OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4777

Numerical Computation

Wednesday **21 JUNE 2006** Afternoon 2 hours 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.

COMPUTER RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities during the examination..

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes. You should note the following points.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.

  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.

  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(i)** A sequence of numbers $x_1, x_2, x_3, \ldots$ is such that

$$x_{r+1} - \alpha \approx k(x_r - \alpha)$$

for some constants k and α.

Show that α may be estimated as $\dfrac{x_1^2 - x_0 x_2}{2x_1 - x_0 - x_2}$. [5]

(ii) An attempt is made to solve the equation $e^x = \tan x$ using the iterative formula

$$x_{r+1} = \ln(\tan x_r).$$

Show that the equation has a root in the interval $[1, 1.5]$. Demonstrate that the given iteration diverges for starting values in this interval.

Use the method based on the formula obtained in part **(i)** to obtain the root correct to 5 decimal places. [11]

(iii) Show that the equation $e^{-x} = \tan x$ has a root that is just slightly greater than π. Demonstrate that the iteration

$$x_{r+1} = -\ln(\tan x_r)$$

fails to converge to this root.

Show that the approach used in part **(ii)** will give convergence to the required root, but that a very accurate starting value is required. Give the root correct to 5 decimal places. [8]

2 **(i)** Explain briefly the advantage, relative to other methods of interpolation, of using divided differences. [2]

The function $f(x)$ has known values as given, correct to 2 decimal places, in the table.

x	1	1.5	2	2.5	3	3.5	4	4.5	5
$f(x)$	–3.00		−6.50	−8.03		−6.66	−2.25	5.65	

(ii) Draw up a divided difference table to produce a sequence of estimates, linear, quadratic, cubic and quartic, for $f(1.5)$. Discuss briefly the accuracy to which it is possible to estimate $f(1.5)$. [10]

(iii) Modify your routine from part **(i)** to produce estimates of

(A) f(3),

(B) f(5).

In each case discuss briefly the likely accuracy of your estimate. [6]

(iv) There is a root of the equation $f(x) = 0$ at x just greater than 4. Modify your routine to estimate values of $f(x)$ near $x = 4$. Hence, by trial and error, determine the root correct to 2 decimal places. [6]

[Turn over

3 The second order differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 2y^2 = 2e^{-kx}$$

with initial conditions $x = 0$, $y = 0$, $\frac{dy}{dx} = 0$, is to be solved, for various values of k, using finite difference methods. The value of y when $x = 1$ is required. This value is denoted by β.

(i) Consider first the case $k = 1$.

Show that, in the usual notation,

$$y_{r+1} = \frac{1}{1-2h}\left(2h^2e^{-x_r} - 2h^2y_r^2 + 2y_r - (1+2h)y_{r-1}\right),$$

and that

$$y_1 = h^2.$$

Show that, with $h = 0.1$, the estimate of β is a little greater than 4.25.

Obtain further estimates of β for $h = 0.05, 0.025, 0.0125$. Hence demonstrate that the method has second order convergence. Determine β correct to 2 decimal places. [17]

(ii) Modify the routines developed in part **(i)** to find estimates of β, correct to 1 decimal place, for $k = -5, -4, \ldots, 4, 5$. Use the spreadsheet to produce a graph of β as a function of k. [7]

4 **(i)** A set of simultaneous linear equations are to be solved using the Gauss-Seidel iterative method. Explain what diagonal dominance is, and how it relates to the convergence of the method.

Show by means of the equations with augmented matrix

$$\left(\begin{array}{ccc|c} 5 & 3 & 3 & 1 \\ 4 & 7 & 4 & 1 \\ 5 & 5 & 9 & 1 \end{array}\right)$$

that diagonal dominance is *not* a necessary condition. [7]

(ii) Modify the routine developed in part **(i)** to solve the equations with augmented matrix

$$\left(\begin{array}{ccc|c} 6-a & 3 & 3 & 1 \\ 4 & 8-a & 4 & 1 \\ 5 & 5 & 10-a & 1 \end{array}\right)$$

for user-specified values of a.

Demonstrate that the Gauss-Seidel iteration converges for $a = 3$ but diverges for $a = 4$. Determine to 1 decimal place the largest value of a for which the Gauss-Seidel iteration converges. [11]

(iii) Modify the routine in part **(ii)** so that it now implements the Gauss-Jacobi method. Show that the iteration now does *not* converge for $a = 0$. Explain how this result relates to the condition of diagonal dominance. [6]

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