# Mathematics (MEI) 

Advanced GCE A2 7895-8

## Reports on the Units

## June 2006

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Any enquiries about publications should be addressed to:
OCR Publications
PO Box 5050
Annersley
NOTTINGHAM
NG15 0DL
Telephone: 08708706622
Facsimile: 08708706621
E-mail: publications@ocr.org.uk

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## PURE MATHEMATICS

## OCR (MEI) Mathematics GCE

## Rules for crossed out and/or replaced work.

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.

## Legacy units

Centres are reminded that under no circumstances can 'legacy' units be used in the 'new' specification from January 2007 onwards. The only units that are acceptable for this specification are units 4751-4777.

## 4751 - Introduction to Advanced Mathematics (C1)

## General Comments

A full spread of marks was seen, but it was perhaps slightly harder to obtain 65+ marks on this paper than on some past C 1 papers.

There were excellent scripts seen from the candidates who were able to approach this examination with confidence in their mastery of the content and the skills required for this paper. However, examiners were concerned at the long tail of weak candidates seen this session. Centres are also reminded that calculators are not allowed in this paper. Examiners found some instances of calculators being used, which were reported as suspected malpractice.

There were some very sketchy attempts at question 13 and some did not attempt it, suggesting that perhaps some candidates had time problems and a few may have failed to turn the page. However, some candidates had several goes at parts of other questions where they had had difficulty, indicating that unfamiliarity with the graph of a reciprocal function may also have been a cause.

## Comments on Individual Questions

## Section A

1) This rearrangement of the formula was well done, but many candidates left their answers as a triple-decker fraction.
2) Most candidates knew that they had to substitute $x=-2$ and many successfully found the value of $a$.
3) Many candidates did not identify the gradient of the line correctly, although most then went on to use the correct method for a line through $(2,10)$.
4) There was often no working by the candidates to support their decisions, but examiners suspected that many had only a limited understanding of the meaning of the implication symbols being tested. Few candidates gained both marks here.
5) Relatively few candidates chose the most straightforward method of substitution. There were many errors, both algebraic and numeric, especially when elimination was attempted or fractions were used.
6) Most candidates could solve the quadratic equation but far fewer could go on to solve the inequality correctly.
7) Only a minority completed part (i) successfully and many showed several lines of ineffective working. The second part was better, but there were many errors in squaring $3 \sqrt{ } 5$.
8) This was the first C 1 paper on which the ${ }^{n} \mathrm{C}_{r}$ notation has been explicitly tested and many candidates were clearly unfamiliar with it, although knowledge of it is a statement in the specification. Many candidates started again in finding the coefficient of $x^{3}$ in the binomial expansion, using Pascal's triangle and not realising that any result for ${ }^{6} \mathrm{C}_{3}$ that they had found already had any relevance.
9) There were many errors in coping with the indices here, although weaker candidates who knew the rules of indices could pick up marks in the second part very easily, and some of them did. In the first part, most obtained $16^{1 / 2}=4$, but then often did not know how to proceed. Some tried to cube 81 and then despaired at finding the fourth root of that, whilst correctly finding $3^{3}$ but evaluating that as 9 was a common error. Better candidates usually sailed through with no problems.
10) Most candidates knew they had to substitute for $y$ and solve the resulting equation, but common errors were $3 x^{2}$ instead of $(3 x)^{2}$, or thinking that $x^{2}+y^{2}=25$ leads to $x+y=5$. Later in the question it was common to omit the negative root.

## Section B

11 (i) Overall, question 11 was the question which caused the greatest problems to candidates in section B. However, most candidates had a reasonable attempt at this first part, usually understanding that it was necessary to find the gradients and show that the product was -1 . Some made arithmetic slips in their calculations (usually with signs) yet managed to believe that the product was -1 . A few calculated the gradient as change in $\mathrm{x} \div$ change in y , but were still "successful" in showing the product to be -1 . A few used the alternative method of Pythagoras' theorem instead of gradients.
(ii) Part (ii) was done correctly and elegantly by some candidates, but in general there were many errors. Expressions like $(x-a)^{2}+(y-b)^{2}=r^{2}$ showed that many of the candidates know the form of equation required for a circle. However, some candidates failed to recognise that the midpoint of AC was the centre of the required circle, and many failed to calculate the radius of the circle correctly, often finding $A C^{2}$, but concluding that $r=\sqrt{ }(85 / 2)$ rather than $\sqrt{ } 85 / 2$. Few candidates gained the final mark in part (ii) either because they made no attempt or because their figures were not going to work out. A few showed no working in the belief that their calculation must be right! Only a handful used the geometric property to gain this mark, which could have been gained even if the equation of the circle had been wrong.
(iii) By this stage some candidates had had enough of this question. There were surprisingly few correct answers in evidence for part (iii) - many candidates appeared to have made no attempt. However, those candidates who drew a diagram often saw quite quickly how easy this part really was. Some weak candidates who floundered in part (ii) were able to succeed here. Many candidates wasted time on this question doing unnecessary and involved algebra.

12 (i) -(iii) Question 12 was much more to the candidates' liking than question 11 and the majority showed a good understanding of the factor theorem, as well as the technique of algebraic division. As a consequence the first three parts were often correct.
(iv) The curve sketching was not so well done though most arrived at a curve that was basically a cubic, even if it was a 'mirror image'. The labelling of the axes was not always done fully; it was not uncommon for the intercept on the $y$-axis to be missed.
(v) This final part was not done as well as the rest of the question. Only the better candidates simplified and then fully factorised the expression in order to yield the full solution. Many continued to try to determine the roots using the factor theorem - usually being satisfied once they had found one solution. Candidates often realised the connection with the previous part to obtain $x=0$. Some candidates who did factorise, cancelled out the $x$ factor and so lost the solution $x=0$.
(i) Many candidates did not seem to realise the need to draw the line $y=2 x+3$ on the graph. Those who did know the technique usually used it correctly and obtained full marks here and in part (v). A few drew the line correctly but did not give the solutions to the equation in part (i).
(ii) Many candidates did well on (ii), usually generating a quadratic equation and making a good attempt to apply the formula. However, a number of careless slips were in evidence. Some did not know how to cope with the $1 / x$ term and attempted to apply the quadratic formula to a non-quadratic equation.
(iii) The curve drawing here was not done too well, the lower part of the graph causing more problems than the top, often being translated 2 down instead of 2 up. Many candidates were content with rough sketches, in contrast to some who showed considerable care in translating the graph of $y=1 / x$ that was already drawn for them on the grid.
(iv) This part was done independently of part (i), with many candidates finding the solution algebraically or by inspection and often arriving at just one of the roots.

## 4752 - Concepts for Advanced Mathematics (C2)

## General Comments

The paper was well received. All questions were accessible but there was wide-spread misunderstanding of the requirements in question 3 and the diagram in question 10. Although good candidates had no trouble completing the paper others could not manage all the work in questions 11 and 12, perhaps due to lack of knowledge, perhaps having spent too much time on earlier problems.
There were many excellent scripts and just a few from candidates totally unprepared at this level.

## Comments on Individual Questions

## Section A

1) This was often totally correct. Surprisingly some could evaluate the first expression but not the second.
2) This was very well done by the majority. Just a few thought that the sum was $a /(r$ 1). Many could not do the simple algebra needed to solve $10=8 /(1-r)$.
3) 

It must be made clear to candidates, as part of the preparation for this paper, that a request for 'the exact value' of anything usually implies that calculators must not be used. To put 0.25 into a calculator on one function and take it out on another gives the answer of 0.258 in a matter of a few seconds; but 0.258 is not exact, nor is 0.2581988897 . The few who put $\sin \theta=1 / 4$ onto a sketch of a right angled triangle were usually successful. The few who put it into $\sin ^{2} \theta+\cos ^{2} \theta=1$ were less successful as this method was more prone to errors such as $\sin \theta+\cos \theta=1$ at some stage, or, $1-(1 / 4)^{2}$ onwards being done by calculator.
4) This was well done by the majority. Some had difficulty integrating the second or third term but the general method for definite integration was well known. Weaker candidates substituted into the function without integrating, or substituted after differentiating.
5) Many knew that this was a cubic curve of the form $y=\mathrm{f}(x)+\mathrm{c}$ and they successfully calculated the value of c for full marks. Many did not know the process and they worked with $y=m x+c$ with $m$ as $3-6^{2}=-33$ or with $m=d y / d x$ to get $y=\left(3-x^{2}\right) x+c$; neither of these methods attracted any marks.
6) Candidates who knew how to work with arithmetic progressions fell into two distinct classes; those who could cope with " $51^{\text {st }}$ to $100^{\text {th }}$ inclusive" and those who could not. The former scored 5 marks, the latter only 3 marks. There were very many who calculated $S_{100}-S_{51}$ or who summed the terms starting at the $51^{\text {st }}$ thinking there were only 49 of them. Weaker candidates used incorrect formulae or G.P. formulae or could not understand the suffix notation.
7) (i) The sketches of $\cos x$ and $\cos 2 x$ were often essentially correct and received full marks even when they were not things of beauty. We did need an indication that the amplitude was 1 . The common errors with $\cos 2 x$ were to double the amplitude or double the period.
(ii) Many correctly found the solutions to the equation, many only two. Some converted $\cos 2 x=0.5$ to $\cos x=0.25$.
8) This was very well done indeed. There were two marks for dealing with $6 x^{3}$. There was a mark for using $\sqrt{ } x$ as $x^{1 / 2}$ and two for differentiating it correctly. There was a penalty for failing to differentiate the constant correctly but this was rarely applied.
9)

The vast majority scored full marks here. There were various initial moves, the commonest was $3 x \log 5=\log 100$. A neat move seen a few times was $x \log 125=$ $\log 100$. Those who started $3 x=\log _{5} 100$ seemed to be inviting trouble but their calculators seemed to cope with base 5 .

## Section B

10) (i) In spite of hard evidence and some strong clues against it, angle SLT was taken to be $105^{\circ}$ by perhaps half the entry. As 105 does appear in the space between the arms of that angle, this misinterpretation was not heavily punished. There were two method marks for applying the cosine rule and one for the sine rule, whatever number they took for angle SLT. The bearing mark was allowed if it correctly followed from the candidate's value for angle LTS or angle LST. Many failed to calculate the required bearing.
(ii) Some had difficulty finding the distance travelled, $24 \times 26 / 60=10.4$, especially if they converted 26 minutes to 0.43 hours as a first step. The formula $s=r \theta$ was well known, just a few used $s=r^{2} \theta / 2$. Most earned the mark for converting their 2 radians to degrees. Many candidates could not resist the temptation to see the diagrams as being symmetrical about the N/S line. Again not many successfully found the required bearing.
11) (i) Most knew exactly what they had to do here to earn the five marks and those who took care not to make silly mistakes did earn them. Some made a slip with the derivative, $2 x^{2}-6 x$ perhaps. Some factorised $3 x^{2}-6 x$ as $3 x(x-3)$ or worse. Weaker candidates did not use $y^{\prime}=0$ to find the stationary points, some tried $y=0$ or $y^{\prime \prime}=0$. The commonest test used was 'for a maximum, $y$ " is negative'.
(ii) Many candidates produced excellent work here. Candidates essentially had to solve $y^{\prime}=9$, pick the root $x=3$ for point P , find $y(3)$, find the gradient of the normal and then the equation of the normal. That was quite a series of operations and many were successful. The work came off the rails when they thought that $P$ was the point where the tangent at $(-1,-3)$ met the curve again. Candidates earned a mark for finding the two intercepts made by their normal. For the final mark their normal and intercepts had to be correct forming a triangle of area 8. A few took a rather longer route using definite integration for the normal from 0 to 12.
12) (i) There was a full range of responses here; everyone knew they had to log both sides but some were more successful than others. The commonest error was loga x bt $\log 10$. Many did not convert bt $\log 10$ to bt.
(ii) Most could see that 2 significant figures were insufficient so they gave 3 but there was much carelessness with the rounding; 3.897 became 3.89 and 4.107 became 4.10 or 4.12 . They received a mark for plotting their values with a 1 mm tolerance and they received a mark for a reasonable and ruled line of best fit, using their points.
(iii) Many candidates found the gradient of their line and wrote down their intercept but were not sure whether to use them in a formula for P or for $\log \mathrm{P}$. Two marks were awarded if they were seen, two more if they were correctly used.
(iv) As many candidates arrived at a prediction by extending their graph and axes these two marks were awarded for any answer in a reasonable range; a single mark was awarded for a wider range. There were many excellent attempts scoring full marks, weaker candidates just scored on the graph or not at all.

## 4753 - Methods for Advanced Mathematics (C3) (Written Examination)

## General Comments

The paper attracted the usual range of responses, with many scripts scoring 60 or more, and a substantial 'tail' of weak candidates scoring less than 20. There was some excellent work seen, with well presented, accurate solutions. In general, it appears that the standard calculus techniques are well understood by candidates, but some of the more peripheral topics in the syllabus, like modulus notation and inverse trigonometric functions, are less secure.

Algebra, as usual, is a stumbling block: for example, basic errors in expanding brackets were seen in questions 5 and 7 (ii). Despite the fact that most candidates for this summer paper will have done substantially more work on trigonometry from studying the C 4 module, the exact values of trigonometric ratios for $60^{\circ}$ and $30^{\circ}$, required for questions 2 and 3 , were sometimes not known.

It would be worth mentioning to candidates for this paper that although graph paper is supplied on demand, it is hard to conceive of a question where it should be used - graphs are almost always better drawn as sketches on their answer paper.

Question 8 was found be the hardest question, so it is difficult to know if weak attempts to this could be ascribed to a lack of time. However, all but a few candidates seemed to have sufficient time to attempt all the questions.

## Comments on Individual Questions

## Section A

Knowledge of the modulus notation was quite variable and centre-dependent. Algebraic errors abounded and $x=1$ was quite often the only solution. $4 \mathrm{x}=2$ rather too frequently led to $\mathrm{x}=2$ ! Also $|x|=1$ or $x= \pm 1$ were common errors. Candidates who mis-interpreted the question as an inequality gained no marks.

2 Nearly all the candidates recognised this question as an integration by parts, with $u=x$ and $\mathrm{d} v / \mathrm{d} x=\sin 2 x$. However, only strong candidates managed to derive the exact result. Quite a few candidates made errors in $v$ such as wrong sign, omitted factor of $1 / 2$, etc. Many candidates failed to substitute exact values for $\sin \pi / 3$ and $\cos \pi / 3$, and verified the given result numerically.

3
This question received a mixed response. Many candidates gained the first three marks without difficulty, but the misconception that $\sin ^{-1} x=1 / \sin x$ was seen quite frequently. In part (ii), many found $y=\pi / 6$ ( $30^{\circ}$ was condoned here), but either gave the answer as cos $y$ or failed to give an exact result.

4 Weaker candidates failed to differentiate $V$ correctly, and a surprising number opted to use the product rule rather than expanding the brackets. In part (ii), $\mathrm{d} V / \mathrm{d} t=0.02$ was an easy mark, and most candidates wrote a successful form of the chain rule and substituted correctly.

5
Part (i) was answered competently by many candidates, though weaker candidates failed to expand $\left(t^{2} \pm 1\right)^{2}$ correctly, or just verified the result for $t=1$. In part (ii), virtually all worked out $c=29$ correctly, and there were quite a few who argued the proof by contradiction convincingly. A common source of error here was to start by taking $t=20$ instead of $2 t=20$.
$6 \quad$ In part (i), most of the sketches were of the correct shape, but many omitted the intercept $M_{0}$ on the vertical axis. Part (ii) was well answered. In part (iii), many candidates failed to substitute $M / M_{0}=1 / 2$, for example starting $M=1 / 2 M_{0} \mathrm{e}^{-k t}$. Using a particular value for $M_{0}$, such as 1 , gained 1 out of 3 marks, even if they took $M_{0}=14$, which was a surprisingly popular choice! Part (iv) was a mark for all and sundry.

## Section B

7 Even the weakest candidates scored marks in this question, and there were many strong solutions
(i) This was well answered; errors included $y=1$.
(ii) The quotient rule was well known and usually correct, and sound candidates achieved these 6 marks without difficulty. However, it was disappointing to see so many candidates making algebraic slips with the brackets in the numerator, and having to 'fudge' the resulting quadratic.
(iii) Well prepared candidates derived the stated transformed integral without difficulty. There was some sloppy notation - missing du's or dx's, limits inconsistent with $x$ or $u$, omitted integral signs, etc. Most of this was condoned, but to achieve full marks, students needed to state that $\mathrm{d} u / \mathrm{d} x=1$ (or $\mathrm{d} u=\mathrm{d} x$ ). The integration was very poorly done, with the majority of candidates failing to integrate the $4 / u$ term as $4 \ln u$.
(iv) This was a more demanding test. The implicit differentiation was mixed, and some left dy/dx as a 'double-decker' fraction, which lost an A mark. Some then substituted $y=7$ instead of $\ln 7$ into the derivative. Some tried logarithmic differentiation, but then differentiated $\ln \left(x^{2}+3\right)$ incorrectly as $1 /\left(x^{2}+3\right)$.

8 Solutions may have suffered from lack of time in some cases. However, this was clearly the most demanding question on the paper, and few candidates scored well.
(i) Some sketch graphs were very hard to decipher, especially when all three graphs were shown super-imposed. In general, the translation in (B) was perhaps better answered than the stretch in (A). We wanted to see the diminishing amplitudes preserved, but allowed some leeway here. Domain errors were common.
(ii) The product rule was quite well answered, and many factored out the $\mathrm{e}^{-x / 5}$ term. However, only good candidates derived $\tan x=5$ successfully - some resorted to verifying numerically. The calculation of $x$ and $y$ should have been straightforward, but many used degrees for $x$ instead of radians. It should be noted that $x=\arctan 5$ was not allowed - we require candidates to evaluate this.
(iii) This was by far the least successful question on the paper. There were many attempts to 'fudge' the result given, mainly because candidates made errors with the $\sin (x+\pi)$ part - say $\sin x+\sin \pi$, for example - and could not see where the negative sign came from. The final 4 marks were rarely achieved, with lots of unconvincing work based on transforming the graph. Candidates needed to show $\int f(u+\pi) d u$ with evidence of transformed limits to gain the marks. The interpretation of the results required candidates to relate this to the areas of successive sections of the curve.

## 4754 - Applications of Advanced Mathematics (C4)

## General Comments

This was the third time this paper had been set and the second time in the Summer session. This proved to be the most difficult paper yet and high total scores were rare. In particular, question 6 proved to be very low scoring, even for good students, and many candidates made no attempt at this question. Question 7 had some confused solutions which might partially have been through lack of time caused by the problems in question 6 . Section A proved to be quite well answered. The Comprehension was generally successful with a good range of marks.

Candidates would be advised to use methods suggested by 'hence' in questions. These methods are usually the easiest. Candidates also need to read questions carefully. For instance, the need for exact answers was often overlooked. In the longer questions, few candidates gave sufficient explanations to support their working. In this way, they risked losing method marks.

## Comments on Individual Questions

## Paper A

## Section A

1) The first part -using the ' $R$ ' method- was well answered. Most errors arose from attempts to quote results rather than working from first principles. Too many used degrees rather than the required radians.
In the second part several candidates tried to find the turning point by differentiation (of both the original expression and that found in the first part) which is more difficult and longer than using knowledge of when trigonometric curves have maxima. The need for exact co-ordinates was often missed.
2) (i) This was well answered by many. Some started badly and lost marks by assuming that $\mathrm{A}=0$ and using that to find B and C . Other errors arose from incorrect clearing of the fractions such as including an extra factor of ( $1+\mathrm{x}$ ) on the right hand side only or writing $(1+x)^{2}$ as $\left(1+x^{2}\right)$. There were also some timeconsuming approaches.
(ii) The first three marks were usually obtained. Few realised that they could use their partial fractions in part (ii) and proceeded to expand

$$
(3+2 x)^{2}(1+x)^{-2}(1-4 x)^{-1} \text {. }
$$

Many made this more difficult by retaining higher powers of $x$ than was necessary in their calculations.
4) (a) Although there were many completely correct solutions here, there were also many candidates who did not appear to realise that an equals sign was required in an equation. Some included a $t$ on the right hand side, $\ldots=t \sqrt{ } x$ being common.
(b) There were some good solutions here. Most attempted to separate the variables but too many had $1 / \sqrt{ } y$ in their integral instead of $\sqrt{ } y$. Those who separated the variables correctly usually integrated correctly. If an arbitrary constant was included in the integration and found immediately, it too, was often correct. Those that rearranged their equation first made errors such as

$$
\begin{aligned}
& y^{3 / 2}=15000 t+c \\
\Rightarrow \quad & y=(15000 t)^{2 / 3}+c^{2 / 3}
\end{aligned}
$$

Very many candidates overlooked the explicit requirement to 'find $y$ in terms of $t$ ' but the majority substituted $t=10$ correctly.

The integration by parts was usually successful. Only a few made the incorrect choices for $u$ and $d v$. There were some sign errors.
(ii) Those who could square $\mathrm{x}^{1 / 2} \mathrm{e}^{-x}$ correctly were usually successful here. There were some candidates who did not spot the connection with part (i) and some failed to substitute the lower limit but this question was usually well answered.

## Section B

6) 

(i) Very few realised that at $\mathrm{E}, \mathrm{y}=0 \Rightarrow 1-\cos \theta=0 \Rightarrow \theta=0$ or $2 \pi$. Many started by attempts to eliminate $\theta$ from the parametric equations or to substitute $x$ into $y$ or to assume the arch was part of an ellipse or of a circle. Candidates often resorted to calculus to find the maximum height instead of realising that $1-\cos \theta$ is a maximum when $\cos \theta=-1$. Some continued to use degrees here instead or radians. For instance, lengths being given as 360a instead of $2 \pi \mathrm{a}$. This part was the most successful in this question although usually only the first two marks were scored. a was often treated as a variable and $\theta$ as a constant. The process of division was usually correct.
(iii) Very few connected the gradient of the line $A B$ with the gradient of the curve at B. The gradient of the tangent at $B=\tan 30^{\circ}=1 / \sqrt{3}$ was not appreciated. This was usually missed out.
Those few trying to verify that $2 \pi / 3$ was a solution of the given equation usually used a calculator rather than an exact form. $0.866=0.866$ was a common response here.
Only a small number of candidates proceeded beyond this point.
(iv) For those that did attempt this part, AF was sometimes found correctly. A very small minority achieved the final result.
7) (i) Well answered and generally correct.
(ii) Most candidates correctly found the vector equation of the line (although often disappointingly missing the $\underline{r}=\ldots$ ). Very few candidates, even the most able, realised how to find the co-ordinates of $D$.
(iii)\&(iv) Solutions of parts (iii) and (iv) were very confused. In many cases mixtures of the same methods were used for both parts. These included

- starting from the vector equations of the planes and eliminating parameters to get the equations and hence the normals
- the use of scalar products (too often without working shown) between the normal vector and (usually only one) vector in the plane
- verification with points
- using the vector cross product to find the normal vector and hence the equation
In some cases long methods were used and these were often presented badly. It may be that some candidates were running out of time at this stage.
(v) This was usually correct when attempted. There were occasional uses of the wrong vectors or the incorrect formula.


## Paper B

## Comprehension

Most candidates achieved respectable marks on this section and there was a good range of marks.
1)

Usually correct but sometimes over-rounded. A few could not deal with the 385 yards.
2) Most substituted $\mathrm{R}=0$ (although some incorrectly substituted a negative value, say -1). Many candidates found the year was 2563.9 but then said 2564 would be the year, failing to see that the answer lay in 2563.
3) Many suggested there was an asymptote at $\mathrm{R}=120$ failing to see the significance of the 120.5 in the equation. There were some pleasing proofs by contradiction, showing that a time of 120 required the logarithm of a negative number.
4) Too many candidates approached this from the graph rather than substituting into the equation. Of those that did consider the equation at $t=0$ and as $t$ tended to infinity there were some good solutions. This question discriminated well between the candidates.
5) Usually done well but some did not realise this was an increasing curve.
6) $t=104$ was not common but the correct method was usually used for the final stage.

## 4755 - Further Concepts for Advanced Mathematics (FP1)

## General Comments

Many strong candidates took this paper and many scored very highly. However, an encouraging number of centres had candidates scoring marks across the full grade range. This seems to indicate that centres are beginning to enter candidates across a wider ability range, taking advantage of the new style AS Further Mathematics and resulting in an increasing entry.

A small proportion of candidates were not well prepared for a paper of this nature.
The overall standard of scripts was very good and many candidates showed a pleasing level of algebraic competence. However, missing brackets, imprecise explanations and poor use of notation were apparent on a significant number of scripts.

Candidates must be sure to label their diagrams clearly. Ambiguous labelling was particularly apparent in question 4.

Some candidates, both weak and strong, seemed to spend an inappropriate amount of time for questions worth few marks. This was particularly the case for 5 (ii).

## Comments on Individual Questions

## 1) Matrices

Surprisingly this was one of the least well answered questions on the paper. Most candidates got part (i) correct but many gave an incorrect matrix for part (ii). A common mistake in part (iii) was to multiply the two matrices in the wrong order.
2) Roots of a cubic equation

Both parts of this question were well answered. A few candidates made sign errors in part (i). Most of those who attempted part (ii) were successful but some did not attempt it.
3) Identity

This question was answered correctly by almost all candidates.
4) Loci of complex numbers on the Argand diagram

While most candidates scored fairly well on this question, a significant few seemed totally unprepared for a question of this type.

In part (i) the commonest mistakes were incorrect or missing shading and the use of an incorrect point for the centre.

The commonest mistake in part (ii) was to fail to exclude the circumference of the inner circle.

Part (iii) produced many mistakes: drawing a whole line rather than just half; drawing it at the wrong angle and using the wrong end point. Several candidates also drew circles or sectors.

Many candidates did not label their diagrams clearly, particularly the boundaries in part (ii).
5) Invariant points

Most candidates got part (i) right but there were also several mistakes on the inverse matrix such as multiplying by the determinant and misplacing the numbers inside the matrix.

Part (ii) proved to be a good discriminator for the stronger candidates, many of whom got it right whilst most candidates either omitted it, or spent considerable time without making any worthwhile progress. A common mistake among some stronger candidates was to post-multiply by $\mathrm{T}^{-1}$ instead of pre-multiplying, or to pre-multiply one side of the equation and post-multiply the other. Several candidates tried to work from the general matrix $\left(\begin{array}{ll}a & c \\ b & d\end{array}\right)$; although this can lead to a correct answer, the working is not simple and candidates who tried this method were invariably unsuccessful. There is a lesson to be learnt from the contrast between the simplicity of using matrix algebra and the difficulty of any other method.
6) Proof by induction

The general standard of the answers to this question was better than in earlier papers. However, some candidates did not convey the logic of the method, for example writing "Let $n=k$ " instead of "Assume the result is true for $n=k$ ". In the central working $3.2^{k}$ and $3.2^{k+1}$ were often changed into $6^{k}$ and $6^{k+1}$ and it was apparent that several candidates were not really sure why $3.2^{k}-3+3.2^{k}$ was equal to $3\left(2^{k+1}-1\right)$. Many failed to explain the steps in their proof adequately and so did not convince that they understood the structure of the proof. Since this question is about proof, such errors incurred a substantial mark penalty.

## 7) Graph

By far the majority of candidates were well prepared for this question. It was unusual, for example, to see the horizontal asymptote stated as $y=0$ instead of $y=1$ in part (i).
(i) The vast majority of candidates answered this correctly.
(ii) A common mistake in part (ii) was to write $\frac{\infty^{2}}{\infty^{2}}$. Many candidates stated that for large negative $x$ the curve approached the asymptote from above, even though their working had shown them otherwise. Many candidates showed no workings to justify their answers and so forfeited at least one mark.
(iii) Most candidates obtained the correct branches for the curve but a significant few drew the left hand branch appearing from the wrong end of the $x=-1$ vertical asymptote. This may have been because candidates mistakenly believed that a graph cannot cross an asymptote, whereas this graph crosses the horizontal asymptote from above, before tending to it from below for large negative values of $x$.

A few candidates lost marks by not labelling the asymptotes on their graphs.
(iv) In part (iv), many candidates made the mistake of giving the answer as $-1>x>2$. It cannot, of course be expressed as a single compound inequality of this type.

Many candidates spent a considerable time on workings for this part of the question when they could have written the answers down from looking at their graph.
8) Complex roots of an equation

This question was very well answered with many candidates obtaining full marks.
(i) In part (i), most candidates knew to substitute the given root $2+\mathrm{j}$ into the equation and to expect an answer of zero. Those who made careless mistakes tended to go back and correct them.
(ii) Very few candidates failed to get part (ii) right.
(iii) Part (iii) was also well answered. About half the candidates answered using the sum of the roots or the product of the roots, and about half found the quadratic factor and then the linear one. Of those using the factor method, some candidates gave $(x-3)$ rather than $(2 x-3)$ as the third factor. It was also not uncommon for candidates to say that $\left((x-2-\mathrm{j}) \times(x-2+\mathrm{j})=x^{2}-4 x+3\right)$ instead of $x^{2}-4 x+5$ leading to a third factor of $(2 x-5)$. Several candidates stated that their third factor was the root.
9) Summation of series

This question was well answered. Many of the stronger candidates obtained full marks on it. However, it was clear that quite a few candidates were in a rush to finish.
(i) A substantial number of candidates made sign errors and then claimed to have arrived at the given answer. A number showed scant regard for the need for brackets and paid the penalty by claiming $3 r^{2}+r$ was equal to $3 r(r+1)$.
(ii) The method of differences was generally well done, though there was much slack use of brackets. A significant number of candidates missed out the factor of $\frac{1}{3}$. Some candidates gave their answer in terms of $r$ instead of $n$. Several candidates did not obey the instruction to use the method of differences and so were awarded no marks for this part.
(iii) Most candidates knew just what to do and came out with the required result. There were a few algebraic errors, especially involving fractions and factorisation. A number of candidates spent considerable time trying to match an incorrect result from part (ii), in some cases having actually achieved the right answer in part (iii). A small number of candidates had difficulty recalling the correct expression for $\sum r$.

## 4756 - Further Methods for Advanced Mathematics (FP2)

## General Comments

The overall standard of work on this paper was generally good. Most candidates presented their work clearly and demonstrated their familiarity with the standard results and techniques. There were some excellent scripts, with about $15 \%$ of the candidates scoring 60 marks or more (out of 72). However, there were also a significant number who appeared to be under prepared and who failed to score marks on some straightforward parts of questions. About $20 \%$ of the candidates scored less than 30 marks.
Candidates did not always read the questions sufficiently carefully, for example the range of values of $\theta$ given in Q.1(a), 'Use the Maclaurin series' in Q.1(b)(ii), 'Integers' in Q.2(b)(iii), and 'Exact' in Q.4(ii) and (iv).
Many candidates would have done better had they seen the connections between different parts of questions, such as Q.1(b)(i) and (ii), Q.2(a)(i) and (ii), Q.2(b)(ii) and (iii), Q.4(i) and (ii), and Q.4(iii) and (iv). Parts of questions labelled (i), (ii), ... are always intended to be connected in some way.
In Section A, Q. 1 was the best answered question, with an average mark of about 12 (out of 18), and Q. 3 was the worst answered, with an average mark of about 10. In Section B, Q. 4 was chosen by almost all the candidates, and the average mark was about 11.

## Comments on Individual Questions

## 1) Polar curve and Maclaurin series

In part(a)(i) the sketch of the curve was usually correct, although some candidates included an extra loop (corresponding to values of $\theta$ outside the given range). In part (a)(ii) most candidates used $\int \frac{1}{2} r^{2} \mathrm{~d} \theta$ with the correct limits, and the subsequent evaluation was quite frequently carried out accurately. Most of the mistakes made were careless slips such as sign errors, or the factor $a^{2}$ being lost or ending up as a. However, a substantial number were unable to integrate $\cos ^{2} \theta$.
In part (b), the methods for obtaining a Maclaurin series, and using it to evaluate an integral approximately, were very well known. However, finding the second derivative of $\tan \left(\frac{1}{4} \pi+x\right)$ caused a surprising amount of difficulty. The correct answer appeared in a great variety of forms, from the expected $2 \sec ^{2}\left(\frac{1}{4} \pi+x\right) \tan \left(\frac{1}{4} \pi+x\right)$ to
$4 \sin \left(\frac{1}{2} \pi+2 x\right) /\left(1+\cos \left(\frac{1}{2} \pi+2 x\right)\right)^{2}$ and even more complicated expressions, and very many candidates failed to obtain a correct expression. Some candidates attempted integration by parts instead of applying their Maclaurin series to the final integral.
2) Complex numbers

In part (a)(i) most candidates were able to write down the required expressions, or find them after a few lines of working, but some had little idea of what to do, usually giving up after some manipulation of fractions.
In part (a)(ii), common errors included algebraic slips in the expansion of $(z-1 / z)^{4}(z+1 / z)^{2}$, replacing $z^{6}+1 / z^{6}$ with $\cos 6 \theta$ instead of $2 \cos 6 \theta$, and, especially, omission of the factor 64. A fair number expressed $(z-1 / z)^{4}$ and $(z+1 / z)^{2}$ separately in terms of multiple angles, then multiplied the results, but only a tiny fraction of these could deal successfully with the resulting $\cos 4 \theta \cos 2 \theta$ term.
In part (b)(i) almost all candidates found the modulus and argument correctly.
Part (b)(ii) was very often answered correctly and efficiently. The modulus ( $\sqrt{2}$ ) of the fifth roots was given in a variety of correct forms, including $32^{0.1}$ and $\sqrt[5]{4 \sqrt{2}}$. The arguments given were sometimes outside the required range, and a fairly common error was to give
the arguments as $\frac{1}{4} \pi+\frac{2}{5} k \pi$ instead of $\frac{1}{20} \pi+\frac{2}{5} k \pi$. The great majority of candidates knew that the roots should appear as the vertices of a regular pentagon on the Argand diagram. Part (b)(iii) was often omitted. Many candidates did realise that they needed to select one of the fifth roots found earlier, but very often an inappropriate root was chosen, giving values of $p$ and $q$ which were clearly not integers. Some ignored the connection with the previous part and expanded $(p+q \mathrm{j})^{5}$; very occasionally the correct solution $p=-1$, $q=-1$ was spotted from the resulting equations.

## 3) Matrices

In part (i) almost all candidates knew a method for finding the inverse matrix, and the process was very often completed accurately. By far the most common approach was to use cofactors; common errors included arithmetic slips, forgetting to transpose the matrix of cofactors, forgetting to change the sign of some minors to obtain the cofactors, and multiplying cofactors by their corresponding elements. A few candidates used elementary row operations.
In part (ii), those who put $k=7$ into the inverse matrix and then used it to find the solution were usually successful. Some candidates started again and obtained a correct inverse of the matrix with $k=7$ (possibly by calculator), even if their part (i) had been incorrect. Very many candidates worked from the three equations, eliminating variables. Careless slips were very common with this method, even when a systematic approach, starting by eliminating one variable in two different ways, was used. The work often occupied several pages, typically eliminating $x$, then $y$, then $z$, but never reaching helpful results.
Part (iii) was very often omitted. There were some very efficient solutions, in which candidates usually eliminated one variable in two different ways, compared their equations to find $p$, and then found $x, y$ and $z$ in terms of a parameter. A variety of other methods were used; the correct value of $p$ was found fairly frequently, but the correct general solution was much rarer.
4) Hyperbolic functions

The proof in part (i) was usually fully correct. Common errors included confusing the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, and failing to expand $\left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)^{2}$ correctly.
In part (ii) most candidates used the result in part (i) to obtain a quadratic in $\sinh x$, leading frequently to a fully correct solution, although some thought that the solution $\sinh x=-1$ should be rejected. Some candidates wrote the equation in exponential form, obtaining a quartic in $\mathrm{e}^{x}$; usually no further progress was made, but a few of these spotted the factor $\left(\mathrm{e}^{x}-2\right)$.
In part (iii) most candidates used the result in part (i) to obtain a form which could be integrated, although some preferred to write it in terms of exponentials, and the integration was usually performed correctly. Very many candidates did not earn the marks for the evaluation; because the answer is given, just stating $\sinh (2 \ln 3)=40 / 9$ is not sufficient. In part (iv), candidates who used the substitution $x=3 \cosh u$ very often answered this correctly and efficiently. A factor of 3 often went astray; and, especially if the upper limit was left as $\operatorname{arcosh}(5 / 3)$ instead of $\ln 3$, the close connection with part (iii) was not always noticed.
5) Investigation of curves

This question was attempted by less than $2 \%$ of the candidates, and only three of these scored more than half marks.

## 4757 - Further Applications of Advanced Mathematics (FP3)

## General Comments

There were some excellent scripts, with about $15 \%$ of candidates scoring more than 60 marks (out of 72), and a wide range of performance; about $20 \%$ of the candidates scored less than 30 marks. Questions 1 and 2 were the most popular, and questions 3 and 5 were the least popular. Some candidates indicated that they were running out of time, and very few presented answers to more than the required three questions. The average marks for the questions were about 14 (out of 24 ) for questions 1, 2 and 3 ; about 16 for question 4 and about 18 for question 5.

## Comments on Individual Questions

## 1) Vectors

The techniques required in this question were generally well known, although weaker candidates often had difficulty selecting which vectors to use in the formulae, for example using one of the position vectors when a displacement vector such as $\overrightarrow{\mathrm{CD}}$ was required. Parts (i) and (ii)( $A$ ) were almost always answered correctly.
Part (ii)(B) was often answered correctly, although many candidates tried to use the formula for the distance between skew lines.
Part (ii)(C) was answered well.
In part (iii) some candidates continued to take $k=1$, but the method was very well understood. Many had problems simplifying the answer, especially when the factor ( $k-1$ ) had not been taken out of the direction vector for the common perpendicular.
Part (iv) was well answered. The value of $k$ could be deduced from part (iii), but this was not always easy when the answer to (iii) was wrong or unsimplified. Even so, it was not difficult to start afresh and find the point of intersection, and $k$, from the three component equations, and many candidates did this successfully.
2) Multi-variable calculus

Part (i) was almost always answered correctly. Many candidates gave the equation of the normal line as their final answer, instead of the normal vector, but this was not penalised. Part (ii) was usually answered correctly, and part (iii) was also well answered, although $\delta z$ was often taken to be $h$ instead of $-h$.
In part (iv) the condition for the normal line to be parallel to the $z$-axis was usually stated correctly, although some thought that it was $\frac{\partial \mathrm{g}}{\partial z}=0$. Quite a number asserted that the simultaneous equations $2 x-4 y=0$ and $-4 x+6 y=0$ were inconsistent, when they are clearly satisfied by $x=y=0$. Very many candidates completed this part correctly.
There were few correct solutions to part (v). Most candidates wrote $2 x-4 y=5$, $-4 x+6 y=-6,-4 z=2$ instead of $2 x-4 y=5 \lambda,-4 x+6 y=-6 \lambda,-4 z=2 \lambda$, and these could score only 2 out of the 8 marks.
3) Differential geometry

Parts (i), (ii) and (iii) were very often answered correctly.
In part (iv) many candidates differentiated correctly, but further progress depended on dividing by $\left(1+t^{2}\right)$ to obtain $x=8 t^{3}$. Very few obtained the correct answer.
In part ( v ) candidates generally knew that they should substitute $x=64$ into the answer to part (iv). A few tried to find the centre of curvature at a general point, seldom successfully.

## 4) Groups

In part (i) the completion of the table was generally well done. It was often completely correct, and the most common error was to mix up $\mathbf{J}$ and $-\mathbf{J}$.
Parts (ii) and (iii) were well answered.
In part (iv) about half the candidates gave a satisfactory explanation. They were expected to say that any set of four elements must include (at least two) elements of order 4, and that a subgroup of order 4 containing an element of order 4 must be cyclic.
In part (v) the four proper subgroups were usually given, but \{l\} was very often given as well; this resulted in the loss of one mark.
In part (vi) candidates usually referred to elements of order 2, but did not always clearly identify at least two such elements in the symmetry group.
5) Markov chains

Most of the candidates who chose this question showed a very good understanding of the topic and were able to use their calculators to manipulate the matrices efficiently. No part of the question caused particular difficulty, and about one third of the attempts scored full marks.
In part (iii) the equilibrium probabilities could be found either by solving simultaneous equations or by considering a high power of the transition matrix; the first of these approaches was slightly more commonly used.

## OCR (MEI) Mathematics GCE

## Rules for crossed out and/or replaced work.

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.

## Legacy units

Centres are reminded that under no circumstances can 'legacy' units be used in the 'new' specification from January 2007 onwards. The only units that are acceptable for this specification are units 4751-4777.

## 4758 - Differential Equations (Written Examination)

## General Comments

The standard of work was generally good. Questions 1 and 4 were attempted by almost all of the candidates. Most then chose question 2 rather than question 3. Candidates often produced accurate work; however errors in integration were common.

## Comments on Individual Questions

1 (i) This was often completely correct.
(ii) Many correct solutions were seen, but some candidates could not state the correct complementary function associated with a repeated root of the auxiliary equation. When considering the behaviour as $t$ tends to infinity, it was
recognised that candidates may not know the behaviour of the $t \mathrm{e}^{-3 t}$ term, and so credit was given on the basis of how they dealt with the other term.
(iii) When considering the complementary function in the general case, many candidates omitted to consider complex roots of the auxiliary equation. When considering the real roots, candidates often did not explain why both roots must be negative.

2 (i) This was often done well, except for slips in the integration. However a minority of students ignored the suggestion to separate variables and when using the integrating factor method found the resulting integral difficult.
(ii) The integrating factor method was usually applied and understood, but errors were common, in particular with the integration by parts and either omitting the constant or failing to divide it by the integrating factor when expressing $y$ in terms of $x$.
(iii) The Euler calculation was often done well but some worked in degrees and others produced unrecognisable figures with no indication of method.

3 (i) Most candidates were able to use Newton's second law to obtain the differential equation, but explanations of the signs were often vague.
(ii) Candidates tried various methods to solve the differential equation, and it was common for separation of variables to be ignored. Even those who used the correct method often made errors in integration.
(iii) This differential equation also caused problems for candidates. Many did not identify the correct method, and even among those who did, correct solutions were extremely rare.
(iv) Many were able to deduce the terminal velocity. The standard of graph sketching was very variable. Some did not consider the entire motion, some ignored the initial conditions, but some produced good sketches. Even some of those who had been unable to solve the differential equations gained full credit here by deducing the key features of the graph from the given information.

4 (i) The elimination of $y$ was often done well, although a few differentiated the first equation with respect to $x$ rather than $t$.
(ii) The solution was often done well, although minor slips were common.
(iii) Many candidates correctly used the first equation. Some tried to set up and solve a differential equation for $y$; such attempts never tried to relate the arbitrary constants in the two solutions.
(iv) The limiting expressions were usually well done, but many candidates did not make clear conclusions in either case.

## MECHANICS

## OCR (MEI) Mathematics GCE

## Rules for crossed out and/or replaced work.

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

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If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.

## Legacy units

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## General Comments

Most candidates found something that they could do and quite a few could do most of the paper but there were several widespread difficulties. First there was a part question that very few could do: Q 7 (iii) proved inaccessible to most candidates, partly because the response we were looking for was too sophisticated and partly because many of the candidates were not able to express their (possibly correct) ideas clearly. Secondly, many candidates did not realise that Q 2 (iii) was a question about vectors. Thirdly, the techniques required for Q 4 were not widely known. Account was taken of these problems when setting grade thresholds at the Award.

There were also some quite marked differences to the responses of candidates in recent sessions.
There were rather more very low scores.
Quite a few candidates showed low levels of ability when dealing with arithmetical and algebraic
expressions. For example: $21 \cos \theta=28$ gives $\theta=\frac{28}{21 \cos }\left(\right.$ which is then evaluated as $\left.\frac{28}{\cos 21}\right)$ :
$\sqrt{(-6)^{2}+13^{2}}$ written and evaluated as $\sqrt{-6^{2}+13^{2}}$.
There seemed to be more scripts that were poorly presented (for instance: parts of a question unlabelled and mixed in together: $3^{\circ}$ written badly in the working and then misread as 30 ).
There were more examples of attempts at the projectile problem based on the assumption that the position of a projectile could be determined by assuming that the motion was in the straight line defined by the initial velocity.
There were more examples of constant acceleration problems worked (at least in part) as if they were zero acceleration problems by using distance $=$ speed $\times$ time .

The ability of candidates to answer parts of the questions sometimes seemed to depend more on the centre than on the ability of the candidates, as measured by responses to the other questions.

There were, of course, many scripts that were beautifully presented with clear, logical working and accuracy shown throughout and which demonstrated a good understanding of the principles and techniques used in this component.

## Comments on Individual Questions

## Section A

## 1 Kinematics of a particle moving in a vertical line

This rather standard question was done quite well by many candidates except that there was widespread confusion about when to use the time of flight and when to use half of this. Many candidates used the same time for the half and the full motion. There seemed to be fewer sign errors than seen in similar problems in recent sessions.

2 Magnitude of a vector, the difference of two vectors, the direction of a vector as a bearing and some simple dynamics and kinematics using vectors
(i) Almost every candidate found the magnitude of the vector correctly.
(ii) Most candidates found the difference of the two vectors correctly. Most then correctly used a trigonometric ratio but could not generate the required bearing from their angle. A clear diagram would have helped most of those that got it wrong.
(iii) Few candidates realised that this was a vector question and so gave only scalar answers. Typically, whole centres set about this the right way with some of the candidates from such a centre scoring marks on this part while struggling elsewhere.

## Newton's second law applied to a train, tension in a coupling and movement on a slope

(i) Almost all of the candidates realised that they should apply Newton's second law. A surprising number applied it only to the engine. A number of candidates think that the law is $F-m g=m a$, presumably from applying it to vertical motion.
(ii) Most candidates produced an answer consistent with their answer to part (i). Quite a few wasted time by starting the problem again from scratch instead of arguing that $4000-R=3500$.
(iii) Almost everyone who knew Newton's second law correctly managed to answer this question with relatively few wrongly applying the driving force to the truck - a considerable improvement on the answering of similar problems in recent sessions.
(iv) Many candidates failed to make much progress with this part. The most common reason was the inability of candidates to write down an expression for the component of the weight down the slope; many omitted $g$ and others used cosine instead of sine. Quite a few candidates misread their own (untidy) writing and turned $3^{\circ}$ into 30 . Those who realised that the extra force had the same magnitude as the component of weight down the slope were usually successful although some wrongly added in the resistance term. Those who applied Newton's second law again did not usually fare so well. There were many sign errors and missing terms and many candidates took the resistance to be $(500+150) \mathrm{N}$. I am not aware of any candidate who applied Newton's second law in any direction but that of the slope who obtained the correct answer.

## Motion of a particle in a plane in vector form and the Cartesian equation of its path

It seemed that many candidates were not familiar with much of the content examined in this question.
(i) Only a few candidates tackled this with confidence and skill. Many thought that the $\mathbf{i}$ component should be zero; many came to this conclusion by the argument that they wanted the $\mathbf{j}$ component zero so they substituted $\mathbf{j}=\mathbf{0}$ giving $\mathbf{r}=3 t i$. Those who worked with $18 t^{2}-1=0$ often either found only one root or failed to explain why the negative root could be ignored. Some candidates showed a poor grasp of the nature of vectors by trying to solve $3 t+18 t^{2}-1=0$.

Those who tried to show that the path crossed the $x$ axis by substituting values for $t$, usually did not make it clear that they were looking for a sign change in the $\mathbf{j}$ component.
(ii) Most candidates correctly differentiated but many lost the $\mathbf{i}$ or the $\mathbf{j}$ or both in the process. Unlike similar questions in recent sessions, the candidates were told to find a velocity before asking them a question about the direction of motion and so most were trying to do the right thing. Many clearly stated that for motion in the $\mathbf{j}$ direction the $\mathbf{i}$ component must be zero but this is not possible as it is constant at 3: others were not so clear and the answers were interpreted generously.
(iii) The minority of the candidates who knew what to do, mostly did it well. The usual errors were to omit the brackets and so substitute $\frac{x^{2}}{3}$ instead of $\frac{x^{2}}{9}$ or fail to simplify the term from $\frac{18}{9}$ to 2.

5 A projectile question that works from maximum height and speed of projection to angle of projection and horizontal range

Many candidates scored well on this question and there seemed to be fewer errors with signs than in recent sessions. However, many candidates used a sequence of formulae when one would have done and used sledgehammer methods on simple equations, for instance using the quadratic formula to solve a quadratic equation with no constant term; one can see from the scripts how poor technique of this type takes up a lot of the time allocation and how the extra working has increased the chance of errors.
(i) Many candidates obtained the given result with a satisfactory method. Candidates who take $u=0$ and $g=9.8$ should explain why this gives $v$ that is the initial vertical component of velocity.
(ii) Most candidates knew what to do. Common mistakes were to confuse sine and cosine and, rather surprisingly, think that the angle of projection was an angle with the upward vertical.
(iii) A lot of candidates knew what to do and many of them did it well. A common mistake was to find only half the time of flight.

## Motion in a straight line involving a $v$ - $\boldsymbol{t}$ graph, constant acceleration formulae, calculus and interpretation of a model

Many candidates answered most of this question very well. The most common errors from strong candidates were found in their interpretation in part (vii) and omission of any working showing consideration of an arbitrary constant in part (vi). Many weaker candidates tried to use the constant acceleration formulae inappropriately and some confused differentiation with integration.
(i) Most candidates answered this part correctly but some included an extra $2 \times 12$ term.
(ii) The only common error here was to give the time it would have taken to travel the distance instead of how much less time.
(iii) Many obtained the correct answer; a positive answer was quite common.
(iv) Most of the candidates used the uvast results and got the right answer; those who tried to use a velocity - time graph approach often confused themselves. A quite large number of candidates used speed $=\frac{\text { distance }}{\text { time }}$ as part of their calculation.
(v) This was generally done well but quite a few candidates evaluated $v(1)$ and some integrated.
(vi) Most candidates knew they should integrate and many of them did so accurately. However, many failed to show any arbitrary constant or failed to show it was zero, some leaving $+C$ in the expression and then correctly obtaining the 32 with a separate definite integration from 0 to 8 .
(vii) Very many candidates thought that there was zero motion because the $v-t$ graph above showed this and supported their argument by showing $v(2)=v(4)=0$. Some came to the same conclusion without appealing to the graph above. However, many candidates knew exactly what was required of them and worked on the right lines. Relatively few both supported their arguments by calculations (either showing that the displacement was negative or the velocity was negative between the given times) and also finished by saying that the car was reversing (or similar words).

## The static equilibrium of connected bodies

For many candidates this proved the hardest question on the paper but others seemed to do well at much of it without too much difficulty. In parts (i) and (ii), many candidates found the angle and then worked out its sine and cosine, often introducing rounding errors.
(i) Many candidates did not realise that they should resolve vertically. Those who used force triangles were especially prone to resolving the 147.
(ii) The lack of clarity in some of the working suggested over reliance on the given answer.
(iii) This was found to be quite the most difficult question to answer correctly on the paper. Very few candidates gave a clear reason and fewer a complete reason. A common statement seen was that 'the forces were still in equilibrium' without any reference to attributes of the system that had not changed.
(iv) The diagrams were often sound but many had wrong labelling (e. g. both strings marked with $T$ ) or (some) arrows missing. Many candidates gave up at this point. However, many continued and spotted how the tension must be related to the other forces acting on the system; others correctly resolved horizontally and vertically and then solved. Others found just one equation and substituted the given value of $T$, deducing the angle but not checking in a second equation.
(v) Well done by many of the candidates, indeed by more than expected. However, quite a few made no progress or simply found the tension in the string (quite often thinking that the 216 had to be resolved). Any equations found were often inaccurate because of sign errors, resolving the tension or using a mass instead of a weight component.

## General Comments

Many candidates could make some progress with at least some part of every question and gain some credit for their work. There were many candidates who had more difficulty with questions 2 and/or 3 than with questions 1 or 4 . The standard of presentation of most scripts was pleasing but, as has happened in other sessions, in some cases diagrams were too poor to be useful. Many candidates do not appreciate the value of a diagram in finding a solution or in conveying to an examiner information relevant to the answer. Labelling of forces should be clear and unambiguous and the diagram large enough for the information to be clearly shown. Candidates who state in a solution which process or principle is being employed tend to be more successful than those who omit this information - stating the principle being employed seems to help the candidate to use it properly.
The standard of algebraic manipulation was, for some candidates, so low as to make it hard for the candidate to demonstrate knowledge of mechanics; this was seen particularly in the attempts to solve simultaneous equations or to rearrange formulae. Efforts were not helped by the use of some clumsy notation with regard to trigonometric functions. e.g. it was common to see expressions such as $\cos (\arcsin 12 / 13)$ with no attempt being made to simplify this.

## Comments on Individual Questions

## 1 Impulse and Momentum

Many candidates gained good marks on this question. The main cause of error was in failure to indicate clearly the direction of the vector answer.
(a) (i) (A) This part posed few problems for the majority of candidates.
(B) Many completely correct solutions were seen to this part. Errors were mainly arithmetic.
(ii) (A) It was pleasing to see many completely correct solutions to this part. Candidates who did not draw a fully labelled diagram were less successful than those who did. Without a diagram, the sign convention was not always as clear as it had to be and errors occurred, usually in the application of Newton's experimental law.
(B) Only the most able (identified by their success on the paper overall) gained full credit for this part. Many candidates failed to indicate direction and a few thought that impulse was equal to change in kinetic energy.
(b) A large number of candidates did well on this part of the question. Most appreciated that there would be no change in speed parallel to the plane and could hence calculate the speed of the rebound. Almost as many could find the coefficient of restitution either by analysing the motion perpendicular to the plane and applying Newton's experimental law or by using $\tan \beta=\mathrm{e} \tan \alpha$.

## Moments and Resolving

Unfortunately many candidates did not apply Newton's third law properly, hence they could not gain the credit for the diagrams required in part (i) and part (iii). However, the majority of candidates understood that moments were required along with some resolution of forces and could gain at least some credit in the other parts of the question.
(i) Many candidates could correctly calculate the force exerted on AB by the hinge at B and could go on to use this to attempt to evaluate the force at C . Of those candidates who failed to obtain the correct answer those who tackled the problem by looking at rods AB and BC separately tended to gain more credit than those who treated AC initially as a single rigid rod.
(ii) The majority of candidates encountered little difficulty with this part.
(iii) Many candidates applied Newton's third law correctly in the horizontal direction giving $U$ in the opposite direction to the diagram for rod AB but failed to do so in the vertical direction and drew a diagram in which $V$ acted vertically upwards as in the diagram for $\operatorname{rod} \mathrm{AB}$.
(iv) A large number of candidates made a good attempt at setting up a second equation in $U$ and $V$ and then went on to try to find a solution to the simultaneous equations. The majority of these then correctly deduced the size of the frictional force.
(i) The amount of explanation required to establish the given result was not appreciated by
the majority of candidates. It was common to see $\mu=5 / 12$ hence $\mu \geq 5 / 12$ without any supporting reasoning. Persistent use of the notation $\sin (\arccos 12 / 13)$ without showing or stating that this was $5 / 13$ meant that many candidates presented an incomplete argument in support of their answers. Very few candidates mentioned the maximum value of the frictional force either implicitly or explicitly and some used rounded decimal fractions throughout and then claimed this was $5 / 12$.
(ii) Many good attempts were seen to this part of the question. Errors usually involved one
of the terms in the work-energy equation. Those candidates who used Newton's second law and the constant acceleration formulae were not usually as successful as those using the work- energy principle.
(iii) Again, some attempts worthy of significant credit were seen for this part, with errors of a similar type to those in part (ii). Candidates who made their method clear usually gained more credit than those who attempted to write down single terms independently and then use these in an equation. Sign errors were common.

## 4 <br> 4

## Work and Energy

It was very satisfying to see that the majority of candidates tried to use work-energy methods throughout this question and did so with some success. Correct solutions were not obtained in some cases because of the absence of a diagram or the omission of an indication about the sign convention adopted.
Almost all of the candidates appreciated the need to use $P=F v$ and very many of them obtained full credit for that part.别

## Centres of Mass

This question was attempted well by almost all candidates with a large number of candidates gaining significant credit. Some excellent solutions were seen.
(i) Most candidates had little or no difficulty with this part.
(ii) Few encountered problems with this part.
(iii) The majority of candidates could obtain some credit for this part with errors being mainly arithmetic. A few candidates attempted to take moments about the edge through O rather than the edge through D but forgot to include the term containing the reaction at the edge through D.
(iv) Many of the diagrams offered here were poor with forces either omitted or unlabelled. A large number of candidates did not appreciate that the reaction would act vertically upwards at some point on the edge through $\mathrm{O} z$ but showed it acting along the edge through A. Others omitted it altogether.
(v) Errors in this part tended to be only arithmetic and it was pleasing to see that many candidates could produce a coherent argument to support their conclusion.

## General Comments

The standard of work on this paper was very good indeed, with about half the candidates scoring 60 marks or more (out of 72), and only about $10 \%$ of the candidates scoring less than 30 marks. The only topics which caused widespread difficulty were radial and transverse components of circular motion in Q.2(b) and some aspects of simple harmonic motion in Q.3.

## Comments on Individual Questions

## 1) Dimensional Analysis and Elastic Energy

The average mark on this question was about 15 (out of 18).
The dimensional work in part (a) was well understood. The most common error was getting the dimensions of power wrong; this was 'followed through' and most of the following marks were usually earned.
The application of elastic energy in part (b) was also well done, the main errors being slips in finding the extension of the string in the extreme position, and forgetting about the initial elastic energy (despite this having been found in part (ii)). Some candidates treated the stiffness as if it were the modulus of elasticity.
2) Circular Motion

This was the worst answered question, with an average mark of about 11.
For the conical pendulum in part (a), many candidates tried to find the tension by resolving along the string $\left(T=m g \cos 55^{\circ}\right)$ instead of vertically $\left(T \cos 55^{\circ}=m g\right)$, and the radius was often taken to be 2.8 instead of $2.8 \sin 55^{\circ}$.
In part (b)(i) the radial component $F_{2}$ was usually given correctly, but only about half the candidates gave the transverse component $F_{1}=m r \alpha$ correctly; the mass or the radius was very often omitted. Some thought that $F_{1}$ was vertical and wrote $F_{1}=m g$. Correct solutions to part (b)(ii) were rare; the majority wrote $F_{2}=\mu \mathrm{mg}$ instead of
$\sqrt{F_{1}{ }^{2}+{F_{2}}^{2}}=\mu \mathrm{mg}$. The final part (b)(iii) was well understood.
3) Elasticity and Simple Harmonic Motion

The average mark on this question was about 14.
Parts (i), (ii) and (iv) were answered correctly by the great majority of candidates.
In part (iii), most candidates were able to obtain the correct equation of motion, although sign errors were quite common, as was omission of the weight.
In part (v), most candidates knew that they should apply $v^{2}=\omega^{2}\left(A^{2}-x^{2}\right)$, but not all could find the appropriate values for $A$ and $x$.
In part (vi), most candidates used $x=A \sin \omega t$, rather than the more efficient $x=A \cos \omega t$, and the correct answer was often obtained. The main difficulties lay in finding appropriate values for $x$.
4) Centres of Mass

This was the best answered question, with an average mark of about 15.
In part (i) the centre of mass of the solid of revolution was found confidently and correctly by almost every candidate.
In part (ii), the principles were well understood, and most candidates managed to obtain the given answer, sometimes after one or two false starts.
In part (iii)(A), most candidates knew what to do and did it correctly; a fairly common error was to take the length of the object to be 4 instead of 3 . Most candidates could also cope with the refinement in part (iii)(B); the majority took moments again, but some did realise that they could just add 3 N to each of the previous values.

## General Comments

The standard of work varied widely, but most candidates were able to demonstrate some understanding of the principles involved in this unit.

## Comments on Individual Questions

1 Variable mass
(i) Most candidates were able to show that the rate of change of $r$ was constant, but sometimes made errors with the role of density in their equations. Some candidates made very heavy weather of finding the mass, setting up and solving a differential equation for mass, rather than simply using the formula for volume of a sphere.
(ii) Although some candidates were able to do this part efficiently, many candidates did not seem to realise that they could integrate the Newton's second law equation directly to get $m v$, but instead expanded $\frac{\mathrm{d}}{\mathrm{d} t}(m v)$ and then set up a differential equation which they were not always able to solve correctly. Some candidates confused their symbols for velocity and volume.

## 2 Stability

(i) This was usually done correctly. Candidates who used diagrams were generally able to make their method clearer than those who relied on words.
(ii) There were many good solutions to this part. Although some errors occurred in differentiation, most errors occurred when solving the resulting equation, with many candidates missing the solution $\frac{1}{3} \pi$ and some introducing solutions representing physically impossible situations. Some candidates mixed up the conditions for stability. Some candidates did not show sufficient working to indicate how they formed their conclusions.

## 3 Power and variable acceleration

(i) This was often done well, but slips in integration were common. Some candidates, after getting their expression for $v$, did not clearly show that the aeroplane took off successfully.
(ii) This was also often done well, but some omitted the constant of integration. The comments about validity were usually good.
(iii) Good solutions were not common. Many candidates wrongly assumed constant power meant constant acceleration. Many candidates did not use their answer to part (i) to find $x$ after 11 seconds, but either took the longer method of integrating the solution to part (ii) or assumed $x$ to be zero.
Rotation
This question was found hard; many candidates made little progress beyond part (i).
(i) Most candidates were able to derive the moment of inertia and centre of mass correctly.
(ii) Many did not use energy here and attempted to use Newton's second law but were unable to solve the resulting equation. Of those who did use energy, many made errors in the potential energy term, particularly by not considering the position of the centre of mass.
(iii) There were few good solutions. Most candidates did not use moment of impulse, but attempted to equate impulse to change in angular momentum. Many candidates spent time calculating the angle, whereas the question did specify that the flagpole stops when it is horizontal.
(iv) It is regretted that this part required a technique not explicitly in the specification. Full account was taken of this in the setting of grade boundaries.

## STATISTICS

## OCR (MEI) Mathematics GCE

## Rules for crossed out and/or replaced work.

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.

## Legacy units

Centres are reminded that under no circumstances can 'legacy' units be used in the 'new' specification from January 2007 onwards. The only units that are acceptable for this specification are units 4751-4777.

## 4766 - Statistics 1

## General Comments

The paper attracted a wide range of responses, from those who were able to produce good answers to almost all questions to those who appeared to be unfamiliar with much of the basic content of the Statistics 1 material. On the whole, candidates seemed to have sufficient time to attempt all questions. Answers were usually well presented with methods and working clear. However, many candidates do not appreciate the implications of using rounded answers in subsequent calculations; nor do they seem to have a good understanding of the use of significant figures.

Most candidates gave good answers to and were able to earn substantial marks from questions $1,3,5,6$ and 7 i. Question 2 was not well answered; candidates were evidently unclear about how to manipulate probabilities in a Venn diagram and many scored at best 2 out of 8 marks. The performance on question 4 was also variable, and the latter parts of Question 7 were not well answered. As mentioned in the January 2006 report, candidates must define $p$ in words in the context of any question on hypothesis testing. Many candidates are still not meeting this requirement. In addition in hypothesis testing on the binomial distribution the use of point probabilities rather than tail probabilities was seen in many scripts. This essentially renders any solution invalid and centres are encouraged to emphasise this point to candidates.

A surprising observation was the number of occasions in which a 'probability' of greater than 1 or even a negative 'probability' was allowed to stand unchallenged as a final answer. This was particularly in evidence in question 6 (iv) and (v), with candidates attempting to use these values in their explanations for part (vi).

## Comments on Individual Questions

## Section A

1) Attempts at a quiz; line graph, skewness, mean and mean squared deviation, calculation of new mean.
(i) The line graph was usually drawn clearly and most candidates scored 2 marks. The few who did not, usually lost marks for omitting labels or adding a curve/polygon to the tops of the lines.
(ii) A number of candidates wrongly (or randomly) believed the data were positively skewed.
(iii) The mean was usually correct. The most common error was the use of divisor $\mathrm{n}=$ 7 instead of 25 . However only about half of the candidature found the mean squared deviation completely correctly. Many square rooted and/or divided by 24 instead of 25 . In addition the usual errors were seen such as the confusion of $\sum x^{2}$ with $\left(\sum x\right)^{2}$. Those who attempted a solution using $\sum(x-\bar{x})^{2} f$ often omitted the frequencies $f$. As indicated in the January report candidates are strongly recommended in this type of question to first calculate the sum of squares $S_{x x}$ and it is pleasing to note that many candidates had been taught to use this approach. A perfectly acceptable alternative method, which was rarely seen, was the use of calculator statistical functions.
(iv) Whilst some candidates coped easily, this part proved to be beyond a significant proportion of candidates, who did not even attempt it. Many used the right method but stopped at 32 , stating 'he needs to average 32 over the next 6 days'. Some found $32 / 5$ rather than $32 / 6$. A common wrong method was to say ( 4.92 $+\bar{x}) / 2=5, \Rightarrow \bar{x}=5.08$. Many believed that the final answer must be a whole number and rounded to 5 or 6 , for which they were penalised. Some used trial and improvement but only tried whole numbers - again getting 5 or 6.
2) Football scoring; conditional probability, Venn diagram, independence, decision making with conditional probabilities.
(i)

Most candidates were able to calculate $\mathrm{P}(A \cap B)=\frac{7}{10} \times \frac{3}{7}=\frac{3}{10}$, but two alternatives were fairly common: $\frac{3}{7} \div \frac{7}{10}=\frac{30}{49}$ and stating that $P(B)=\frac{3}{7} \times \frac{7}{10}$ $=0.3$, followed by $\mathrm{P}(A \cap B)=\mathrm{P}(A) \times \mathrm{P}(B)=0.21$.
(ii) The Venn diagram was rarely correct. Occasionally, 0.4 and/or 0.2 were positioned in the correct space, but completely correct diagrams were rare. 0.7 was often placed where 0.4 should have been and 0.5 was placed where 0.2 should have been. To some extent this may have been a result of candidates having become confused in part (i), but it does suggest that centres should place much more emphasis on this topic.
(iii) Some candidates knew the rule for independence but were reluctant to back it up with numerical values as evidence. Simply quoting the independence rule did not secure any credit. Many candidates used completely erroneous values when trying to determine whether the events were independent.
(iv) Again, some candidates knew that $\mathrm{P}(B \mid A)>P(B)$ was required but could not substantiate it with numerical values. Candidates are strongly advised to give numerical evidence in questions of this type and not to rely on qualitative arguments or stories.

Biased four faced dice modelled a discrete random variable; justifying the value of $k$; calculation of $E(X)$ and $\operatorname{Var}(X)$.
This was perhaps the best answered question on the paper. Candidates were comfortable with the underlying concepts and scored highly.
(i) The majority of candidates established the value of $k$ convincingly but some candidates who substituted the given value of $k$ then failed to add up the resulting figures to justify a sum of unity.
(ii) The calculation of $\mathrm{E}(X)$ and $\operatorname{Var}(X)$ was very well done with only the occasional case where the candidate subtracted $\mu$ rather than $\mu^{2}$ or alternatively omitted the subtraction altogether.
4) Three course meal; various choices made by Peter and Esther.
(i) Many arrived at the correct answer of 60 here, often as $4 \times 5 \times 3$. The most common alternative seen was $4+5+3=12$. Some used factorials $5!\times 4!\times 3!=$ 17280 although candidates might have been alerted to their error by such a high figure from a choice of just 4 starters, 5 main courses and 3 sweets.
(ii) (A) Many used ${ }^{4} \mathrm{C}_{2}$ or equivalent to arrive at the given answer of 6 or simply listed the different combinations but there were many incorrect attempts seen such as 3 ! or ${ }^{3} \mathrm{C}_{1} \times{ }^{2} \mathrm{C}_{1}$.
(B) This proved more challenging and few correct responses were seen with two common errors being ${ }^{12} \mathrm{C}_{6}=924$ and the addition of the binomial coefficients ${ }^{4} \mathrm{C}_{2}$ $+{ }^{5} \mathrm{C}_{2}+{ }^{3} \mathrm{C}_{2}=19$.
(iii) In (A) many candidates gave the incorrect response of $1 / 5 \times 1 / 5=1 / 25$ failing to realise that Peter could choose any dish to begin with as long as Esther chose the same dish thus giving a probability of $1 \times 1 / 5=1 / 5$. Part (B) was also poorly answered and relatively few correct solutions were seen, with $1-p($ same $)$ $=1-(1 / 4)^{2}-(1 / 5)^{2}-(1 / 3)^{2}=0.786$ often wrongly submitted. Equally common was the attempt at $(24 / 25) \times(15 / 16) \times(8 / 9)=4 / 5$, although special consideration was given to candidates who chose this latter route. It was very rare to see the correct response of $3 / 4 \times 4 / 5 \times 2 / 3=2 / 5$.
5) Darts modelled by the binomial distribution, expected value. This question was generally well answered and candidates were able to demonstrate a sound understanding of binomial probabilities. However, in both parts (ii) and (iii), some candidates used rounded values from part (i) to produce inaccurate solutions. As a guiding rule, candidates should be advised to avoid premature approximation and to work with full accuracy, using the storage facility on their calculator if necessary.
(i) A small number of candidates omitted the binomial coefficient of ${ }^{3} \mathrm{C}_{2}$ from their answer.
(ii) Most candidates correctly multiplied their answer to part (i) by 50, although a few were confused by the concept of expected value and chose to work with a value of 0.87 . Some used a multiplier of 150 (darts) instead of 50 (sets of 3 darts). As always, many incorrectly believed that the mean or expected value had to be a whole number.
(iii) Many correct solutions were seen but some candidates reverted to 0.87 in place of 0.2952 , or used incorrect powers.

## Section B

6) Genuine and fake paintings; tree diagram, positive and correct test results, conditional probability calculations; further test to eliminate fakes and associated probability calculation.
This question was generally well answered and provided a rich source of marks for
all but the weakest candidates. Some excellent responses were seen.
(i) The tree diagram was almost invariably correct.
(ii) Again, very well answered, although a common mistake was to add wrongly the decimals 0.855 and 0.02 to give 0.857 instead of 0.875 .
(iii) Usually correctly answered but a number of candidates chose to leave this part blank - perhaps unclear as to how to interpret the word 'correct' in the context of the question.
(iv) Often candidates were able to apply the correct conditional probability formula and many correct answers were seen. A common erroneous method was 0.875/ 0.9. Another common error was the use of $P(A \cap B)=P(A) \times P(B)$ in finding the numerator of the conditional probability expression. This of course is almost never valid in a question of this sort, since the probabilities concerned are rarely independent. There were also a number of candidates who seemed content to give probabilities greater than 1 as their answer.
(v) Although slightly more difficult than part (iv), since neither element of the fraction had already been calculated, most candidates scored the same number of marks here as in part (iv), making equivalent errors (or none).
(vi) The difference between positive given genuine and genuine given positive was not clearly understood. The words 'positive' and 'genuine' were often wrongly considered to be interchangeable, as were 'negative' and 'fake'. A small number of candidates were able to give excellent responses, ably supported by their correct numerical results.
(vii) Most candidates attempted and made some progress with this part but completely correct solutions were often restricted to the best candidates. The most common error was to multiply the correct triple product $0.9 \times 0.05 \times 0.96$ by three, leading to 0.1296 , instead of cubing it.
7) Splitting rocks to find fossils. The binomial distribution, $P(X \geq 1)$, least value of $\boldsymbol{n}$; hypothesis test on the binomial distribution.
(i) Part ( $A$ ) was usually correct, most candidates choosing to use the formula to calculate ${ }^{20} \mathrm{C}_{1} \times 0.1 \times 0.9^{19}$ rather than using tables. Some candidates gave an answer of 0.3917 which was $\mathrm{P}(X \leq 1)$.
Part (B) once again was well answered with the most common errors being $\mathrm{P}(X \geq$
8) $=1-P(X \leq 1)$ or $P(X \geq 1)=1-P(X=1)$ instead of the correct $P(X \geq 1)=1-P(X$ $=0$ ).
(ii) Some very good solutions were seen from high scoring candidates who obtained $n=16$ very efficiently by trial and improvement or by use of logs. Other attempts faltered because of poor manipulation of the inequality or by the use of $0.8^{n}$ rather than $0.9^{n}$. Some candidates wrote down $n=16$ without working or thought that 1 $0.9^{16}=0.8147$ on its own was sufficient justification. In questions of this type, full justification is required, ie the probabilities associated with the least value of $n$ which meets the requirement and also of the greatest value of $n$ which does not meet the requirement must both be explicitly stated.
(iii) (A) The hypotheses were usually correct although a number of candidates gave the hypotheses in words, which was not penalised on this occasion. Candidates should learn and use the correct symbolic notation, which is $\mathrm{H}_{0}: p=0.1$. They should realise that answers such as $\mathrm{H}_{0}=0.1$ or $\mathrm{H}_{0}: \mathrm{P}(X)=0.1$ will be penalised. Very few candidates defined the parameter $p$ in this part. This is now a mandatory requirement, as pointed out in the January 2006 report. Few centres seemed to have taken heed of this as yet.
(B) Many candidates correctly evaluated $\mathrm{P}(X \leq 0)$ and compared this with 0.05 , scoring 2 marks, but unfortunately a large proportion of candidates then mistakenly calculated the point probability $\mathrm{P}(X=1)$ instead of the tail probability $\mathrm{P}(X \leq 1)$ and so their result for the critical region was invalid. It is worth stressing that many candidates erroneously try to argue that a probability is in the critical region, instead of saying it is less than $5 \%$ or whatever the significance level may be, which suggests that their understanding of critical regions is limited.
(C) Since the critical region had already been established in part $B$, all that was required in part C was for candidates to state clearly that ' 2 does not lie in the critical region' and then conclude that there is insufficient evidence to reject the null hypothesis (and as always with a hypothesis test, to express this conclusion in context). However most candidates did not appear to know how to use the critical region since they calculated the point probability $\mathrm{P}(X=2)$ and then incorrectly compared this to 0.05 , thus denying themselves any credit. Those who used the tail probability $\mathrm{P}(X \leq 2)$ were able to gain full credit although they wasted time by unnecessarily calculating this probability. Candidates who did reach a conclusion were sometimes unclear about the appropriate statistical language, with statements such as 'reject the alternative hypothesis' being seen. It is important that candidates realise that $\mathrm{H}_{0}$ is either accepted (or more properly it is stated that 'there is insufficient evidence to reject $\mathrm{H}_{0}{ }^{\prime}$ ) or that $\mathrm{H}_{0}$ is rejected in favour of $\mathrm{H}_{1}$.

## 4767 - Statistics 2

## General Comments

The majority of candidates seemed to be well prepared and took the opportunity to display their knowledge of the specification with much success. Candidates were consistently very successful handling the variety of calculations needed and able to structure clearly their answers to hypothesis tests, but found interpretive parts of questions more of a challenge; in particular Q3 (iii) and Q4 (ii). The majority of candidates showed sufficient accuracy in their working, resisting the temptation to round-off values to make working easier. Candidates appeared to have ample time to complete the paper within the time allowed.

## Comments on Individual Questions

## Section A

1) (i) Well answered, mostly producing full marks. The most frequent error was to use the Poisson distribution.
(ii)A Well answered with most candidates scoring full marks. Very few failed to obtain the correct Poisson mean.
(ii)B Well answered. Some candidates used 1-P $(X \leq 8)$ which scored no marks.
(iii) Most candidates scored full marks. Frequently seen errors include comments relating to general conditions for a Poisson model, and "p is close to 0 or 1 ".
(iv) Well answered. Many candidates were penalised for stating $\sigma^{2}=12$.
(v) Many candidates failed to recognise the need for a continuity correction, and were penalised, but went on to secure the remaining marks.
(vi) Well answered with many candidates scoring full marks. Frequently seen errors include using -2.326, and inappropriate rounding for the final (integer) answer.
2) (i)A Well answered with most candidates working to a suitable level of accuracy and finding a probability using the correct tail of the Normal distribution. Some candidates lost the accuracy mark through premature rounding or spurious attempts at inappropriate continuity corrections.
(i) $B \quad$ Again, well answered. It was pleasing to see most candidates working with the correct form of calculation and to a suitable level of accuracy.
(ii) Well answered. A common error was to use $B(3, p)$ - this was treated as a misread. Candidates could obtain full marks by using their answer to part (i)B in place of 0.7256 .
(iii) Reasonably well answered with good attempts at solving simultaneous equations regularly seen. Common mistakes included using 0.5244 in place of -0.5244 despite this producing a negative value for $\sigma$. Other candidates used ( $1-$ 0.5244 ) in place of -0.5244 . Some failed to obtain $z$-values in their equations and scored no marks.
(iv) Very few candidates picked up the mark for stating that $\mu$ represents the population mean. With the remaining marks, many candidates scored highly. Some lost the final mark for failing to provide a conclusion in context. Many candidates were penalised for calculating their test statistic using a divisor of 1.6 rather than $1.6 / \sqrt{ } 10$.
3) (i) Well answered. Most scored full marks. Some candidates lost the final accuracy mark through miscalculation.
(ii) Very few candidates obtained the mark for identifying $\rho$ as the population correlation coefficient. Some candidates were penalised for using a one-tailed test. Most identified the correct critical value but a few candidates failed to make an explicit, sensible comparison before coming to their conclusion. Those candidates who failed to relate their conclusions to the context of the question lost the final mark.
(iii)A Poorly answered. Not many convincing answers were seen. Many candidates made vague, general comments about the area of the critical region.
(iii)B Poorly answered. Incorrect comments such as "it is more accurate" were common.
(iv) Most candidates did better on this part of the question than the previous part. Many lost the final mark through stating that the procedure could be improved by repeating the test. Many candidates realised that ignoring the first result was not a good idea but failed to provide a suitable reason.
4) (i) Well answered. Apart from slips in calculations, marks were commonly lost through failing to give hypotheses in context, rounding expected frequencies to the nearest integer, using the wrong value for $v$ and/or the wrong critical value. Candidates mentioning correlation were penalised. Some candidates ignored the instruction to show a table of contributions of each cell to the test statistic these candidates were automatically penalised in part (ii)
(ii) Poorly answered. Very few candidates referred to the contributions to the test statistic despite the request in the question. Commonly, candidates made references only to expected and observed frequencies which could gain at most two marks.

## General Comments

The overall standard of the scripts seen was pleasing: many candidates appeared well prepared for this paper. However, as in January, the quality of their comments, interpretations and explanations was consistently below that of the rest of the work. In particular, in Question 3 the standard of writing was felt to be disappointingly poor. Invariably all four questions were attempted. Question 2 was found to be very accessible and most candidates scored full or nearly full marks. The other Questions, 1, 3 and 4, were well answered, with many candidates scoring relatively high marks, though Question 3 was not quite as well answered as the others. There was no evidence to suggest that candidates found themselves short of time at the end.

## Comments on Individual Questions

1) Continuous random variables; Chi-squared hypothesis test for the goodness of fit; simulating loads on structures.
In this question many candidates lost marks needlessly through carelessness and a lack of good exam technique.
(i) The mean of the distribution was usually found with little difficulty. On the other hand, when they came to consider the mode, many candidates found that $\mathrm{f}(x)$ possessed two stationary points but failed to provide any explanation at all for choosing one rather than the other. Some form of discrimination was required for full marks.
(ii) It was disappointing just how often candidates' work for the cumulative distribution function was deficient. A valid method should involve a definite integral (complete with limits) or include " $+c$ " and an attempt to evaluate it. Also the verification of the values of $F(x)$ left very much to be desired. In far too many cases the work presented showed no evidence whatsoever that the candidate had bothered to perform the calculation. For instance candidates who write " $F\left(\frac{1}{4}\right)=3\left(\frac{1}{4}\right)^{4}-8\left(\frac{1}{4}\right)^{3}+6\left(\frac{1}{4}\right)^{2}=\frac{67}{256}$ as required" (sic) should not be surprised to receive no credit.
(iii) In this part the expected frequencies and the calculations up to and including the test statistic were usually correct. Most also went on to compare it to a critical value taken from the correct Chi-squared distribution. However very few appreciated that the test statistic was so large that it would be considered significant at every level of test available to them and that therefore this meant that the evidence should be described as "very highly significant" or words to that effect.
As in the past with questions of this type many candidates neglected to discuss their conclusions, as requested in the question. Perhaps they have not realised that the conclusion of the test informs but is not part of this discussion. On this occasion a comparison of the observed and expected frequencies should have led to an explanation of the exceptional size of the test statistic.
2) Combinations of Normal distributions; journey times on bus routes.

This question was very well answered with very many scoring full marks. Candidates seemed well prepared for it and understood what was expected. Those who take the trouble to provide simple sketch graphs of the standard Normal distribution do much to enhance the quality of their responses. There was evidence from some quarters of effective use of the built in functions on graphics calculators.
(i) This part was almost always correct.
(ii) And so was this part.
(iii) If this part went wrong, as it did just occasionally, it was as a result of an incorrect variance (use of 0.85 instead of $0.85^{2}$ ).
(iv) Again this part was done well. There were more errors with the variance than in part (iii). The question indicated that the scheduled time should be "given as a whole number of minutes" which meant that the final answer should have been rounded up. Quite a few candidates neglected to do this and so lost the final mark.
(v) Answers to this part were mostly correct, but a large minority used the wrong percentage point in their calculation. This was usually because they thought mistakenly that a small sample necessitated the use of a $t$ distribution, and failed to realise that the standard deviation given was for the population.
3)

Sampling; the $t$ distribution: hypothesis test for the population mean; confidence interval for a population mean; survey of staff opinions and wealth.
The answers to the early parts of this question left much to be desired. It was often not clear that candidates understood the different types of sampling. For example, their comments about one type were likely to be so vague that they could equally well apply to any other type and contained nothing that was characteristic of the type under consideration. Concepts of "fair", "representative" and "bias" seemed poorly understood.
(i) A common criticism offered for random sampling was that there would be more chosen from the largest group (administrative) than from the other two groups. Few seemed to realise that random sampling carried the risk of picking an exceptional sample. Judged by what they wrote candidates seemed to want equal numbers to be chosen from the three groups in order for the sample to be representative.
(ii) As mentioned above it was sometimes not possible to tell that the candidate was talking about a systematic sample, nor that he/she knew what a systematic sample involved. Responses were often confined to repeating the criticism of unrepresentativeness made about random sampling in part (i).
(iii) Led by the question, perhaps, candidates commented favourably about stratified sampling as the desirable method, to the effect that it would be representative because the proportions were the same as in the population (which was not always consistent with what they had said in part (i)).
(iv) This part was almost always correct.
(v) The calculations for the test and of the confidence interval were usually performed competently. Sometimes the final conclusion of the test was not as carefully expressed as is required.
4)

Wilcoxon paired sample test for a difference in population medians; paired sample confidence interval for a difference in population means using a $t$ distribution; installation of new fences at factories.
(i) This part was very well answered by very many candidates who worked through the Wilcoxon test with obvious ease. Occasionally there were errors in the ranking process or with the critical value or with the wording of the final conclusion.
(ii) The calculation of the confidence interval was usually correct, though quite often the wrong number of degrees of freedom and/or the wrong percentage point were used. The "wordy" bits of this question were regularly missing or incomplete. Candidates should note that Normality of the differences is required; Normality of the two separate populations is neither necessary nor sufficient for this. The word "population" is a necessary part of both the assumption for using the $t$ distribution and the explanation for not using the Normal distribution.

## General Comments

This is the first time that the new-specification Statistics 4 module has been sat. It is now the highest module in the statistics strand of the MEI specification; its content is a selection of material from the higher modules of the old specification. The new rules under which the present specification must operate mean that opportunities to proceed to high levels in the applied mathematics strands are very limited; so it is good to see that numbers proceeding to the highest level in statistics are holding up well.

There was some very good work, but also some candidates who were perhaps not quite ready to take the examination.

The paper consists of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted. All four questions received many attempts another encouraging feature, as it indicates that centres and candidates are spreading their work over all the options.

There were in fact several candidates who attempted all four questions. Sometimes they deleted one but, whether they did that or not, all four were marked and the best three counted. It needs to be said, and not for the first time, that in general it is not a wise policy to attempt all the questions. Of course it sometimes happens that a solution "goes wrong" and the candidate decides to give it up and proceed to another question. As a tactic, that is acceptable. But candidates should not set out with the strategy of expecting to attempt more questions than are required. It is far better to try to produce the required number of nearly-complete answers than a surfeit of fragments.

Another general point must again be made. There were again several instances of "fudging" of answers that were provided in the question. Such answers are provided partly as a reassurance and check on work so far, but mainly so that they can be used in the rest of the question even by candidates who could not derive them. It is no shame whatever to use a given answer in this way. A number of candidates did so in an honest and open way. In some cases, it appeared that they did not know how to derive the given answer at all, and in other cases they were let down by algebraic errors, commonly noting (sometimes in a humorous way) that something must have gone wrong somewhere. This is entirely acceptable as examination technique. What is not acceptable is faking the answer: commonly done using the magic disappearing minus sign, or by arriving at an incorrect algebraic statement (often gross/y incorrect) and then merely stating that it equals the given answer as though it is hoped that the examiner won't notice.

## Comments on Individual Questions

1) This was on the "estimation" option. The first part required a maximum likelihood estimator to be found; the remaining parts sought its mean and variance, followed by comparison with another estimator using the efficiency criterion.

There were many excellent solutions to the first part, but a substantial minority of candidates had problems here. Some clearly did not know what to do at all; others had some idea but ran into problems right from the beginning; and others made a good start but were then let down by poor technique. Maximum likelihood was of course in the sixth statistics module of the old specification; candidates who wish to offer a solution to this option now must ensure that they are adequately prepared and practised in the work. Maximum likelihood is a fairly advanced concept but not particularly difficult technically provided one is careful and thorough in one's work.

The middle parts of the question were usually well done, even by candidates who could not derive the maximum likelihood estimator in part (i) [with reference to remarks above, note that this is a case where the answer was deliberately given so that it could be used]. Most, but not all, candidates knew how to work out the relative efficiency in part (iv), but there was insecurity in part (v). It was not enough just to aver that $y \leq 1$; the question says "show that", and some sort of convincing explanation was required. Many candidates understood that this result meant that the maximum likelihood estimator was preferable due to having smaller variance, but in some scripts the explanation was not fully complete.
2) This was on the "generating functions" option. It was primarily about obtaining and using a moment generating function.

Many candidates carefully and thoroughly obtained the given answer in part (i), and other candidates made a good start but then made mistakes so that the answer did not come out. However, this was one of the places where there was quite a lot of faking.

In part (ii), most candidates knew that the convolution theorem gave the moment generating function of the $Y$ variable straight away. Obtaining the mean and variance of $Y$ from its moment generating function was also usually done well, though many candidates were clumsy in their technique for differentiation - disappointingly so, in what must be a "Further Mathematics" module. Note that the question includes the explicit word "hence"; other methods of finding the mean and variance were not acceptable.

Very few candidates were able to answer part (iii). The result (see the published mark scheme), which is very simple, seemed not to be known.

In part (iv), candidates' commentaries on the accuracy of the Normal approximation were often very insightful. Some reference to the value of $n$ was expected if full marks were to be obtained. The published mark scheme is based on "remarkably good agreement"; some candidates, having made earlier errors, did not get good agreement here, but their work was followed through.
3) This question was on the "inference" option. It included unpaired and paired $t$ tests.

There were a few candidates who used some form of Wilcoxon test in one part of the question or the other, despite the explicit instruction in each part to use a $t$ test. Unfortunately these candidates lost marks quite heavily. This also applied to candidates who used a wrong type of $t$ test (e.g. an unpaired test in part (ii)).

Usually the work was well done. There was some insecurity in stating assumptions and hypotheses. In part (iii), the point being looked for in the discussion was that the pairing eliminated variability between workers; many candidates made that point, but others lost their way in statements about the nature of the estimated standard deviations.
4) This was on the "design and analysis of experiments" option. Most candidates realised that the required design was a Latin square and produced an example of one; a few candidates however were besotted with randomised blocks. In the next part, the formal statement of the model was sometimes very carefully set out, but many candidates were not quite complete in this.

The analysis in the last part was usually done well. However, another point that can again be made is that there were many candidates who were very inefficient in their calculations. This appeared to have been getting better over the last few years of the old specification, but this year has taken a turn for the worse again. What might be called the $" s_{b}{ }^{2} / s_{w}{ }^{2}$ " method is extremely cumbersome for hand calculation. It is intricate, takes a great deal of time, and is liable to produce errors. The "squared totals" method (as exhibited, somewhat in summary form, in the published mark scheme) is very much better for hand calculation.
[Incidentally, it also appeared that there were candidates who were able to read the required values directly from their calculators. These candidates must be careful to get the values right (i.e. no keying errors), for no method marks can be given if there is only an unsupported numerical answer that happens to be wrong.]

## DECISION MATHEMATICS

## OCR (MEI) Mathematics GCE

## Rules for crossed out and/or replaced work.

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.

## Legacy units

Centres are reminded that under no circumstances can 'legacy' units be used in the 'new' specification from January 2007 onwards. The only units that are acceptable for this specification are units 4751-4777.

## 4771 - Decision Mathematics 1

## General Comments

Performances on parts of this paper were disappointing. In particular many candidates were unable successfully to execute the algorithm in question 3 and many were unable to deal with the simulation in question 6. Conversely the graph question (question 2 ) and the CPA question (question 4) were both done well.

## Comments on Individual Questions

## Q 1 Networks

(i) A straightforward application of Dijkstra. As always there were some candidates, rather too many on this presentation, who either did not know the algorithm or could not convince the examiners that they knew it.
(ii) The instruction "... explain how your working ..." caused difficulties. All that was required was a reference to working values, or to shortest paths to neighbouring vertices.

Q 2 Graphs
Candidates scored well on this question. It was pleasing to see this modelling dealt with so successfully.

Q 3 Algorithms
This was very badly done.
(i) At step 1 most candidates correctly computed $M={ }^{(0+2)} / 2=1$. At step 2 most correctly had $f(M)=1^{2}-2=-1$. At step 3 the vast majority incorrectly put $L$ equal to -1 (i.e. $f(M)$ ) instead of the correct value of 1 (i.e. $M$ ). It is difficult to see why this should have been the case, and it was costly.
(ii) Many candidates seemed to think that an adequate objective for the algorithm was "to make L equal to R". Some others, who had much more idea, thought that it was finding the square root of $R$.
(iii) All that was required was the observation that the algorithm was missing a stopping condition - candidates did not need to provide such a condition.

Q 4 CPA
(i) \& (ii) These parts were well done. Again, it was pleasing to see an aspect of modelling being tackled so well.
(iii) Not many candidates knew what a resource histogram is.
(iv) The question was constructed so that candidates would be able to use their answer to part (iii) to help them in part (iv). Some were able to argue through to the correct answer having not succeeded with part (iii).
(v) Many candidates gratefully accepted the 5 marks which were on offer here.

Q 5 LP
(i) How do we persuade candidates to define their variables properly? It was anticipated that the time constraint would create problems for some candidates, and indeed many gave $0.5 b+0.75 \mathrm{~s}<5$ instead of $1.5 \mathrm{~b}+1.75 \mathrm{~s}<5$.
(ii) Some very tiny graphs were seen. Many candidates either overlooked or ignored the instruction to give the coordinates of their feasible region.
(iii) This revealed modelling weaknesses. Many gave "b=2s" or equivalent.
(iv) Too many candidates did not show how they were solving the LP. The instruction in part (ii), that they give the coordinates of their feasible region, was intended to help with this.
(v)\&(vi) Many candidates showed weaknesses in interpretation in these parts. Good candidates answered them quickly and efficiently.

## Q6 Simulation

(i) The vast majority of candidates clearly did not read the question. They saw a set of probabilities and set off in a knee-jerk routine, simulating a supposed probability distribution with 6 outcomes, despite the fact that the probabilities did not add up to 1 , and despite the fact that nothing much made sense thereafter.
(ii) It might have been thought that the provided tables would have helped candidates to understand this question, but that seemed not to be the case. This is well illustrated by the number of candidates who filled every cell with either a tick or a cross, and then gave an "out of 60" probability.
(iii) Very few candidates were able, in their comments, to distinguish between "reality" and the simulation model. Most comments parroted the question in referring to replacing the component, rather than to continuing the simulation.
(iv) A substantial minority of candidates thought that the reliability could be improved by using 3 -digit random numbers.

## 4772 - Decision Mathematics 2

## General Comments

Performances on this paper were mostly at least good. Candidates were able and wellprepared.

## Comments on Individual Questions

## Q 1 Logic

Most candidates collected the first 10 marks. After that the difficulty level rose in the second requirement of part (ii), but a pleasing number still managed a sophisticated two step proof. Against that, far too many candidates started with what they were required to prove and deduced a true statement, the "howler" of logic.
In part (iii) very many candidates insisted on inserting their own linguistic flourishes, and often revealed their own prejudices.

## Q 2 Networks

This question worked well in that candidates who did make slips, understandable in applying Floyd, were able to recover.
Not much space was provided on the insert for parts (ii) and (iii), since long essays were not required. However, long essays were often provided!
The most difficult aspect of part (i) was ending with a " 2 " in the first row and third column of the route matrix. In part (ii) this gives the "2" in the route $1-2-4-3$. Many candidates, as expected, ended up with a " 4 " in that position of the route matrix, yet still gave the correct route in part (ii). Markers, however, followed through the incorrect "4" to the route 1-4-3 in part (ii).

## Q 3 Decision Analysis

Many candidates worked with very strange payoffs in part (i), sometimes delivering negative EMVs. Many also thought that finding the value and marking the decision at the decision node on their diagram in part (i) provided an answer to the interpretation part of part (ii). It did not.
More candidates than has been the case in the past were able to handle utilities in part (iii), though there were still some who applied the utility function to calculated EMVs instead of to payoffs.
There were some good answers seen to part (iv), but some candidates got into a terrible tangle with the order of their branching.
Some candidates overcomplicated their computations in part (iii) in allowing for the cost of a blood pressure test by subtracting "x" from the relevant payoffs. Had the computations involved utilities that would have been necessary, but in the question as posed all that was needed was the difference in EMVs between having the check and not having the check.

## Q 4 LP

Candidates were very well prepared for this question, with many succeeding in accurate implementations of the simplex algorithm.
In part (i) too many candidates echoed the error seen often in D1 - failing to identify variables.
The question as to why the analyst did not simplify the "eyes" inequality turned out to be the most difficult on the paper. Very few answered it correctly. Many thought that dividing both sides by two would allow toys to be provided with an odd number of eyes. Contrastingly it was very pleasing to see quite a number of candidates correctly answering part (ii) - examiners had thought that this would be as difficult. As mentioned above, the algorithm was well applied in part (iii), but interpretation was less good. Listing the values taken by the variables does not constitute interpretation examiners needed to know what was to be made at what profit, and what would be left over. Similar comments could be applied to part (v), though marking was more lenient at that point.
Part (iv) was answered well, although it seemed to consume quite a lot of energy in setting it up.

## 4773 - Decision Mathematics Computation

## General Comments

Candidates should ensure that all necessary printed computer outputs are supplied and appropriately labelled. In a number of cases candidates were not awarded marks because they had failed to show sufficient evidence of methods used. In a few cases candidates referred to computer output in their written work, but the relevant sheets were not attached.

## Comments on Individual Questions

## Q 1 LP modelling

This question was generally poorly done.
In part (i) the majority of candidates did not appreciate the need for a variable for each investment type for each potential start year and therefore were not able to formulate the LP correctly.
Some candidates did not appreciate the "compound interest" nature of the annual investments and hence did not realise that both $A$ and $C$ were higher yield than $B$ over a 3 -year period. However, those that did were often able to give a reasoned argument as to what yield $B$ needed to succeed in part (iii).

Q 2 Recurrence relations
Other than a few candidates who used $\mathrm{u}(0)=0$ and started the Fibonacci sequence at $\mathrm{n}=1$, part (i) was straightforward and done well.
Good candidates were able to complete the proof in part (ii), whilst others struggled to solve the simultaneous equations.
Part (iii) was occasionally lacking in evidence of method.
In part (iv) a significant number of candidates failed to generate/demonstrate the ratios in the 21 rows of their spreadsheet.

## Q 3 Networks

Some candidates complicated this question by using a separate letter of the alphabet for each Warehouse/Shop pairing, making it difficult for them to cross check their LP against the data in the question.
Candidates with a working LP generally gave clear interpretation of the solution in part (ii).

In part (iii) candidates either needed to add a new set of variables for the Transportation from Shop to Customer, or "double up" their original variables, one set of 12 for each Customer. In reading that the shop requirements no longer applied, many candidates lost sight of the fact that a shop cannot supply more crates than it receives and therefore did not include the relevant transhipment constraints in their formulation. Consequently their results did not correspond to a practical solution.

## Q 4 Simulation

Most candidates achieved full marks in part (i).
Whilst some excellent solutions were seen from some candidates in the rest of the question, others failed to show appropriate evidence of their work. Often spreadsheets were poorly laid out, printed across multiple pages, and with little annotation.
A significant proportion of candidates appeared to be unaware that Excel can cope with testing text cells, and so coded the 3 weather types numerically, necessitating additional processing, and making their work less readable.

## NUMERICAL METHODS

## OCR (MEI) Mathematics GCE

## Rules for crossed out and/or replaced work.

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

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If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.

## Legacy units

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## 4776 - Numerical Methods (Written Examination)

## General Comments

Once again, candidates varied greatly in their level of preparation for this examination. The best showed an admirable grasp of the techniques and approaches required in numerical mathematics. Some, however, had little knowledge of even the most elementary material. Candidates generally were happier with the calculations than with any analysis or interpretation of results.

## Comments on Individual Questions

## 1) Error analysis

This question proved to be beyond the grasp of the majority of candidates. Only a handful could produce the simple argument that the gradient, $f^{\prime}(x)$ is approximately equal to $k / h$. The requirement to use the given result was ignored by most. No credit could be given for any other approach to the error in the function.
2) Solution of an equation

This question was tackled far more successfully, though some candidates used methods other than that required, such as the secant method or the Newton-Raphson method. In part (ii) a substantial minority did not appreciate the need to show a change of sign in the interval $(0.1995,0.2005)$. Simply showing that $f(0.200)$ is very near to zero does not establish that 0.200 is correct to $3 \mathrm{~d} . \mathrm{p}$. , and it scores no marks.
3) Numerical integration

Most candidates did this question well, though some lost marks by working with too few decimal places. The relationships between the methods were better understood than in recent examinations. The final answer can be justified as 3.256 or 3.2560 ; most, but by no means all, candidates with the correct working gave one or other of these values.
4) Numerical differentiation

The three estimates were obtained correctly by almost all candidates, but the comments on their likely accuracy often missed the point. Numerical differentiation with very small $h$, as in this case, involves the subtraction of nearly equal numbers and so gives rise to a progressive loss of precision.
5) Newton's forward difference method

The majority understood that, as the second differences are almost constant, the data can be well approximated by a quadratic. Estimating $g(1.5)$ caused little difficulty, though some who had the correct formula substituted erroneous values into it.
6) Location of roots, solution of an equation

Identifying the root in part (i) of this question was found easy, but many then carried out the bisection process inefficiently, often performing several iterations more than required. A substantial number of candidates did not work with the maximum possible error and so had no clear idea of when to stop. Some assumed that the maximum possible error at the start of the process would be 0.5

In part (ii), most could show that there is no change of sign in f between $x=7.7$ and 7.9. The explanations of why this might be were generally poor. Often it was stated that there must be two (or more) roots. Very few correctly identified an asymptote as being responsible.

In part (iii), the point estimate of 7.8 was obtained by almost everyone, but many did
not then correctly determine whether 7.8 is correct to 1 d.p. Many simply argued that the root could be anywhere in the interval $(7.7,7.9)$ so that 7.8 was not secure to 1 d.p.
7) Numerical differentiation and integration, errors

The value of $D$ was generally correct, but the value of / was often based on a single interval rather than on two. The majority of candidates handled Lagrange's formula correctly, though in some centres it appeared to be unknown. Further estimates of $D$ and / were usually obtained correctly from the polynomial. Comparisons between the estimates were generally rather vague and uninformative. Candidates were required to make comments to the effect that though the absolute differences in the values of $D$ and / were of the same order, the relative difference in the values of $/$ was less than that for $D$. It was rare to see any such analysis.

## 4777 - Numerical Computation

## General Comments

The candidature for this paper was, once again, small. Standards varied greatly; the best scored full marks, but at the other extreme there was little knowledge of the necessary theory and little familiarity with the techniques. Candidates all seemed at home in the use of a spreadsheet, however.

## Comments on Individual Questions

## Section A

1) Solution of an equation; acceleration

This question was frequently well done. In part (i) the algebra was often convoluted and sometimes unconvincing. Parts (ii) and (iii) were generally correct, though a few candidates did not appreciate that, when Aitken's acceleration formula is used to produce an improved estimate, that value should be used to re-start the process.
2) Divided differences

Parts (i) and (ii) were generally done correctly with candidates showing a good grasp of theory and practice. In part (iii), a common error was failing to re-order the data points relative to the point at which the function is to be evaluated. In part (iv) it was often unclear what candidates were doing. Using trial and error it is possible to show that the function, estimated by a quartic, changes sign between $x=4.165$ and 4.175. Hence the root is estimated as 4.17 to 2 d.p.
3) Second order differential equation

This was the least popular question, but those who tackled it mostly scored highly. The algebra in part (i) was done well, as were the solution and the graph in parts (ii) and (iii).
4) Iterative solution of a system of linear equations

This, too, was a question on which most candidates scored high marks. The level of understanding of the methods was good, though some confused the Gauss-Seidel and Gauss-Jacobi versions. Those who worked with the equations as separate entities fared a little better than those who used decomposition on the coefficient matrix.

## Coursework Report, Summer 2006

We felt that Centres had administered the coursework process effectively this session except for the continued failure to provide the Centre Authentication Form CCS160. Some Examination Officers seemed to have been unaware of the letter sent by the Board to say that grades for the unit would not be awarded without it.
Many Centres change from option A to option B without informing the moderator. Centres are reminded that once the paperwork has been completed by the Board, the moderator will be expecting to receive coursework and should be told of the change.

## Core, C3-4753

Moderation was straightforward, the proportion of changes reduced from $29 \%$ to $22 \%$.
Common problems continue to occur. Clearly they will continue to do so with different candidates - our concern is that assessors continue to credit work that does not meet the criteria of the investigation.

The most common problem encountered this session was inadequate graphical illustrations. They need to agree with calculated iterates and have both annotation and explanation. Autograph outputs particularly need annotating to be convincing.

Cof S
The main problems here were that candidates do not give iterates to demonstrate failure and the use of trivial equations.
There continue to be several instances where $f(x)=0$ is shown in the table for failure. The method has found the root and so this cannot be counted as a failure and so should not be credited.

Newton-Raphson method
The main problem in this domain is the failure to establish and justify error bounds.
Rearrangement method
The main problem in this domain is that the rearrangement formula was not stated, particularly with the demonstration of failure. The comparison of $\mathrm{g}^{\prime}(x)$ and the gradient of $y=x$ is also inadequate.

Comparison of methods
Often the descriptions here are not precise enough. Candidates also use different starting values.

Written Communication
The allocation or not of this mark has been more consistent this time. We still find, however, a comment on the cover sheet such as "excellent in every respect" and the mark is awarded only to turn the page to discover that the candidate is going to solve the equation $x^{3}+x-7$.

## Differential Equations - 4758

The number of Centres which required their marks changing is higher than usual ( $30 \%$ compared to about $20 \%$ ). This could be due to the fact that the students may not have had the experience of modelling in Mechanics 2.

The most common investigations were 'Aeroplane Landing' and 'Cascades'. For the former the weak areas tended to be, in domain 2, not considering the initial model for the whole motion (ie resistance proportional to $V+B$ ). The source of the data was not always clear, and for full marks in this particular domain, it is expected that the source be made clear, the data given in table and graph form, with a discussion of its accuracy together with the point where the motion changes. In scheme A, in general, candidates often do not address the variation of parameters; this affected marks in two domains.

Since the object of the coursework is to test the student's understanding of the modelling cycle, the role of the assumptions is crucial. Consequently supplying a list of assumptions and not relating them to the model or the experiment shows a lack of understanding. Also the justification of the model is an essential piece of the 'jigsaw'. Often the differential equations seem to be plucked out of the air, or come about through curve fitting.

Where possible, one would like to see comparisons of data in both graph and numeric form.
It is good to see some Centres attempting to model real life situations which are different from the ones usually submitted. This is to be encouraged. However, care must be taken when setting these tasks that the solution of the differential equations reflects the methods taught in this module. Also, that there is scope for modifying the model, not just modifying the parameters.

## Numerical Methods - 4776

This was the largest cohort to date for this unit. Many candidates submitted work of high quality, which was carefully and accurately assessed. However, in a significant minority of cases, minor adjustments were necessary, and in a small number of cases the adjustments made were substantial.

Major problems arose when the task was unsuitable, and some lightweight tasks were inappropriately given very high marks. For instance, finding the three roots of a cubic by the Newton-Raphson method does not give scope for a substantial application, or for development in the final two domains. In a small number of cases candidates put their problem in the context of a "real life" scenario, and took measurements from a map, which they then used to estimate an area using one of the methods they were familiar with. Unfortunately they did not appreciate that inaccuracies in their measurements led to all sorts of problems with further iterations and the resulting error analysis. This approach is therefore not advisable.

In some cases all the candidates from a centre tackled the same problem in much the same way. In such situations it is expected that this would result in the second mark in the first two domains being withheld.

In domain 2, replication of bookwork is unnecessary - a simple explanation of why the chosen strategy has been selected is all that is required.

In domain 3, a substantial application can be regarded, for instance, as applying the Trapezium Rule with $1,2,4,8,16,32$ and 64 strips. This should be enough in most cases to identify a suitable ratio of differences for extrapolation, or to decide on how much iteration is needed to achieve the desired precision.

In domain 4, an annotated print out of spreadsheet cell formulae works well to explain the use of technology. A clear commentary in the text may also suffice.

In domain 5, it is unlikely that comparing obtained results with the "true value" will be creditworthy. Candidates are expected to achieve convergence by iteration or extrapolation, and to justify their quoted level of accuracy without reference to any known value - whether it be $\pi$, or a solution obtained by a graphical calculator.

In domain 6, the first two marks are easily accessible - a formal statement of the solution, to a (justified) accuracy of six significant figures (or better) is expected. For the last two marks, it is usually necessary to refer to the error analysis. A consideration of any discrepancy between the observed and theoretical ratio of differences is usually appropriate.

The most successful candidates had chosen definite integrals to find with unbounded error terms, whose convergence was slower than expected. This gave plenty of scope for development in the final two sections.

## 7895-8, 3895-3898 AS and A2 MEI Mathematics

## June 2006 Assessment Series

## Unit Threshold Marks

| Unit |  | Maximum <br> Mark | A | B | C | D | $\mathbf{E}$ | $\mathbf{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| All units | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| $\mathbf{4 7 5 1}$ | Raw | 72 | 53 | 45 | 37 | 30 | 23 | 0 |
| $\mathbf{4 7 5 2}$ | Raw | 72 | 55 | 48 | 41 | 34 | 27 | 0 |
| $\mathbf{4 7 5 3}$ | Raw | 72 | 51 | 44 | 38 | 31 | 24 | 0 |
| $\mathbf{4 7 5 3 / 0 2}$ | Raw | 18 | 14 | 12 | 10 | 9 | 8 | 0 |
| $\mathbf{4 7 5 4}$ | Raw | 90 | 57 | 49 | 41 | 33 | 26 | 0 |
| $\mathbf{4 7 5 5}$ | Raw | 72 | 58 | 50 | 43 | 36 | 29 | 0 |
| $\mathbf{4 7 5 6}$ | Raw | 72 | 52 | 45 | 38 | 31 | 25 | 0 |
| $\mathbf{4 7 5 7}$ | Raw | 72 | 51 | 44 | 38 | 32 | 26 | 0 |
| $\mathbf{4 7 5 8}$ | Raw | 72 | 62 | 54 | 46 | 37 | 28 | 0 |
| $\mathbf{4 7 5 8 / 0 2}$ | Raw | 18 | 14 | 12 | 10 | 9 | 8 | 0 |
| $\mathbf{4 7 6 1}$ | Raw | 72 | 55 | 47 | 40 | 33 | 26 | 0 |
| $\mathbf{4 7 6 2}$ | Raw | 72 | 43 | 37 | 31 | 25 | 20 | 0 |
| $\mathbf{4 7 6 3}$ | Raw | 72 | 60 | 52 | 44 | 36 | 29 | 0 |
| $\mathbf{4 7 6 4}$ | Raw | 72 | 46 | 40 | 35 | 30 | 25 | 0 |
| $\mathbf{4 7 6 6}$ | Raw | 72 | 54 | 47 | 40 | 33 | 27 | 0 |
| $\mathbf{4 7 6 7}$ | Raw | 72 | 58 | 51 | 44 | 37 | 30 | 0 |
| $\mathbf{4 7 6 8}$ | Raw | 72 | 59 | 51 | 43 | 36 | 29 | 0 |
| $\mathbf{4 7 6 9}$ | Raw | 72 | 52 | 45 | 38 | 32 | 26 | 0 |
| $\mathbf{4 7 7 1}$ | Raw | 72 | 53 | 46 | 39 | 33 | 27 | 0 |
| $\mathbf{4 7 7 2}$ | Raw | 72 | 57 | 49 | 41 | 34 | 27 | 0 |
| $\mathbf{4 7 7 3}$ | Raw | 72 | 48 | 42 | 36 | 30 | 25 | 0 |
| $\mathbf{4 7 7 6}$ | Raw | 72 | 51 | 44 | 37 | 30 | 23 | 0 |
| $\mathbf{4 7 7 6 / 0 2}$ | Raw | 18 | 13 | 11 | 9 | 8 | 7 | 0 |
| $\mathbf{4 7 7 7}$ | Raw | 72 | 55 | 47 | 39 | 32 | 25 | 0 |

## Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

|  | Maximum <br> Mark | A | B | C | D | E | U |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{7 8 9 5 - 7 8 9 8}$ | 600 | 480 | 420 | 360 | 300 | 240 | 0 |
| $\mathbf{3 8 9 5 - 3 8 9 8}$ | 300 | 240 | 210 | 180 | 150 | 120 | 0 |

The cumulative percentage of candidates awarded each grade was as follows:

|  | A | B | C | D | E | U | Total Number of <br> Candidates |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{7 8 9 5}$ | 40.4 | 61.2 | 77.2 | 89.2 | 96.9 | 100 | 9024 |
| $\mathbf{7 8 9 6}$ | 60.2 | 77.5 | 88.7 | 95.6 | 99.0 | 100 | 1237 |
| $\mathbf{7 8 9 7}$ | 70.5 | 90.9 | 90.9 | 93.2 | 95.5 | 100 | 44 |
| $\mathbf{7 8 9 8}$ | 100 | 100 | 100 | 100 | 100 | 100 | 5 |
| $\mathbf{3 8 9 5}$ | 27.7 | 43.6 | 57.9 | 71.2 | 82.0 | 100 | 11502 |
| $\mathbf{3 8 9 6}$ | 50.9 | 68.6 | 82.4 | 90.0 | 95.6 | 100 | 1247 |
| $\mathbf{3 8 9 7}$ | 80.7 | 86.8 | 94.0 | 98.8 | 98.8 | 100 | 83 |
| $\mathbf{3 8 9 8}$ | 58.8 | 64.7 | 76.5 | 88.2 | 94.1 | 100 | 17 |

# OCR (Oxford Cambridge and RSA Examinations) 

## 1 Hills Road

Cambridge
CB1 2EU

## OCR Information Bureau

## (General Qualifications)

Telephone: 01223553998
Facsimile: 01223552627
Email: helpdesk@ocr.org.uk
www.ocr.org.uk

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