect - and there were also sign errors. Most knew that they had to equate *dy/dx* to zero but made errors in their simplification to the given equation.

(iii) Some omitted this or tried to factorise and then abandoned the attempt. Of those that did use the formula, a common mistake was to solve the quadratic equation for *cos θ* but then to use this as *θ* in the expression for *y.*

(iv) This was disappointing. The first part was usually correct but a significant number failed to integrate the polynomial. Of those that did integrate, many surprisingly made numerical errors when substituting the limits.

8 (i) Most candidates correctly found the distance AB.

(ii) Many failed to find the required angle ABC.

(iii) This proved to be very successful for many. Those that gave the required vector equation in terms of *a* and *b*, however, could rarely make progress. A few found *a* and *b* successfully without explicitly writing down the equation of the line.

(iv) Once again too many candidates failed to realise that in order to prove that a vector is perpendicular to a plane it is necessary to show it is perpendicular to **two** vectors in the plane. Others did not evaluate their dot product, merely stating it was zero. Most used the Cartesian form of the equation with success. There were still some candidates who approached this from the vector equation of the plane and they were more likely to make errors.

Paper B

Comprehension

1 The tables in (i) and (ii) were usually correct but there were occasional slips. In (iii) candidates often failed to calculate using Benford's Law. It was unclear what their methods were in (iii) but they may have been trying to use Fig.9.

2 This was often successful but it was not always clear which tables the candidates were referring to.

3 Some failed to explain about the multiplication of leading digits. For those that did, the multiplication factor quoted did not always work for the complete range. Multiplying 3,4 and 5 by 3.5 or 4 was commonly seen.

4 Usually correct although log (n+1) –log n=log(n+1)/log n was seen.

5 The approach encouraged by the question was not always used. There were some very long and often confused solutions involving changing all ‘L’ expressions to strings of ‘p’ equations and eliminating.

6 Candidates often seemed not to have read this question carefully. There were many good solutions, but too often the proportions were calculated rather than using the frequencies in the table.