RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4757

Further Applications of Advanced Mathematics (FP3)

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Option 1: Vectors

1 Four points have coordinates $A(-2, -3, 2)$, $B(-3, 1, 5)$, $C(k, 5, -2)$ and $D(0, 9, k)$.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{CD}$. [4]

(ii) For the case when AB is parallel to CD,

(A) state the value of k, [1]

(B) find the shortest distance between the parallel lines AB and CD, [6]

(C) find, in the form $ax + by + cz + d = 0$, the equation of the plane containing AB and CD. [3]

(iii) When AB is not parallel to CD, find the shortest distance between the lines AB and CD, in terms of k. [4]

(iv) Find the value of k for which the line AB intersects the line CD, and find the coordinates of the point of intersection in this case. [6]

Option 2: Multi-variable calculus

2 A surface has equation $x^2 - 4xy + 3y^2 - 2z^2 - 63 = 0$.

(i) Find a normal vector at the point (x, y, z) on the surface. [4]

(ii) Find the equation of the tangent plane to the surface at the point $Q(17, 4, 1)$. [4]

(iii) The point $(17 + h, 4 + p, 1 - h)$, where h and p are small, is on the surface and is close to Q.

Find an approximate expression for p in terms of h. [4]

(iv) Show that there is no point on the surface where the normal line is parallel to the z-axis. [4]

(v) Find the two values of k for which $5x - 6y + 2z = k$ is a tangent plane to the surface. [8]

Option 3: Differential geometry

3 The curve C has parametric equations $x = 2t^3 - 6t$, $y = 6t^2$.

(i) Find the length of the arc of C for which $0 \leqslant t \leqslant 1$. [6]

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant t \leqslant 1$ is rotated through 2π radians about the x-axis. [5]

(iii) Show that the equation of the normal to C at the point with parameter t is

$$y = \frac{1}{2}\left(\frac{1}{t} - t\right)x + 2t^2 + t^4 + 3.$$ [4]

(iv) Find the cartesian equation of the envelope of the normals to C. [6]

(v) The point P$(64, a)$ is the centre of curvature corresponding to a point on C. Find a. [3]

Option 4: Groups

4 The group G consists of the 8 complex matrices $\{\mathbf{I}, \mathbf{J}, \mathbf{K}, \mathbf{L}, -\mathbf{I}, -\mathbf{J}, -\mathbf{K}, -\mathbf{L}\}$ under matrix multiplication, where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{J} = \begin{pmatrix} \mathrm{j} & 0 \\ 0 & -\mathrm{j} \end{pmatrix}, \ \mathbf{K} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \mathbf{L} = \begin{pmatrix} 0 & \mathrm{j} \\ \mathrm{j} & 0 \end{pmatrix}.$$

(i) Copy and complete the following composition table for G. [6]

	I	**J**	**K**	**L**	**–I**	**–J**	**–K**	**–L**
I	**I**	**J**	**K**	**L**	**–I**	**–J**	**–K**	**–L**
J	**J**	**–I**	**L**	**–K**	**–J**	**I**	**–L**	**K**
K	**K**	**–L**	**–I**					
L	**L**	**K**						
–I	**–I**	**–J**						
–J	**–J**	**I**						
–K	**–K**	**L**						
–L	**–L**	**–K**						

(Note that $\mathbf{JK} = \mathbf{L}$ and $\mathbf{KJ} = -\mathbf{L}$.)

(ii) State the inverse of each element of G. [3]

(iii) Find the order of each element of G. [3]

(iv) Explain why, if G has a subgroup of order 4, that subgroup must be cyclic. [4]

(v) Find all the proper subgroups of G. [5]

(vi) Show that G is not isomorphic to the group of symmetries of a square. [3]

Option 5: Markov chains

5 A local hockey league has three divisions. Each team in the league plays in a division for a year. In the following year a team might play in the same division again, or it might move up or down one division.

This question is about the progress of one particular team in the league. In 2007 this team will be playing in either Division 1 or Division 2. Because of its present position, the probability that it will be playing in Division 1 is 0.6, and the probability that it will be playing in Division 2 is 0.4.

The following transition probabilities apply to this team from 2007 onwards.

- When the team is playing in Division 1, the probability that it will play in Division 2 in the following year is 0.2.
- When the team is playing in Division 2, the probability that it will play in Division 1 in the following year is 0.1, and the probability that it will play in Division 3 in the following year is 0.3.
- When the team is playing in Division 3, the probability that it will play in Division 2 in the following year is 0.15.

This process is modelled as a Markov chain with three states corresponding to the three divisions.

(i) Write down the transition matrix. [3]

(ii) Determine in which division the team is most likely to be playing in 2014. [6]

(iii) Find the equilibrium probabilities for each division for this team. [3]

In 2015 the rules of the league are changed. A team playing in Division 3 might now be dropped from the league in the following year. Once dropped, a team does not play in the league again.

- The transition probabilities from Divisions 1 and 2 remain the same as before.
- When the team is playing in Division 3, the probability that it will play in Division 2 in the following year is 0.15, and the probability that it will be dropped from the league is 0.1.

The team plays in Division 2 in 2015.

The new situation is modelled as a Markov chain with four states: 'Division1', 'Division 2', 'Division 3' and 'Out of league'.

(iv) Write down the transition matrix which applies from 2015. [3]

(v) Find the probability that the team is still playing in the league in 2020. [5]

(vi) Find the first year for which the probability that the team is out of the league is greater than 0.5. [4]

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