OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4756

Further Methods for Advanced Mathematics (FP2)

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a\left(\sqrt{2} + 2\cos\theta\right)$ for $-\frac{3}{4}\pi \leqslant \theta \leqslant \frac{3}{4}\pi$, where a is a positive constant.

(i) Sketch the curve. [2]

(ii) Find, in an exact form, the area of the region enclosed by the curve. [7]

(b) **(i)** Find the Maclaurin series for the function $\mathrm{f}(x) = \tan\left(\frac{1}{4}\pi + x\right)$, up to the term in x^2. [6]

(ii) Use the Maclaurin series to show that, when h is small,

$$\int_{-h}^{h} x^2 \tan\left(\tfrac{1}{4}\pi + x\right)\mathrm{d}x \approx \tfrac{2}{3}h^3 + \tfrac{4}{5}h^5.$$ [3]

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \dfrac{1}{z^n}$ and $z^n - \dfrac{1}{z^n}$ in simplified trigonometric form. [2]

(ii) By considering $\left(z - \dfrac{1}{z}\right)^4\left(z + \dfrac{1}{z}\right)^2$, find A, B, C and D such that

$$\sin^4\theta\cos^2\theta = A\cos 6\theta + B\cos 4\theta + C\cos 2\theta + D.$$ [6]

(b) **(i)** Find the modulus and argument of $4 + 4\mathrm{j}$. [2]

(ii) Find the fifth roots of $4 + 4\mathrm{j}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

Illustrate these fifth roots on an Argand diagram. [6]

(iii) Find integers p and q such that $(p + q\mathrm{j})^5 = 4 + 4\mathrm{j}$. [2]

3 **(i)** Find the inverse of the matrix $\begin{pmatrix} 4 & 1 & k \\ 3 & 2 & 5 \\ 8 & 5 & 13 \end{pmatrix}$, where $k \neq 5$. [6]

(ii) Solve the simultaneous equations

$$\begin{aligned} 4x + y + 7z &= 12 \\ 3x + 2y + 5z &= m \\ 8x + 5y + 13z &= 0 \end{aligned}$$

giving x, y and z in terms of m. [5]

(iii) Find the value of p for which the simultaneous equations

$$\begin{aligned} 4x + y + 5z &= 12 \\ 3x + 2y + 5z &= p \\ 8x + 5y + 13z &= 0 \end{aligned}$$

have solutions, and find the general solution in this case. [7]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$1 + 2\sinh^2 x = \cosh 2x. \qquad [3]$$

(ii) Solve the equation

$$2\cosh 2x + \sinh x = 5,$$

giving the answers in an exact logarithmic form. [6]

(iii) Show that $\int_0^{\ln 3} \sinh^2 x \, dx = \frac{10}{9} - \frac{1}{2}\ln 3$. [5]

(iv) Find the exact value of $\int_3^5 \sqrt{x^2 - 9} \, dx$. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations

$$x = q - k\sin q, \quad y = 1 - \cos q,$$

where k is a positive constant.

(i) For the case $k = 1$, use your graphical calculator to sketch the curve. Describe its main features. [4]

(ii) Sketch the curve for a value of k between 0 and 1. Describe briefly how the main features differ from those for the case $k = 1$. [3]

(iii) For the case $k = 2$:

(A) sketch the curve; [2]

(B) find $\frac{dy}{dx}$ in terms of q; [2]

(C) show that the width of each loop, measured parallel to the x-axis, is

$$2\sqrt{3} - \frac{2p}{3}.$$ [5]

(iv) Use your calculator to find, correct to one decimal place, the value of k for which successive loops just touch each other. [2]