OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4773

Decision Mathematics Computation

Thursday **15 JUNE 2006** Afternoon 2 hours 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.
- You are permitted to use a graphical calculator in this paper.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

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1 An investor is considering three investment opportunities over the next five years. He wishes to maximise the amount of money he has at the end of those five years.

Investment A is a one-year investment. It is available in each of the years and may be started at the beginning of any year. At the end of a year it will return £1.15 for every £1 invested.

Investment B is a three-year investment. It may be started at the beginning of year 1, year 2, or year 3. It will return £1.55 for every £1 invested.

Investment C is another one-year investment, but it is not available until the start of year 3. It will return £1.20 per annum for every £1 invested.

The investor has £50000 to invest.

(i) Define appropriate variables and formulate the investor's problem as an LP. [10]

(ii) Solve your LP using your LP package, and interpret your solution.

You should enclose printouts of your formulation and your output with your solution. [4]

(iii) You should have found that it is not worth investing in B. By experimenting with your LP, or otherwise, find what the return on B would have to be to make it worthwhile investing in it. [3]

2 The Fibonacci recurrence relation is $u_{n+2} = u_{n+1} + u_n$, with $u_0 = 1$ and $u_1 = 1$.

(i) Build a spreadsheet with two columns, the first giving the numbers 0, 1, 2, …, and the second giving the corresponding Fibonacci numbers.

Print out the first 20 Fibonacci numbers. [3]

(ii) Write down and solve the auxiliary equation for the Fibonacci recurrence relation. Hence find an expression for the n^{th} Fibonacci number, and show that it can be expressed in the form

$$u_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right). \quad [9]$$

(iii) Verify that the formula is correct by coding the formula into the third column of your spreadsheet.

Print out your spreadsheet formula and print out your spreadsheet. [2]

(iv) In the fourth column of your spreadsheet compute the Fibonacci ratios R_n, where R_{n+1} is the $(n + 1)^{\text{th}}$ Fibonacci number divided by the n^{th} Fibonacci number.

Describe what happens.

Find the exact value of the limit (which is known as the Golden Ratio). [5]

3 Four shops, S1, S2, S3 and S4 are to be supplied with crates of material from three warehouses, W1, W2 and W3. The requirements at the shops are 10 crates at S1, 15 at S2, 12 at S3 and 20 at S4. There are 20 crates available at each warehouse.

The costs of delivering a single case from each warehouse to each shop are shown in Table 3.1.

cost (£)	S1	S2	S3	S4
W1	2	2	1	5
W2	3	2	2	4
W3	5	5	1	2

Table 3.1

(i) Formulate an LP to solve the problem of moving crates from warehouses to shops at minimum total cost. Produce a printout of your formulation. [7]

(ii) Use your LP package to solve your LP. Produce a printout of your solution.

Interpret your solution. [4]

Two customers, C1 and C2, require 30 and 27 crates respectively. The costs per crate of supplying each of them from each of the shops is shown in Table 3.2.

cost (£)	S1	S2	S3	S4
C1	4	6	3	2
C2	1	4	2	5

Table 3.2

(iii) Formulate, solve and interpret an LP to find the cheapest way of supplying the two customers from the warehouses via the shops. There are still 20 crates available at each warehouse, but the shop requirements no longer apply. [7]

4 The weather in Brighting is either wet, showery or dry. On the day following a wet day there is a 20% chance that it will be wet and a 30% chance that it will be showery. On the day following a showery day there is a 40% chance that it will be wet and a 15% chance that it will be showery. On the day following a dry day there is a 15% chance that it will be wet and a 25% chance that it will be showery.

Today the weather in Brighting is dry.

(i) Find the probabilities of it being wet, showery or dry in Brighting on the day after tomorrow. [4]

(ii) Build a spreadsheet to simulate the weather in Brighting tomorrow, and the day after tomorrow.

(You may wish to set up lookup tables to model the probabilities, and to use "= IF(… ,… ,…)" statements to branch to the appropriate lookup columns.)

Print out the formulae which you use in your spreadsheet. [7]

(iii) Run your simulation 10 times, putting your results into a table.

Estimate the probabilities of it being wet, showery or dry the day after tomorrow. [3]

(iv) Extend your spreadsheet to investigate what happens after 20 days.

Print out your spreadsheet.

Run your simulation 10 times, putting your results into a table.

Estimate the probabilities of it being wet, showery or dry after 20 days. [4]

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