OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Combined Mark Schemes And Report on the Units

January 2006

3895-8/7895-8/MS/R/06J

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MARK SCHEME ON THE UNITS

REPORTS FOR THE UNITS

Mark Scheme 4751
January 2006

Section A

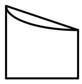
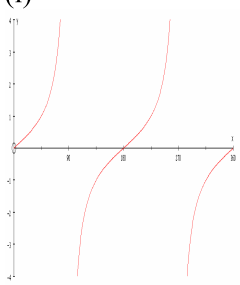
1	$n(n + 1)$ seen = odd × even and/or even × odd = even	M1 A1	<u>or</u> B1 for n odd ⇒ n^2 odd, and comment eg odd + odd = even B1 for n even⇒ n^2 even, and comment eg even + even = even allow A1 for 'any number multiplied by the consecutive number is even'	2
2	(i) translation of $\begin{pmatrix}2\\0\end{pmatrix}$ (ii) $y = \text{f}(x - 2)$	1 1 2	 or '2 to the right' or '$x \to x + 2$' or 'all x values are increased by 2' 1 for $y = \text{f}(x + 2)$	4
3	$16 + 32x + 24x^2 + 8x^3 + x^4$ isw	4	3 for 4 terms correct, 2 for 3 terms correct, or M1 for 1 4 6 4 1 s.o.i. and M1 for expansion with correct powers of 2	4
4	$x > -4.5$ o.e. isw www [M1 for × 4 M1 expand brackets or divide by 3 M1 subtract constant from LHS M1 divide to find x]	4	accept −27/6 or better; 3 for $x = -4.5$ etc or Ms for each of the four steps carried out correctly with inequality [−1 if working with equation] (ft from earlier errors if of comparable difficulty)	4
5	$[C =]\dfrac{4P}{1-P}$ or $\dfrac{-4P}{P-1}$ o.e.	4	M1 for $PC + 4P = C$ M1 for $4P = C - PC$ or ft M1 for $4P = C(1 - P)$ or ft B3 for $[C =]\dfrac{4}{\frac{1}{P}-1}$ o.e. unsimplified	4
6	f(1) used $1^3 + 3 \times 1 + k = 6$ $k = 2$	M1 A1 A1	or division by $x - 1$ as far as $x^2 + x$ or remainder = $4 + k$ B3 for $k = 2$ www	3
7	grad BC = − ¼ soi $y - 3 = -\frac{1}{4}(x - 2)$ o.e. cao 14 or ft from their BC	2 1 2	M1 for $m_1m_2 = -1$ soi or for grad AB = 4 or grad BC = 1/4 e.g. $y = -0.25x + 3.5$ M1 for subst $y = 0$ in their BC	5
8	(i) $30\sqrt{2}$ (ii) $\dfrac{1}{11}+\dfrac{2}{11}\sqrt{3}$ or $\dfrac{3}{33}+\dfrac{6}{33}\sqrt{3}$ or mixture of these	2 3	M1 for $\sqrt{8}=2\sqrt{2}$ or $\sqrt{50} = 5\sqrt{2}$ soi B1 for $6\sqrt{50}$ or other correct $a\sqrt{b}$ M1 for mult num and denom by $6+\sqrt{3}$ and M1 for denom = 11 or 33 B2 for $\dfrac{3+6\sqrt{3}}{33}$ or $\dfrac{1+2\sqrt{3}}{11}$	5
9	(i) $k \le 25/4$ (ii) −2.5	3 2	M2 for $5^2 - 4k \ge 0$ or B2 for 25/4 obtained isw or M1 for $b^2 - 4ac$ soi or completing square accept −20/8 or better, isw; M1 for attempt to express quadratic as $(2x + a)^2$ or for attempt at quadratic formula	5

Section B

10	**i**	(0, 0), $\sqrt{45}$ isw or $3\sqrt{5}$	1+1		2
	ii	$x = 3 - y$ or $y = 3 - x$ seen or used subst in eqn of circle to eliminate variable $9 - 6y + y^2 + y^2 = 45$ $2y^2 - 6y - 36 = 0$ or $y^2 - 3y - 18 = 0$ $(y - 6)(y + 3) = 0$ $y = 6$ or -3 $x = -3$ or 6 $\sqrt{(6 - -3)^2 + (3 - -6)^2}$	M1 M1 M1 M1 M1 A1 A1 M1	for correct expn of $(3 - y)^2$ seen oe condone one error if quadratic or quad. formula attempted [complete sq attempt earns last 2 Ms] or A1 for (6, –3) and A1 for (–3, 6) no ft from wrong points (A.G.)	8
11	**i**	$(x - 3.5)^2 - 6.25$	3	B1 for $a = 7/2$ o.e, B2 for $b = -25/4$ o.e. or M1 for $6 - (7/2)^2$ or $6 - (\text{their } a)^2$	3
	ii	(3.5, −6.25) o.e. or ft from their (i)	1+1	allow $x = 3.5$ and $y = -6.25$ or ft; allow shown on graph	2
	iii	(0, 6) (1, 0) (6, 0) curve of correct shape fully correct intns and min in 4th quadrant	3 G1 G1	1 each [stated or numbers shown on graph]	5
	iv	$x^2 - 7x + 6 = x^2 - 3x + 4$ $2 = 4x$ $x = ½$ or 0.5 or 2/4 cao	M1 M1 A1	 or $4x - 2 = 0$ (simple linear form; condone one error) condone no comment re only one intn	3
12	**i**	sketch of cubic the correct way up curve passing through (0, 0) curve touching x axis at (3, 0)	G1 G1 G1		3
	ii	$x(x^2 - 6x + 9) = 2$ $x^3 - 6x^2 + 9x = 2$	M1 M1	or $(x^2 - 3x)(x - 3) = 2$ [for one step in expanding brackets] for 2nd step, dep on first M1	2
	iii	subst $x = 2$ in LHS of their eqn or in $x(x - 3)^2 = 2$ o.e. working to show consistent division of their eqn by $(x - 2)$ attempted $x^2 - 4x + 1$ soln of their quadratic by formula or completing square attempted $x = 2 \pm \sqrt{3}$ or $(4 \pm \sqrt{12})/2$ isw locating the roots on intersection of their curve and $y = 2$	1 1 M1 A1 M1 A2 G1	or 2 for division of their eqn by $(x - 2)$ and showing no remainder or inspection attempted with $(x^2 + kx + c)$ seen condone ignoring remainder if they have gone wrong A1 for one correct must be 3 intns; condone $x = 2$ not marked; mark this when marking sketch graph in (i)	7 G1

Mark Scheme 4752
January 2006

Section A

1	7/9 or 140/180 o.e.	2	B1 for 180° = π rad o.e. or 0.78 or other approximations	2
2	224	2	M1 for $2^3 + 3^3 + 4^3 + 5^3$	2
3	triangle divided into 2 rt angled tris √3 and 1 indicated 60 indicated	H1 S1 A1		3
4	16.1  overestimate + expn eg sketch	4 1	M3 for ¼{8.2 + 4.2 + 2 (6.4 + 5.5 + 5 + 4.7 + 4.4)} M2 for one slip/error M1 for two slips/errors	5
5	(i)  tan x = ¾ (ii) 36.8 to 36.9 and 216.8 to 216.9	2 M1 A1A1	no numbers required on axes unless more branches shown. G1 for a correct first sweep Allow 37, 217	5
6	$y'' = 2x - 6$ $y'' = 0$ at $x = 3$ $y' = 0$ at $x = 3$ showing y' does not change sign	B1 B1 B1 E1	or that y'' changes sign	4
7	(i) 5 (ii) 5.646... to 2 sf or more	2 3	M1 for 6 = 1.2r M2 for 2 x 5x sin 0.6 or $\sqrt{(5^2 + 5^2 - 2.5.5. \cos 1.2)}$ or 5 sin 1.2/sin 0.971 M1 for these methods with 1 error	5
8	$\frac{2}{3}x^{\frac{3}{2}} - 3x^{-2} + c$ o.e.	5	1 for each element	5
9	(i) $\log_{10} y = 0.5x + 3$ (ii) $y = 10^{0.5x+3}$ isw	B3 2	B1 for each term scored in either part o.e. e.g. $y = 1000 \times 10^{\sqrt{x}}$	5

Section B

10	**i**	$y' = 6 - 2x$ $y' = 0$ used $x = 3$ $y = 16$ (0, 7) (–1, 0) and (7,0) found or marked on graph sketch of correct shape	M1 M1 A1 A1 3 1	condone one error 1 each must reach pos. y - axis	 8
	ii	58.6 to 58.7	3 M1	B1 for $7x + 3x^2 - x^3/3$ [their value at 5] – [their value at 1] dependent on integration attempted	3
	iii	using his (ii) and 48	1		1
11	**i**	$3x^2 - 6$	2	1 if one error	2
	ii	$-\sqrt{2} < x < \sqrt{2}$	3	M1 for using their $y' = 0$ B1 f.t. for both roots found	3
	iii	subst $x = -1$ in their y' [=–3] $y = 7$ when $x = -1$ $y + 3x = 4$ $x^3 - 6x + 2 = -3x + 4$ (2, –2) c.a.o.	B1 M1 A1 M1 A1,A1	f.t. f.t. 3 terms f.t.	6
12	**i**	A 23 B 24 C 480	2 2 2	M1 for 5, 7, 9 etc or AP with $a = 5$, $d = 2$ M1 for $51 = 5 + 2(n - 1)$ o.e. M1 for attempted use of sum of AP formula eg 20/2[10+19×2]	2 2 2
	ii	A 11.78 – 11.80 B $5 \times 1.1^{n-1} > 50$ $1.1^{n-1} > 10$ $(n - 1) \log 1.1 > 1$ $n - 1 > 1/\log 1.1$ n = 26	2 B1 B1 L1 A1 1	 Or other step towards completion (NB answer given) independent	

Mark Scheme 4753
January 2006

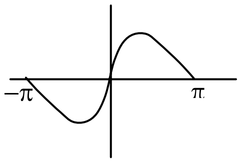
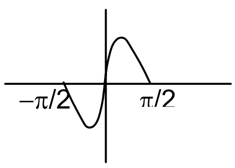
Section A

1 $y = (1 + 6x)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1+6x)^{-2/3}.6$ $= 2(1 + 6x)^{-2/3}$ $= 2[(1 + 6x)^{1/3}]^{-2}$ $= \frac{2}{y^2}$ *	M1 B1 A1 E1	Chain rule $\frac{1}{3}(1+6x)^{-2/3}$ or $\frac{1}{3}u^{-2/3}$ any correct expression for the derivative www
or $y^3 = 1 + 6x$ $\Rightarrow x = (y^3 - 1)/6$ $\Rightarrow dx/dy = 3y^2/6 = y^2/2$ $\Rightarrow dy/dx = 1/(dx/dy) = 2/y^2$ *	M1 A1 B1 E1	Finding x in terms of y $y^2/2$ o.e.
or $y^3 = 1 + 6x$ $\Rightarrow 3y^2\, dy/dx = 6$ $\Rightarrow dy/dx = 6/3y^2 = 2/y^2$ *	M1 A1 A1 E1 [4]	together with attempt to differentiate implicitly $3y^2\, dy/dx$ $= 6$
2 (i) When $t = 0$, $P = 5 + a = 8$ $\Rightarrow a = 3$ When $t = 1$, $5 + 3e^{-b} = 6$ $\Rightarrow e^{-b} = 1/3$ $\Rightarrow -b = \ln 1/3$ $\Rightarrow b = \ln 3 = 1.10$ (3 s.f.) **(ii)** 5 million	M1 A1 M1 M1 A1ft B1 [6]	substituting $t = 0$ into equation Forming equation using their a Taking lns on correct re-arrangement (ft their a) or $P = 5$
3 (i) $\ln (3x^2)$ **(ii)** $\ln 3x^2 = \ln(5x + 2)$ $\Rightarrow 3x^2 = 5x + 2$ $\Rightarrow 3x^2 - 5x - 2 = 0$* **(iii)** $(3x + 1)(x - 2) = 0$ $\Rightarrow x = -1/3$ or 2 $x = -1/3$ is not valid as $\ln(-1/3)$ is not defined	B1 B1 M1 E1 M1 A1cao B1ft [7]	$2\ln x = \ln x^2$ $\ln x^2 + \ln 3 = \ln 3x^2$ Anti-logging Factorising or quadratic formula ft on one positive and one negative root

4 (i) $\frac{dV}{dt}=2$ **(ii)** tan 30 = $1/\sqrt{3}$ $= r/h$ $\Rightarrow$ $h = \sqrt{3}\, r$ $\Rightarrow$ $V=\frac{1}{3}\pi r^2.\sqrt{3}r=\frac{\sqrt{3}}{3}\pi r^3$ * $\frac{dV}{dr}=\sqrt{3}\pi r^2$ **(iii)** When $r = 2$, $dV/dr = 4\sqrt{3}\pi$ $\frac{dV}{dt}=\frac{dV}{dr}.\frac{dr}{dt}$ $\Rightarrow$ $2 = 4\sqrt{3}\pi\, dr/dt$ $\Rightarrow$ $dr/dt = 1/(2\sqrt{3}\pi)$ or 0.092 cm s^{-1}	B1 M1 E1 B1 M1 M1 A1cao [7]	Correct relationship between r and h in any form From exact working only o.e. e.g. $(3\sqrt{3}/3)\pi\, r^2$ or $\frac{dr}{dt}=\frac{dr}{dV}.\frac{dV}{dt}$ substituting 2 for dV/dt and r = 2 into their dV/dr
5(i) $y^3 = 2xy + x^2$ $\Rightarrow$ $3y^2\frac{dy}{dx}=2x\frac{dy}{dx}+2y+2x$ $\Rightarrow$ $(3y^2-2x)\frac{dy}{dx}=2y+2x$ $\Rightarrow$ $\frac{dy}{dx}=\frac{2(x+y)}{3y^2-2x}$ * **(ii)** $\frac{dx}{dy}=\frac{3y^2-2x}{2(x+y)}$	B1 B1 M1 E1 B1cao [5]	$3y^2\frac{dy}{dx}=$ $2x\frac{dy}{dx}+2y+2x$ collecting dy/dx terms on one side www
6(i) $y = 1 + 2\sin x$ $y \leftrightarrow x$ $\Rightarrow$ $x = 1 + 2\sin y$ $\Rightarrow$ $x - 1 = 2\sin y$ $\Rightarrow$ $(x - 1)/2 = \sin y$ $\Rightarrow$ $y=\arcsin(\frac{x-1}{2})$ * Domain is $-1 \le x \le 3$ **(ii)** A is $(\pi/2, 3)$ B is (1, 0) C is $(3, \pi/2)$	M1 A1 E1 B1 B1cao B1cao B1ft [7]	Attempt to invert Allow $\pi/2$ = 1.57or better ft on their A

Section B

7(i) $2x - x\ln x = 0$ $\Rightarrow$ $x(2 - \ln x) = 0$ $\Rightarrow$ $(x = 0)$ or $\ln x = 2$ $\Rightarrow$ at A, $x = e^2$	M1 A1 [2]	Equating to zero
(ii) $\frac{dy}{dx} = 2 - x.\frac{1}{x} - \ln x.1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow$ $\ln x = 1, x = e$ When $x = e$, $y = 2e - e\ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]	Product rule for $x \ln x$ d/dx (ln x) = 1/x $1 - \ln x$ o.e. equating their derivative to zero $x = e$ $y = e$
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]	Substituting x=1 or their e^2 into their derivative -1 and 1 www
(iv) Let $u = \ln x$, dv/dx = x $\Rightarrow v = ½x^2$ $\int x\ln x dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2.\frac{1}{x}dx$ $= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x dx$ $= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$ * $A = \int_1^e (2x - x\ln x)dx$ $= \left[x^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]_1^e$ $= (e^2 - ½e^2\ln e + ¼e^2) - (1 - ½1^2\ln 1 + ¼1^2)$ $= \frac{3}{4}e^2 - \frac{5}{4}$	M1 A1 E1 B1 B1 M1 A1 cao [7]	Parts: $u = \ln x$, dv/dx = $x \Rightarrow v = ½x^2$ correct integral and limits $\left[x^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]$ o.e. substituting limits correctly

8 (i) $f(-x) = \frac{\sin(-x)}{2-\cos(-x)}$ $= \frac{-\sin(x)}{2-\cos(x)}$ $= -f(x)$  	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ Graph completed with rotational symmetry about O.
(ii) $f'(x) = \frac{(2-\cos x)\cos x - \sin x.\sin x}{(2-\cos x)^2}$ $= \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$ $= \frac{2\cos x - 1}{(2-\cos x)^2}$ * $f'(x) = 0$ when $2\cos x - 1 = 0$ $\Rightarrow \cos x = ½$, $x = \pi/3$ When $x = \pi/3$, $y = \frac{\sin(\pi/3)}{2-\cos(\pi/3)} = \frac{\sqrt{3}/2}{2-1/2}$ $= \frac{\sqrt{3}}{3}$ So range is $-\frac{\sqrt{3}}{3} \le y \le \frac{\sqrt{3}}{3}$	M1 A1 E1 M1 A1 M1 A1 B1ft [8]	Quotient or product rule consistent with their derivatives Correct expression numerator = 0 Substituting their $\pi/3$ into y o.e. but exact ft their $\frac{\sqrt{3}}{3}$
(iii) $\int_0^\pi \frac{\sin x}{2-\cos x}dx$ let $u = 2 - \cos x$ $\Rightarrow du/dx = \sin x$ When $x = 0$, $u = 1$; when $x = \pi$, $u = 3$ $= \int_1^3 \frac{1}{u}du$ $= [\ln u]_1^3$ $= \ln 3 - \ln 1 = \ln 3$	M1 B1 A1ft A1cao	$\int \frac{1}{u}du$ $u = 1$ to 3 $[\ln u]$
or $= [\ln(2-\cos x)]_0^\pi$ $= \ln 3 - \ln 1 = \ln 3$	M2 A1 A1 cao [4]	$[k \ln(2 - \cos x)]$ $k = 1$
(iv)  	B1ft [1]	Graph showing evidence of stretch s.f. ½ in x – direction
(v) Area is stretched with scale factor ½ So area is ½ ln 3	M1 A1ft [2]	soi ½ their ln 3

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Section A

1 $\frac{2x}{x-2} - \frac{4x}{x+1} = 3$ $\Rightarrow$ $2x(x + 1) - 4x(x - 2) = 3(x - 2)(x + 1)$ $\Rightarrow$ $2x^2 + 2x - 4x^2 + 8x = 3x^2 - 3x - 6$ $\Rightarrow$ $0 = 5x^2 - 13x - 6$ $= (5x + 2)(x - 3)$ $\Rightarrow$ $x = -2/5$ or 3.	M1 M1 A1 M1 A1 cao [5]	Clearing fractions expanding brackets oe factorising or formula
2 $dx/dt = 1 - 1/t$ $dy/dt = 1 + 1/t$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1+\frac{1}{t}}{1-\frac{1}{t}}$ When $t = 2$, $dy/dx = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$	B1 M1 A1 M1 A1 [5]	Either dx/dt or dy/dt soi www
3 $\overrightarrow{BA} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$ $\overrightarrow{BA}.\overrightarrow{BC} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}.\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = (-4)\times 2 + 1\times 5 + (-3)\times(-1)$ $= -8 + 5 + 3 = 0$ $\Rightarrow$ angle ABC = 90° Area of triangle = ½ × BA × BC $= \frac{1}{2}\times\sqrt{(-4)^2+1^2+3^2}\times\sqrt{2^2+5^2+(-1)^2}$ $= ½ \times \sqrt{26} \times \sqrt{30}$ = 13.96 sq units	B1 M1 A1 M1 M1 A1 [6]	soi , condone wrong sense scalar product = 0 area of triangle formula oe length formula accept 14.0 and √195

4(i) $2\sin 2\theta + \cos 2\theta = 1$ $\Rightarrow$ $4\sin\theta\cos\theta + 1 - 2\sin^2\theta = 1$ $\Rightarrow 2\sin\theta(2\cos\theta - \sin\theta) = 0$ or $4\tan\theta - 2\tan^2\theta = 0$ $\Rightarrow$ $\sin\theta = 0$ or $\tan\theta = 0$, $\theta = 0°, 180°$ or $2\cos\theta - \sin\theta = 0$ $\Rightarrow$ $\tan\theta = 2$ $\Rightarrow$ $\theta = 63.43°, 243.43°$ OR Using $R\sin(2\theta+\alpha)$ $R=\sqrt{5}$ and $\alpha=26.57°$ $2\theta + 26.57 = \arcsin 1/R$ $\theta=0°, 180°$ $\theta=63.43°, 243.43°$	M1 A1 A1 M1 A1, A1 [6] M1 A1 M1 A1 A1,A1 [6]	Using double angle formulae Correct simplification to factorisable or other form that leads to solutions 0° and 180° $\tan\theta = 2$ (-1 for extra solutions in range) (-1 for extra solutions in range)
5 (i) Plane has equation $x - y + 2z = c$ At $(2, -1, 4)$, $2 + 1 + 8 = c$ $\Rightarrow c = 11$. **(ii)** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$ $\Rightarrow$ $7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$ $\Rightarrow$ $2\lambda = -2$ $\Rightarrow$ $\lambda = -1$ Coordinates are (6, 9, 7)	B1 M1 A1 M1 M1 A1 A1 [7]	$x - y + 2z = c$ finding c ft their equation from (i) ft their x-y+2z=c cao
6 (i) $\dfrac{1}{\sqrt{4-x^2}} = 4^{-\frac{1}{2}}(1-\frac{1}{4}x^2)^{-\frac{1}{2}}$ $= \frac{1}{2}[1+(-\frac{1}{2})(-\frac{1}{4}x^2)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{1}{4}x^2)^2+\ldots]$ $= \frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\ldots$ **(ii)** $\int_0^1 \dfrac{1}{\sqrt{4-x^2}}dx \approx \int_0^1 (\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4)dx$ $= \left[\frac{1}{2}x+\frac{1}{48}x^3+\frac{3}{1280}x^5\right]_0^1$ $= \frac{1}{2}+\frac{1}{48}+\frac{3}{1280}$ $= 0.5232$ (to 4 s.f.) **(iii)** $\int_0^1 \dfrac{1}{\sqrt{4-x^2}}dx = \left[\arcsin\frac{x}{2}\right]_0^1$ $= \pi/6 = 0.5236$	M1 M1 A1 A1 M1ft A1 B1 [7]	 Binomial coeffs correct Complete correct expression inside bracket cao

Section B

7(i) $A\hat{O}P = 180 - \beta = 180 - \alpha - \theta$ $\Rightarrow \beta = \alpha + \theta$ $\Rightarrow \theta = \beta - \alpha$ $\tan\theta = \tan(\beta - \alpha)$ $= \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$ $= \frac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10}\cdot\frac{y}{16}}$ $= \frac{16y - 10y}{160 + y^2}$ $= \frac{6y}{160 + y^2}$ * When $y = 6$, $\tan\theta = 36/196$ $\Rightarrow \theta = 10.4°$	M1 M1 E1 M1 A1 E1 M1 A1 cao [8]	Use of sum of angles in triangle OPT and AOP oe SC B1 for $\beta=\alpha+\theta$, $\theta=\beta-\alpha$ no justification Use of Compound angle formula Substituting values for tan α and tan β www accept radians
(ii) $\sec^2\theta\frac{d\theta}{dy} = \frac{(160+y^2)6 - 6y.2y}{(160+y^2)^2}$ $= \frac{6(160 + y^2 - 2y^2)}{(160+y^2)^2}$ $\Rightarrow \frac{d\theta}{dy} = \frac{6(160 - y^2)}{(160+y^2)^2}\cos^2\theta$ *	M1 M1 A1 A1 E1 [5]	$\sec^2\theta\frac{d\theta}{dy} = \ldots$ quotient rule correct expression simplifying numerator www
(iii) $d\theta/dy = 0$ when $160 - y^2 = 0$ $\Rightarrow y^2 = 160$ $\Rightarrow y = 12.65$ When $y = 12.65$, $\tan\theta = 0.237\ldots$ $\Rightarrow \theta = 13.3°$	M1 A1 M1 A1cao [4]	oe accept radians

8 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow$ $dx/dt = -ka(1 + kt)^{-2}$ $= -ka(x/a)^2$ $= -kx^2/a$ * OR $kt=a/x - 1$, $t= a/kx - 1/k$ $dt/dx= -a/kx^2$ $\Rightarrow dx/dt= -kx^2/a$	M1 A1 E1 [3] M1 A1 E1 [3]	Chain rule (or quotient rule) Substitution for x
(ii) When $t = 0$, $x = a \Rightarrow a = 2.5$ When $t = 1$, $x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$ $\Rightarrow$ $1 + k = 1.5625$ $\Rightarrow$ $k = 0.5625$	B1 M1 A1 [3]	$a = 2.5$
(iii) In the long term, $x \to 0$	B1 [1]	or, for example, they die out.
(iv) $\frac{1}{2y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{2-y}$ $\Rightarrow$ $1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = ½$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = ½$ $\Rightarrow \frac{1}{2y-y^2} = \frac{1}{2y} + \frac{1}{2(2-y)}$	M1 M1 A1 A1 [4]	partial fractions evaluating constants by substituting values, equating coefficients or cover-up
(v) $\int \frac{1}{2y-y^2} dy = \int dt$ $\Rightarrow$ $\int [\frac{1}{2y} + \frac{1}{2(2-y)}] dy = \int dt$ $\Rightarrow$ $½ \ln y - ½ \ln(2 - y) = t + c$ When $t = 0$, $y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln y - \ln(2 - y) = 2t$ $\Rightarrow$ $\ln \frac{y}{2-y} = 2t$ * $\frac{y}{2-y} = e^{2t}$ $\Rightarrow$ $y = 2e^{2t} - ye^{2t}$ $\Rightarrow$ $y + ye^{2t} = 2e^{2t}$ $\Rightarrow$ $y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow$ $y = \frac{2e^{2t}}{1+e^{2t}} = \frac{2}{1+e^{-2t}}$ *	M1 B1 ft A1 E1 M1 DM1 E1 [7]	Separating variables $½ \ln y - ½ \ln(2 - y)$ ft their A,B evaluating the constant Anti-logging Isolating y
(vi) As $t \to \infty$ $e^{-2t} \to 0 \Rightarrow y \to 2$ So long term population is 2000	B1 [1]	or $y = 2$

Comprehension

1. It is the largest number in the Residual column in Table 5. **B1**

2. (i)

Acceptance percentage, *a*%		10%	14%	**12%**	**11%**	**10.5%**
Party	**Votes (%)**	**Seats**	**Seats**	**Seats**	**Seats**	**Seats**
P	30.2	**3**	**2**	**2**	**2**	**2**
Q	11.4	**1**	**0**	**0**	**1**	**1**
R	22.4	**2**	**1**	**1**	**2**	**2**
S	14.8	**1**	**1**	**1**	**1**	**1**
T	10.9	**1**	**0**	**0**	**0**	**1**
U	10.3	**1**	**0**	**0**	**0**	**0**
Total seats		**9**	**4**	**4**	**6**	**7**

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

10% & 14% **B1**
Trial **M1**
10.5% (10.3<x≤10.9) **A1**
Allocation **A1**

(ii)

	Round							
Party	1	2	3	4	5	6	7	Residual
P	30.2	**15.1**	**15.1**	**10.07**	**10.07**	**10.07**	**10.07**	**10.07**
Q	11.4	**11.4**	**11.4**	**11.4**	**11.4**	**5.7**	**5.7**	**5.7**
R	22.4	**22.4**	**11.2**	**11.2**	**11.2**	**11.2**	**7.47**	**7.47**
S	14.8	**14.8**	**14.8**	**14.8**	**7.4**	**7.4**	**7.4**	**7.4**
T	10.9	**10.9**	**10.9**	**10.9**	**10.9**	**10.9**	**10.9**	**5.45**
U	10.3	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**	**10.3**
Seat allocated to	**P**	**R**	**P**	**S**	**Q**	**R**	**T**	

Seat Allocation P 2 Q 1 R 2 S 1 T 1 U 0

General method **M1** Round 2 correct **A1** Round 5 correct **A1**(condone minor arithmetic error) Residuals **A1** www Allocation **A1** cso

3. $\frac{11.2}{1+1} < 11 \le \frac{11.2}{1} \quad \Rightarrow 5.6 < 11 \le 11.2$ **M1, A1**

for either or both

M1 only for $5.6<a\leq 11.2$

4. (i) The end-points of the intervals are the largest values in successive columns of Table 5.(or two largest within a column)

B1

So in

2	$16.6 < a \leq 22.2$

22.2 is the largest number in Round 2. 16.6 is the largest number in Round 3.

B1

(ii)

Seats	a		**Seats**	a
1	$22.2 < a \leq 27.0$		5	$11.1 < a \leq 11.2$
2	$16.6 < a \leq 22.2$		6	$10.6 < a \leq 11.1$
3	$13.5 < a \leq 16.6$		7	$9.0 < a \leq 10.6$
4	$11.2 < a \leq 13.5$			

B1

5. (i) ● means $\leq$, ○ means $<$ (greater or less than) **B1**

(ii)
$$\frac{V_k}{N_k+1} < a \qquad a \leq \frac{V_k}{N_k}$$
$$V_k < aN_k + a \qquad aN_k \leq V_k$$
$$V_k - aN_k < a \qquad 0 \leq V_k - aN_k$$
$$0 \leq V_k - aN_k < a$$
B1

(iii) The unused votes may be zero but must be less than *a*. **B1**

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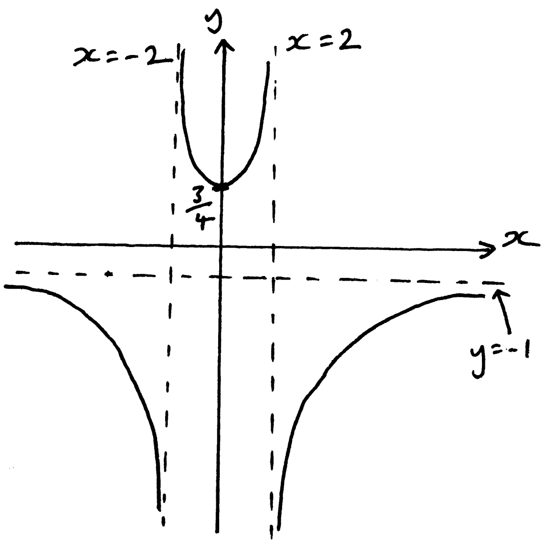
Section A

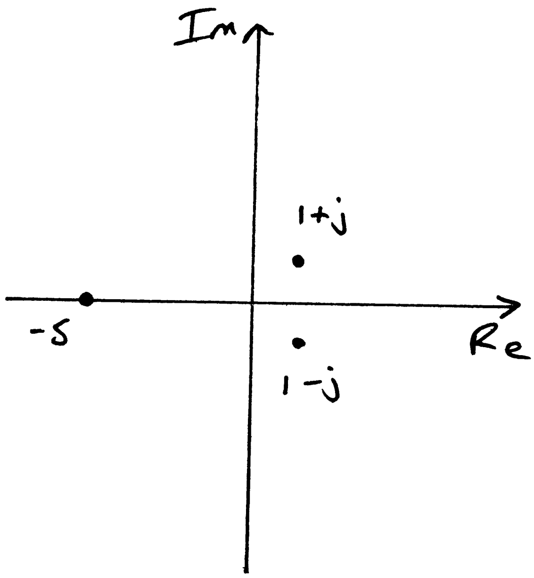
1(i)	$2\mathbf{B} = \begin{pmatrix} 4 & -6 \\ 2 & 8 \end{pmatrix}$, **A + C** is impossible, $\mathbf{CA} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \\ 1 & 2 \end{pmatrix}$, $\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 & 6 \\ 0 & -2 \end{pmatrix}$	B1 B1 M1, A1 B1 **[5]**	**CA** 3×2 matrix M1
1(ii)	$\mathbf{AB} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 11 & 0 \\ 4 & 5 \end{pmatrix}$ $\mathbf{BA} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 8 & 11 \end{pmatrix}$ $\mathbf{AB} \neq \mathbf{BA}$	M1 E1 **[2]**	Or **AC** impossible, or student's own correct example. Allow M1 even if slip in multiplication Meaning of commutative
2(i)	$\lvert z \rvert = \sqrt{\left(a^2 + b^2\right)}$, $z^* = a - b\mathrm{j}$	B1 B1 **[2]**	
2(ii)	$zz^* = (a + bj)(a - bj) = a^2 + b^2$ $\Rightarrow zz^* - \lvert z \rvert^2 = a^2 + b^2 - \left(a^2 + b^2\right) = 0$	M1 M1 A1 **[3]**	Serious attempt to find zz^*, consistent with their z^* ft their $\lvert z \rvert$ in subtraction All correct
3	$\sum_{r=1}^{n}(r+1)(r-1) = \sum_{r=1}^{n}\left(r^2 - 1\right)$ $= \frac{1}{6}n(n+1)(2n+1) - n$ $= \frac{1}{6}n\left[(n+1)(2n+1) - 6\right]$ $= \frac{1}{6}n\left(2n^2 + 3n - 5\right)$ $= \frac{1}{6}n(2n+5)(n-1)$	M1 M1, A1, A1 M1 A1 **[6]**	Condone missing brackets Attempt to use standard results Each part correct Attempt to collect terms with common denominator c.a.o.

4(i)	$6x - 2y = a$ $-3x + y = b$	B1 B1 **[2]**	
4(ii)	Determinant = 0 The equations have no solutions or infinitely many solutions.	B1 E1 E1 **[3]**	No solution or infinitely many solutions Give E2 for 'no unique solution' s.c. 1: Determinant = 12, allow 'unique solution' B0 E1 E0 s.c. 2: Determinant = $\frac{1}{0}$ give maximum of B0 E1
5(i)	$\alpha + \beta + \gamma = -3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -7$, $\alpha\beta\gamma = -1$	B2 **[2]**	Minus 1 each error to minimum of 0
5(ii)	Coefficients *A*, *B* and *C* $2\alpha + 2\beta + 2\gamma = 2 \times -3 = -6 = \frac{-B}{A}$ $2\alpha \times 2\beta + 2\beta \times 2\gamma + 2\gamma \times 2\alpha = 4 \times -7 = -28 = \frac{C}{A}$ $2\alpha \times 2\beta \times 2\gamma = 8 \times -1 = -8 = \frac{-D}{A}$ $\Rightarrow x^3 + 6x^2 - 28x + 8 = 0$	M1 A3 **[4]**	Attempt to use sums and products of roots ft their coefficients, minus one each error (including '= 0' missing), to minimum of 0
	OR		
	$\omega = 2x \Rightarrow x = \frac{\omega}{2}$ $\left(\frac{\omega}{2}\right)^3 + 3\left(\frac{\omega}{2}\right)^2 - 7\left(\frac{\omega}{2}\right) + 1 = 0$ $\Rightarrow \frac{\omega^3}{8} + \frac{3\omega^2}{4} - \frac{7\omega}{2} + 1 = 0$ $\Rightarrow \omega^3 + 6\omega^2 - 28\omega + 8 = 0$	M1 A1 A1 A1 **[4]**	Attempt at substitution Correct substitution Substitute into cubic (ft) c.a.o.

6	$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$		
	$n = 1$, LHS = RHS = $\frac{1}{2}$	B1	
	Assume true for $n = k$	E1	Assuming true for k (must be explicit)
	Next term is $\frac{1}{(k+1)(k+2)}$	B1	$(k + 1)^{\text{th}}$ term seen c.a.o.
	Add to both sides $\text{RHS} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$	M1	Add to $\frac{k}{k+1}$ (ft)
	$= \frac{k(k+2)+1}{(k+1)(k+2)}$ $= \frac{k^2+2k+1}{(k+1)(k+2)}$ $= \frac{(k+1)^2}{(k+1)(k+2)}$		
	$= \frac{k+1}{k+2}$	A1	c.a.o. with correct working
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$.	E1	True for k, therefore true for $k + 1$ (dependent on $\frac{k+1}{k+2}$ seen)
	Since it is true for $k = 1$, it is true for $k = 1, 2, 3$	E1 **[7]**	Complete argument
			Section A Total: 36

Section B

7(i)	$3+x^2 \neq 0$ for any real x.	E1 **[1]**	
7(ii)	$y = -1,\ x = 2,\ x = -2$	B1, B1 B1 **[3]**	
7(iii)	Large positive x, $y \to -1^-$ (e.g. consider $x = 100$) Large negative x, $y \to -1^-$ (e.g. consider $x = -100$)	M1 B1 **[2]**	Evidence of method required From below on each side c.a.o.
7(iv)	Curve 3 branches correct Asymptotes labelled Intercept labelled  	B1 B1 B1 **[3]**	Consistent with (i) and their (ii), (iii) Consistent with (i) and their (ii), (iii) Labels may be on axes Lose 1 mark if graph not symmetrical May be written in script
7(v)	$\frac{3+x^2}{4-x^2} = -2 \Rightarrow 3+x^2 = -8+2x^2$ $\Rightarrow 11 = x^2$ $\Rightarrow x = (\pm)\sqrt{11}$ From graph, $-\sqrt{11} \le x < -2$ or $2 < x \le \sqrt{11}$	M1 A1 B1 A1 **[4]**	Reasonable attempt to solve Accept $\sqrt{11}$ $x < -2$ and $2 < x$ seen c.a.o.

8(i)	$\alpha^2 = (1+\mathrm{j})^2 = 2\mathrm{j}$ $\alpha^3 = (1+\mathrm{j})2\mathrm{j} = -2+2\mathrm{j}$ $z^3 + 3z^2 + pz + q = 0$ $\Rightarrow 2\mathrm{j} - 2 + 3\times 2\mathrm{j} + p(1+\mathrm{j}) + q = 0$ $\Rightarrow (8+p)j + p + q - 2 = 0$ $p = -8$ and $p + q - 2 = 0 \Rightarrow q = 10$	M1, A1 A1 M1 M1 A1 **[6]**	 Substitute their α^2 and α^3 into cubic Equate real and imaginary parts to 0 Results obtained correctly
8(ii)	$1-\mathrm{j}$ must also be a root. The roots must sum to –3, so the other root is z = -5	B1 M1 A1 **[3]**	 Any valid method c.a.o.
8(iii)		B2 **[2]**	Argand diagram with all three roots clearly shown; minus 1 for each error to minimum of 0 ft their real root

Section B (continued)			
9(i)	$(25,50)$	B1 **[1]**	
9(ii)	$\left(\frac{1}{2}y, y\right)$	B1, B1 **[2]**	
9(iii)	$y=6$	B1 **[1]**	
9(iv)	All such lines are parallel to the x-axis.	B1 **[1]**	Or equivalent
9(v)	All such lines are parallel to $y=2x$.	B1 **[1]**	Or equivalent
9(vi)	$\begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$	B3 **[3]**	Minus 1 each error s.c. Allow 1 for reasonable attempt but incorrect working
9(vii)	$\det\begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 1 \end{pmatrix} = 0\times1-0\times\frac{1}{2}=0$ Transformation many to one.	M1 E2 **[3]**	Attempt to show determinant = 0 or other valid argument May be awarded without previous M1 Allow E1 for 'transformation has no inverse' or other partial explanation
			Section B Total: 36
			Total: 72

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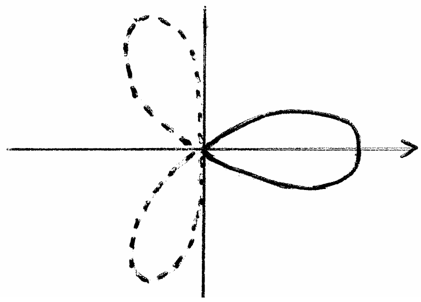
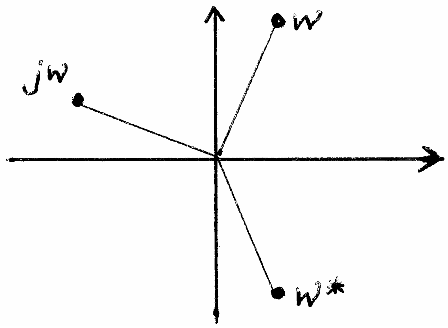
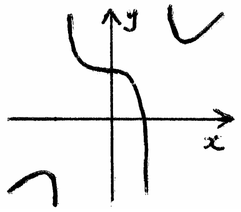
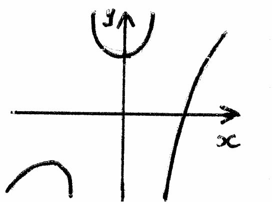
1(a)(i)		B1 B1 B1 **3**	For one loop in correct quadrant(s) For two more loops Continuous and broken lines *Dependent on previous B1B1*
(ii)	Area is $\int \frac{1}{2}r^2\,d\theta = \int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{1}{2}a^2\cos^2 3\theta\,d\theta$ $= \int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{1}{4}a^2(1+\cos 6\theta)\,d\theta$ $= \left[\frac{1}{4}a^2(\theta + \frac{1}{6}\sin 6\theta)\right]_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}$ $= \frac{1}{12}\pi a^2$	M1 A1 M1 A1 B1 **5**	For $\int \cos^2 3\theta\,d\theta$ For a correct integral expression including limits *(may be implied by later work)* For $\int \cos^2 3\theta\,d\theta = \frac{1}{2}\theta + \frac{1}{12}\sin 6\theta$ Accept $0.262a^2$
(b)	$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx = \left[\frac{1}{2}\arcsin\left(\frac{2x}{\sqrt{3}}\right)\right]_0^{\frac{3}{4}}$ $= \frac{1}{2}\arcsin\left(\frac{3}{2\sqrt{3}}\right)$ $= \frac{1}{6}\pi$	M1 A1A1 M1 A1 **5**	For arcsin For $\frac{1}{2}$ and $\frac{2x}{\sqrt{3}}$ *Dependent on previous M1*
	OR Putting $2x = \sqrt{3}\sin\theta$ M1 A1 Integral is $\int_0^{\frac{1}{3}\pi} \frac{1}{2}\,d\theta$ A1 M1 $= \frac{1}{6}\pi$ A1		For any sine substitution For $\int \frac{1}{2}\,d\theta$ For changing to limits of θ *Dependent on previous M1*
(c)	Putting $\sqrt{3}x = \tan\theta$ Integral is $\int_0^{\frac{1}{3}\pi} \frac{1}{\sec^3\theta}\left(\frac{\sec^2\theta}{\sqrt{3}}\right)d\theta$ $= \int_0^{\frac{1}{3}\pi} \frac{\cos\theta}{\sqrt{3}}\,d\theta = \left[\frac{\sin\theta}{\sqrt{3}}\right]_0^{\frac{1}{3}\pi}$ $= \frac{1}{2}$	M1 A1A1 M1 A1 **5**	For any tan substitution For $\frac{1}{(\sec^2\theta)^{\frac{3}{2}}}$ and $\frac{\sec^2\theta}{\sqrt{3}}$ Including limits of θ

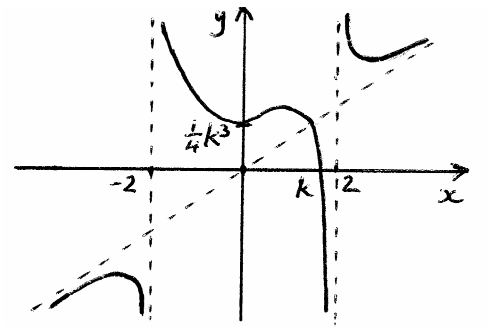
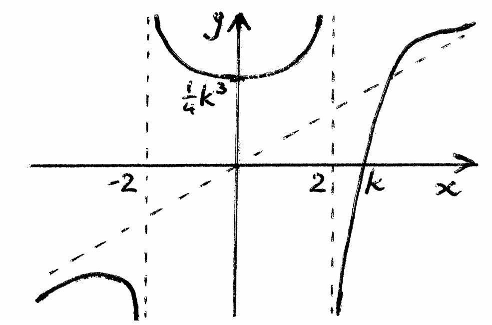
2 (i)	$\lvert w \rvert = \frac{1}{2}$, $\arg w = 3\theta$ $\lvert w^* \rvert = \frac{1}{2}$, $\arg w^* = -3\theta$ $\lvert \mathrm{j}w \rvert = \frac{1}{2}$, $\arg \mathrm{j}w = 3\theta + \frac{1}{2}\pi$ 	B1 B1 ft B1B1 ft B2	**6**	w^* and $\mathrm{j}w$ in correct positions relative to their w in first quadrant Give B1 for at least two points in correct quadrants
(ii)	$(1+w)(1+w^*) = 1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta} + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta} + (\frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta})(\frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})$ $= 1 + \frac{1}{2}(\cos 3\theta + \mathrm{j}\sin 3\theta) + \frac{1}{2}(\cos 3\theta - \mathrm{j}\sin 3\theta) + \frac{1}{4}$ $= \frac{5}{4} + \cos 3\theta$	M1 A1 M1 A1 (ag)	**4**	for $w^* = \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta}$ for $1 + \frac{1}{4}$ correctly obtained for $w = \frac{1}{2}(\cos 3\theta + \mathrm{j}\sin 3\theta)$ for $\cos 3\theta$ correctly obtained
(iii)	$C + \mathrm{j}S = \mathrm{e}^{2\mathrm{j}\theta} - \frac{1}{2}\mathrm{e}^{5\mathrm{j}\theta} + \frac{1}{4}\mathrm{e}^{8\mathrm{j}\theta} - \ldots$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}}{1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta}}$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}{(1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta})(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}{\frac{5}{4} + \cos 3\theta}$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}\theta}}{\frac{5}{4} + \cos 3\theta} \quad \left(= \dfrac{4\mathrm{e}^{2\mathrm{j}\theta} + 2\mathrm{e}^{-\mathrm{j}\theta}}{5 + 4\cos 3\theta} \right)$ $C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$ $S = \dfrac{4\sin 2\theta - 2\sin\theta}{5 + 4\cos 3\theta}$	M1 M1 A1 M1 A1 M1 A1 (ag) A1	**8**	Obtaining a geometric series Summing an infinite geometric series Using complex conjugate of denom Equating real or imaginary parts Correctly obtained

3 (i)	$(1-\lambda)[\,(-3-\lambda)(-4-\lambda)-12\,]$ $-2[\,-2(-4-\lambda)-12\,]+3[\,-4-2(-3-\lambda)\,]=0$ $(1-\lambda)(\lambda^2+7\lambda)-2(2\lambda-4)+3(2\lambda+2)=0$ $\lambda^3+6\lambda^2-9\lambda-14=0$	M1 A1 A1 (ag) **3**	Evaluating $\det(\mathbf{M}-\lambda\mathbf{I})$ *Allow one omission and two sign errors* $\det(\mathbf{M}-\lambda\mathbf{I})$ correct Correctly obtained (=0 is required)
(ii)	When $\lambda=-1$, $-1+6+9-14=0$ $(\lambda+1)(\lambda^2+5\lambda-14)=0$ $(\lambda+1)(\lambda-2)(\lambda+7)=0$ Other eigenvalues are $2,\ -7$	B1 M1 A1 **3**	or showing that $(\lambda+1)$ is a factor, and deducing that -1 is a root for $(\lambda+1)\times$ quadratic factor
(iii)	$x+2y+3z=-x$ $-2x-3y+6z=-y$ $2x+2y-4z=-z$ $z=0$, $x+y=0$ An eigenvector is $\begin{pmatrix}1\\-1\\0\end{pmatrix}$	M1 M1 A1 **3**	At least two equations Solving to obtain an eigenvector
	OR $\begin{pmatrix}2\\2\\3\end{pmatrix}\times\begin{pmatrix}-2\\-2\\6\end{pmatrix}=\begin{pmatrix}18\\-18\\0\end{pmatrix}$ M1 M1 A1		Appropriate vector product Evaluation of vector product
(iv)	$\mathbf{M}\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}6\\0\\2\end{pmatrix}=2\begin{pmatrix}3\\0\\1\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}0\\3\\-2\end{pmatrix}=\begin{pmatrix}0\\-21\\14\end{pmatrix}=-7\begin{pmatrix}0\\3\\-2\end{pmatrix}$	M1 A1A1 **3**	Any method for verifying or finding an eigenvector
(v)	$\mathbf{P}=\begin{pmatrix}1&3&0\\-1&0&3\\0&1&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}-1&0&0\\0&2&0\\0&0&-7\end{pmatrix}^3$ $=\begin{pmatrix}-1&0&0\\0&8&0\\0&0&-343\end{pmatrix}$	B1 ft M1 A1 ft **3**	seen or implied (ft) *(condone eigenvalues in wrong order)* Order must be consistent with **P** (when B1 has been awarded)
(vi)	By CHT, $\mathbf{M}^3+6\mathbf{M}^2-9\mathbf{M}-14\mathbf{I}=\mathbf{0}$ $\mathbf{M}^2+6\mathbf{M}-9\mathbf{I}-14\mathbf{M}^{-1}=\mathbf{0}$ $\mathbf{M}^{-1}=\frac{1}{14}\mathbf{M}^2+\frac{3}{7}\mathbf{M}-\frac{9}{14}\mathbf{I}$	B1 M1 A1 **3**	Condone omission of **I** Condone dividing by **M**

4 (a)	$\frac{1}{2}(e^x - e^{-x}) + 2(e^x + e^{-x}) = 8$ $5e^{2x} - 16e^x + 3 = 0$ $(5e^x - 1)(e^x - 3) = 0$ $e^x = \frac{1}{5},\ 3$ $x = -\ln 5,\quad \ln 3$	M1 M1 M1 A1A1 A1 ft **6**	Exponential form Quadratic in e^x Solving to obtain a value of e^x Exact logarithmic form from 2 positive values of e^x *Dependent on M3*
	OR $\sqrt{c^2 - 1} = 8 - 4c$ $15c^2 - 64c + 65 = 0$ M1 M1 $c = \frac{5}{3},\ \frac{13}{5}$ A1A1 $x = \pm\ln 3,\ \pm\ln 5$ M1 $x = \ln 3,\ -\ln 5$ A1		Obtaining quadratic in c (or s) $(15s^2 + 16s - 48 = 0)$ Solving to obtain a value of c (or s) or $s = \frac{4}{3},\ -\frac{12}{5}$ Logarithmic form (including $\pm$ if c) cao
(b)	$\int_0^2 \frac{1}{2}e^x(e^x - e^{-x})\,dx$ $= \left[\frac{1}{4}e^{2x} - \frac{1}{2}x\right]_0^2$ $= (\frac{1}{4}e^4 - 1) - (\frac{1}{4})$ $= \frac{1}{4}(e^4 - 5)$	M1 M1 A1 A1 **4**	Exponential form Integrating to obtain a multiple of e^{2x}
(c)(i)	$\dfrac{\frac{2}{3}}{\sqrt{1 + (\frac{2}{3}x)^2}} \quad \left(= \dfrac{2}{\sqrt{9 + 4x^2}}\right)$	B2 **2**	Give B1 for any non-zero multiple of this
(ii)	$\left[x\,\text{arsinh}(\frac{2}{3}x)\right]_0^2 - \int_0^2 \frac{2x}{\sqrt{9 + 4x^2}}\,dx$ $= \left[x\,\text{arsinh}(\frac{2}{3}x) - \frac{1}{2}\sqrt{9 + 4x^2}\right]_0^2$ $= \left(2\,\text{arsinh}(\frac{4}{3}) - \frac{5}{2}\right) - (-\frac{3}{2})$ $= 2\ln\left(\frac{4}{3} + \sqrt{1 + \frac{16}{9}}\right) - 1$ $= 2\ln 3 - 1$	M1 A1 ft B1 M1 M1 A1 (ag) **6**	Integration by parts applied to $\text{arsinh}(\frac{2}{3}x) \times 1$ for $\int \frac{x}{\sqrt{9 + 4x^2}}\,dx = \frac{1}{4}\sqrt{9 + 4x^2}$ Using both limits (provided both give non-zero values) Logarithmic form for arsinh (intermediate step required)

5 (i)	$x = 2,\quad x = -2$ $y = x + \dfrac{4x - k^3}{x^2 - 4}$ Asymptote is $y = x$	B1 M1 A1 **3**	 Dividing out or B2 for $y = x$ stated

(ii)	 $k < 2$  $k > 2$	B1 B1 B1 B1 **4**	$k < 2$ for LH and RH sections for central section, with positive intercepts on both axes $k > 2$ for LH and central sections for RH section, crossing x-axis
(iii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(x^2-4)(3x^2)-(x^3-k^3)(2x)}{(x^2-4)^2}$ $= \dfrac{x(2k^3+x^3-12x)}{(x^2-4)^2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$ When $x \approx 0$, $2k^3 + x^3 - 12x > 0$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} < 0$ when $x < 0$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} > 0$ when $x > 0$ Hence there is a minimum when $x = 0$	M1 A1 A1 (ag) M1 A1 (ag) **5**	Using quotient rule (or equivalent) Any correct form Correctly shown or evaluating $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = 0$ or $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{8}k^3 > 0$ when $x = 0$
(iv)	Curve crosses $y = x$ when $x^3 - k^3 = x(x^2-4)$ $x = \frac{1}{4}k^3$ So curve crosses this asymptote	M1 A1 (ag) **2**	

(v)	$k < 2$ 	B2		Asymptotes shown Intercepts $\frac{1}{4}k^3$ and k indicated Minimum on positive y-axis Maximum shown Give B1 for minimum and maximum on central section
	$k > 2$ 	B2	**4**	Asymptotes shown Intercepts $\frac{1}{4}k^3$ and k indicated Minimum on positive y-axis RH section crosses $y = x$ and approaches it from above Give B1 for RH section approaching both asymptotes correctly

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1(i)	$\lambda^2 + 6\lambda + 8 = 0$	M1		
	$\lambda = -2$ or -4	A1		
	CF $I = A\mathrm{e}^{-2t} + B\mathrm{e}^{-4t}$	F1		
	PI $I = a\mathrm{e}^{-t}$	B1		
	$a\mathrm{e}^{-t} - 6a\mathrm{e}^{-t} + 8a\mathrm{e}^{-t} = 6\mathrm{e}^{-t}$	M1	differentiate and substitute	
	$3a = 6$	dM1	compare	
	$a = 2$	A1		
	$I = 2\mathrm{e}^{-t} + A\mathrm{e}^{-2t} + B\mathrm{e}^{-4t}$	F1	CF + PI	
				8
(ii)	$\lambda^2 + 6\lambda + 9 = 0$	M1		
	$\lambda = -3$ (repeated)	A1		
	CF $I = (C + Dt)\mathrm{e}^{-3t}$	F1		
	PI $I = b\mathrm{e}^{-t}$	B1		
	$b\mathrm{e}^{-t} - 6b\mathrm{e}^{-t} + 8b\mathrm{e}^{-t} = 6\mathrm{e}^{-t}$	M1	substitute and compare	
	$b = \frac{3}{2}$	A1		
	$I = \frac{3}{2}\mathrm{e}^{-t} + (C + Dt)\mathrm{e}^{-3t}$	F1	CF + PI	
	$1.5 = \frac{3}{2} + C \Rightarrow C = 0$	M1	condition on I	
	$\dot{I} = -\frac{3}{2}\mathrm{e}^{-t} - 3(C + Dt)\mathrm{e}^{-3t} + D\mathrm{e}^{-3t}$	M1	differentiate	
	$0 = -\frac{3}{2} - 3C + D \Rightarrow D = \frac{3}{2}$	M1	condition on $\dot{I}$	
	$I = \frac{3}{2}\left(\mathrm{e}^{-t} + t\mathrm{e}^{-3t}\right)$	A1	cao	
	as $t \to \infty, I \to 0$	F1	recognise $\mathrm{e}^{-t} \to 0$	
				12
(iii)	$\lambda^2 + 6\lambda + k = 0 \Rightarrow \lambda = -3 \pm \sqrt{9-k}$	M1		
	$0 < k < 9 \Rightarrow \sqrt{9-k} < 3 \Rightarrow$ two negative roots			
	hence $I = A\mathrm{e}^{-\lambda_1 t} + B\mathrm{e}^{-\lambda_2 t} \to 0$	E1		
	$k > 9 \Rightarrow \lambda = -3 \pm \beta j$	M1	complex roots with negative real part (or CF)	
	$\mathrm{e}^{-3t}(A\cos\beta t + B\sin\beta t) \to 0$	E1		
				4

2(i)	$\frac{1}{1-3y}\frac{dy}{dx} = \frac{1}{x^2}$	M1	separate	
	$\int \frac{1}{1-3y}dy = \int \frac{1}{x^2}dx$	M1	integrate	
	$-\frac{1}{3}\ln\lvert 1-3y\rvert = -\frac{1}{x} + c$	A1 A1	± LHS ± RHS	
	$1-3y = Ae^{3/x}$	M1	rearrange	
	$y=0, x=1 \Rightarrow A = e^{-3}$	M1	condition	
	$y = \frac{1}{3}\left(1-\exp(\frac{3}{x}-3)\right)$	A1		
	$x=2 \Rightarrow y \approx 0.259$	A1	from correct solution	
				8
(ii)	$\frac{dy}{dx} + \frac{3}{x}y = \frac{1}{x^2}\cos x$	M1	divide	
	$I = \exp(\int \frac{3}{x}dx)$	M1	attempt integrating factor	
	$= x^3$	A1		
	$\frac{d}{dx}\left(x^3 y\right) = x\cos x$	F1	follow their I	
	$x^3 y = \int x\cos x\,dx = x\sin x - \int \sin x\,dx$	M1	integrate (by parts)	
	$= x\sin x + \cos x + B$	A1	RHS (or multiple) constant not required here	
	$y = x^{-2}\sin x + x^{-3}(\cos x + B)$	M1	divide to get y	
	$x=1, y=0 \Rightarrow B = -\cos 1 - \sin 1$	M1	use condition	
	$y = x^{-2}\sin x + x^{-3}(\cos x - \cos 1 - \sin 1)$	A1		
	$x=2 \Rightarrow y \approx 0.00258$	M1 A1	substitute $x=2$ cao	
				11
(iii)	$y' = \left(\cos x - 3x\left(y+0.1y^2\right)\right)x^{-2}$	B1	seen or implied by correct numerical value	
	x y $\dot{y}$			
	1.8 0.034411 –0.12767	M1	use algorithm	
	1.9 0.021644 –0.12380	A1	$y(1.9)$	
	2.0 0.00926(3 …)	A1	$y(2.0)$	
	Using smaller h would give greater accuracy	B1		
				5

3(i)	$F = ma \Rightarrow mv\frac{dv}{dx} = mg - 0.001mv^2$	M1	N2L (accept just *ma* for M1)	
	$\Rightarrow v\frac{dv}{dx} = g - 0.001v^2$	E1		
	down positive so weight positive and resistance negative as it opposes motion	B1		
				3

(ii)	$\int \frac{-0.002v}{g-0.001v^2}\,dv = \int -0.002\,dx$	M1 M1	separate integrate
	$\ln\left\lvert g-0.001v^2\right\rvert = -0.002x + c$	A1	LHS (or multiple)
	$v^2 = 1000\left(g - A\mathrm{e}^{-0.002x}\right)$	M1	rearrange
	$x=0, v=0 \Rightarrow A = g$	M1	use condition
	$v = \sqrt{1000g\left(1-\mathrm{e}^{-0.002x}\right)}$	A1	cao
	$x = 50 \Rightarrow v = 30.54$	E1	must follow correct work
			7
(iii)	$mv\frac{dv}{dx} = mg - 2mv$	M1	
	$v\frac{dv}{dx} = g - 2v$	A1	
	$\int \frac{v}{g-2v}\,dv = \int dx$	M1	separate
	$\frac{1}{2}\int\left(-1+\frac{g}{g-2v}\right)dv = x + c$	A1 M1	$x+c$ attempt to integrate LHS
	$-\frac{1}{2}v - \frac{1}{4}g\ln\lvert g-2v\rvert = x + c$	A1	
	$x = 50, v = 30.54 \Rightarrow c = -74.91...$	M1	use condition (correct value of v at least)
	$v = 5 \Rightarrow x = 76.36$	M1	
	so 26.4 m deep	A1	
			9
(iv)	terminal velocity when acceleration zero $\Rightarrow v = 4.9$	M1 F1	 follow their DE
		B1 B1 B1	increasing from (0,0) decreasing to asymptote at 4.9 (or follow their value) cusp/max at (50, 30.54) (both coordinates shown)
			5

4(i)	$\ddot{x} = \dot{x} - 2\dot{y} + \cos t$	M1	differentiate	
	$= \dot{x} - 2(4x - 3y + \cos t) + \cos t$	M1	substitute	
	$= \dot{x} - 8x + 6y - \cos t$			
	$y = \frac{1}{2}(x + \sin t - \dot{x})$	M1	y in terms of $x, \dot{x}$	
	$\ddot{x} = \dot{x} - 8x + 3(x + \sin t - \dot{x}) - \cos t$	M1	substitute	
	$\ddot{x} + 2\dot{x} + 5x = 3\sin t - \cos t$	E1		
				5
(ii)	$\lambda^2 + 2\lambda + 5 = 0$	M1	auxiliary equation	
	$\lambda = -1 \pm 2j$	M1	solve to get complex roots	
		A1		
	CF $x = e^{-t}\left(A\cos 2t + B\sin 2t\right)$	F1	CF for their roots (for complex roots must be in exp/trig form, not complex exponentials)	
	PI $x = a\sin t + b\cos t$	B1		
	$\dot{x} = a\cos t - b\sin t, \ddot{x} = -a\sin t - b\cos t$	M1	differentiate twice and substitute	
	$-a - 2b + 5a = 3$	M1	compare	
	$-b + 2a + 5b = -1$	M1	solve	
	$a = \frac{1}{2}, b = -\frac{1}{2}$	A1		
	$x = e^{-t}\left(A\cos 2t + B\sin 2t\right) + \frac{1}{2}\left(\sin t - \cos t\right)$	F1	CF + PI	
				10
(iii)	$y = \frac{1}{2}\left(x + \sin t - \dot{x}\right)$	M1	y in terms of $x, \dot{x}$	
	$\dot{x} = -e^{-t}\left(A\cos 2t + B\sin 2t\right) + e^{-t}\left(-2A\sin 2t + 2B\cos\right.$			
	$+\frac{1}{2}\left(\cos t + \sin t\right)$	M1	differentiate x	
	$y = e^{-t}\left((A-B)\cos 2t + (A+B)\sin 2t\right) + \frac{1}{2}\left(\sin t - \cos t\right)$	M1	substitute for $x, \dot{x}$	
		A1	CF part	
		A1	PI part	
				5
(iv)	$x \sim \frac{1}{2}\left(\sin t - \cos t\right)$	F1		
	$y \sim \frac{1}{2}\left(\sin t - \cos t\right)$	F1		
	hence for large $t, x \approx y$	B1	must follow correctly from their solutions	
	but unless $A = B = 0, \quad A - B \neq A$ or $A + B \neq B$ so $x \neq y$	B1	must follow correctly from their solutions	
				4

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Section A

Q 1		mark		Sub
(i)	$\frac{-15}{6} = -2.5$ so -2.5 m s^{-2}	M1 A1	Use of $\Delta v / \Delta t$. Condone use of v/t. Must have - ve sign. Accept no units.	2
(ii)	$\frac{1}{2} \times 10 \times 4 = 20$ m	M1 A1	Attempt at area or equivalent	2
(iii)	Area under graph is $\frac{1}{2} \times 5 \times 5 = 12.5$ (and -ve) closest is $20 - 12.5 = 7.5$ m	M1 A1	May be implied. Area from 4 to 9 attempted. Condone missing –ve sign. Do not award if area beyond 9 is used (as well). cao	2
				6

Q 2		mark		Sub
(i)	Pulley is smooth (and the string is light)	E1	Only require pulley is smooth. Do not accept only 'string is light'.	1
(ii)	$4g = 39.2$ N	B1	Accept either	1
(iii)	Let tension in each string be T $39.2 = 2T\cos 20$ $T = 20.85788\ldots$ so 20.9 N (3 s.f.)	M1 B1 F1	Equating 39.2 to attempt at tensions in **both** BC and BD. Tensions need not be equal. No extra forces. Must attempt resolution. Condone $\sin \leftrightarrow \cos$. For one occurrence of $T\cos 20$ in any equation. Accept reference to only one string. FT **their** $4g$ If Lami's Theorem used: M1 correct format B1 equation correct. FT **their** $4g$ F1 FT **their** $4g$ If Triangle of Forces used: M1 triangle with **their** $4g$ labelled **and** an attempt to use this triangle. Ignore arrows. B1 for correct equation. FT **their** $4g$. F1 FT **their** $4g$.	3
				5

Q 3		mark		Sub
(i)	$\lvert\mathbf{F}\rvert = 12.5$ so 12.5 N bearing is $90 - \arctan\frac{12}{3.5}$ = (0)16.260… so (0)16.3° (3 s. f.)	B1 M1 A1	 Use of arctan with 3.5 and 12 or equiv May be obtained directly as $\arctan\frac{3.5}{12}$	3
(ii)	24/7 = 12/3.5 or …. **G** = 2**F** so $\lvert\mathbf{G}\rvert = 2\lvert\mathbf{F}\rvert$	E1 B1	Accept **statement** following **G** = 2**F** shown. Accept equivalent in words.	2
(iii)	$\frac{9+12}{3.5} = \frac{-18+q}{12}$ so $q = 6\times12+18 = 90$	M1 A1	Or equivalent or in scalar equations. Accept $\frac{21}{q-18}$ or $\frac{q-18}{21}$ = tan (i) or tan(90 - (i)) Accept 90**j**	2
				7

Q 4		mark		Sub
(i)	N2L in direction of motion $D-(100+300)=(900+700)\times1.5$ D = 2800 so 2800 N	M1 A1 A1	Apply N2L. Allow 1 resistance omitted and sign error but total mass must be used. Condone use of $F = mga$. No extra forces. All correct cao	3
(ii)	N2L on trailer $T-300=700\times1.5$ T =1350 so 1350 N	M1 A1	Use either car or trailer. All forces present. No extras. Correct mass and a Allow sign error. Must use $F = ma$. cao	2
				5

Q 5		mark		Sub
(i)	$9\mathbf{i}$ m s^{-2}; $(9\mathbf{i} - 12\mathbf{j})$ m s^{-2}	B1	Award for either. Accept no units. (isw e.g. finding magnitudes)	1
(ii)	N2L $\mathbf{F} = 4\,(9\mathbf{i} - 12\mathbf{j}) = (36\mathbf{i}\ -48\,\mathbf{j})$ N	B1	Accept factored form. isw. FT **a**(3). Accept 60 N or **their** $4\lvert\mathbf{a}\rvert$	1
(iii)	$\mathbf{v} = \int \begin{pmatrix} 9 \\ -4t \end{pmatrix} dt = \begin{pmatrix} 9t + C \\ -2t^2 + D \end{pmatrix}$ Using $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$ when $t = 1$ $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 + C \\ -2 + D \end{pmatrix}$ $\Rightarrow C = -5, D = 4$ so $\mathbf{v} = (9t - 5)\mathbf{i} + (4 - 2t^2)\mathbf{j}$	M1 A1 M1 A1	Integration. At least one term correct. Neglect arbitrary constant(s) Sub at $t = 1$ to find arb const(s) Any form	4
				6

Q 6		mark		Sub
(i)	$14 = 2u + 0.5a \times 4$ $19 = u + 5a$ Solving gives $u = 4$ and $a = 3$	M1 A1 A1 M1 F1	Use of appropriate *uvast* for either equn Any form Any form Attempt at solution of 2 equns in 2 unknowns. At least one value found . Must have complete correct solution to **their** equns. .	5
(ii)	$19^2 = 4^2 + 2 \times 3 \times s$ or $s = 4 \times 5 + 0.5 \times 3 \times 25$ $s =$ 57.5 so 57.5 m	M1 A1	Use of appropriate *uvast* and **their** u, a & $t = 5$. cao [Accept 50 if $t = 7$ instead of $t = 5$ in (i) for 2/2]	2
				7

Section B

Q 7		mark		Sub
(i)	60 N	B1		1
(ii)	$60 + 70\cos 30 = 120.62...$ so 121 N (3 s. f.)	M1 A1	70 cos30 or 70 sin 30 used only with 60N. Accept sign errors. cao. Any reasonable accuracy	2
(iii)	resolve ↑ $R + 70\sin 30 - 50g = 0$ $R = 455$ so 455 N	 M1 A1 A1	Resolve ↑ All forces present. No extras. Allow sign errors and $\sin \leftrightarrow \cos$. All correct. cao	3
(iv)	N2L → $160 - 125 = 50a$ $a = 0.7$ so 0.7 m s^{-2}	 M1 A1	N2l. No extra forces. Accept 125 N omitted but not use of $F = mga$	2
(v)	N2L → $-125 = 50a$ $a = -2.5$ $0 = 1.5^2 + 2 \times -2.5 \times s$ $s = 0.45$ so 0.45 m	 M1 A1 M1 A1	N2L to find new accn. Accept +125 but not $F = mga$. May be implied. Accept +2.5 Appropriate (sequence of) *uvast* using a new value for acceln. Allow use of ± **their new** *a* cao. Signs must be justified.	4
(vi)	N2L → $160 + Q\cos 30 - 115 = 50 \times 3$ $Q = 121.24...$ so 121 (3 s. f.)	 M1 B1 A1 A1	Use of N2L with cpt of Q attempted. Accept 115 omitted or taken to be 125 and *a* wrong. Do not allow $F = mga$. Qcos30 seen in any equn. All correct cao	4
				16

Q 8		mark		Sub
(i)	$x = 14\cos 60t$	M1	Consider motion in *x* direction. Need not resolve. Allow $\sin \leftrightarrow \cos$. Condone +1 seen.	
	so $x = 7t$	A1	Need not be simplified.	
	$y = 14\sin 60t - 4.9t^2 + 1$	M1	Suitable *uvast* used for *y* with *g* $= \pm 9.8, \pm 10, \pm 9.81$ soi. Need not resolve. Allow $\sin \leftrightarrow \cos$.	
		A1	Allow + 1omitted. Any form and 2 s. f. Need not be simplified	
	$y = 7\sqrt{3}t - 4.9t^2 + 1$ ($y = 12.124...t - 4.9t^2 + 1$)	A1	All correct. +1 need not be justified. Accept any form and 2 s. f. Need not be simplified.	5
(ii) (A)	time taken to reach highest point			
	$0 = 7\sqrt{3} - 9.8T$	M1	Appropriate *uvast* . Accept *u* = 14 and $\sin \leftrightarrow \cos$ and $u \leftrightarrow v$. Require *v* = 0 or equivalent. $g = \pm 9.8, \pm 10, \pm 9.81$ soi.	
	so $\frac{5\sqrt{3}}{7}$ s (1.23717…. = 1.24 s (3 s. f.))	A1	cao [If time of flight attempted, do not award M1 if twice interval obtained]	2
(B)	distance from base is $7 \times \frac{5\sqrt{3}}{7} = 5\sqrt{3}$ m	M1	Use of **their** *x* = 7*t* with **their** *T*	
	(= 8.66025… so 8.66 m (3 s. f.))	B1	FT **their** *T* only in *x* = 7*t*. Accept values rounding to 8.6 and 8.7.	2
(C)	**either** Height at this time is $H = 7\sqrt{3} \times \frac{5\sqrt{3}}{7} - 4.9 \times \left(\frac{5\sqrt{3}}{7}\right)^2 + 1$	M1	Subst in **their quadratic** *y* with **their** *T*.	
		A1	Correct subst of **their** *T* in **their** *y* which has attempts at all 3 terms. Do not accept *u* = 14.	
	= 8.5	A1		
	clearance is $8.5 - 6 = 2.5$ m	E1	Clearly shown.	
	or for height above pt of projection $0 = \left(7\sqrt{3}\right)^2 + 2 \times -9.8 \times s$	M1	Appropriate *uvast* . Accept *u* = 14. $g = \pm 9.8, \pm 10, \pm 9.81$ soi	
		A1	Attempt at vert cpt accept $\sin \leftrightarrow \cos$ .Accept sign errors but not *u* = 14.	
	$s = 7.5$	A1		
	so clearance is $7.5 - 5 = 2.5$ m	E1	Clearly shown.	4
(iii)	See over			

Q 8	continued	mark		sub
(iii)	Elim t between $y = 7\sqrt{3}t - 4.9t^2 + 1$ and $x = 7t$ so $y = 7\sqrt{3}\frac{x}{7} - 4.9\left(\frac{x}{7}\right)^2 + 1$ so $y = \sqrt{3}x - 0.1x^2 + 1$	M1 F1	Must see **their** $t = x/7$ fully substituted in **their** quadratic y (accept bracket errors) Accept any form correctly written. FT **their** x and 3 term quadratic y (neither using $u = 14$)	2
(iv)	**either** need $6 = 7\sqrt{3}t - 4.9t^2 + 1$ so $4.9t^2 - 7\sqrt{3}t + 5 = 0$ $t = \frac{5(\sqrt{3} \pm 1)}{7}$ (0.52289…. or 1.95146…) moves by $\left(\frac{5(\sqrt{3}+1)}{7} - \frac{5\sqrt{3}}{7}\right) \times 7$ [(1.95146.. – 1.23717…)×7] = 5 m	M1 M1 A1 M1 A1	**their** quadratic y from (i) = 6, or equivalent. Dep. Attempt to solve this 3 term quadratic. (Allow $u = 14$). for either root Moves by $\lvert$**their** root - **their** (ii)(A)$\rvert \times 7$ or equivalent. Award this for recognition of correct dist (no calc) cao [If new distance to wall found must have larger of 2 +ve roots for 3rd M and award max 4/5 for 13.66]	
	or using equation of trajectory with $y = 6$ $6 = \sqrt{3}x - 0.1x^2 + 1$ Solving $x^2 - 10\sqrt{3}x + 50 = 0$ $x = 5(\sqrt{3} \pm 1)$ (13.660… or 3.6602….) distance is $5(\sqrt{3}+1) - 5\sqrt{3}$ = 5 m	 M1 M1 A1 M1 A1	Equating **their** quadratic trajectory equn to 6 Dep. Attempt to solve this 3 term quadratic. (Allow $u = 14$). for either root distance is $\lvert$**their** root $-$ **their**(ii)(B)$\rvert$ Award this for recognition of correct dist (no calc) Cao [If new distance to wall found must have larger of 2 + ve roots for 3rd M and award max 4/5 for 13.66]	5
				20

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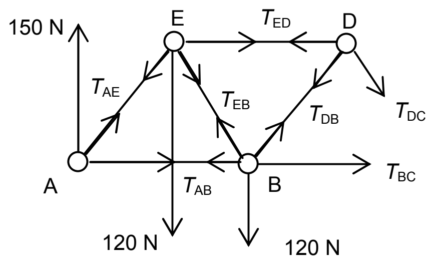
Q 1		mark		Sub
(i)	$16 = 0.4v$ so 40 m s^{-1}	M1 A1	Use of $I = \Delta mv$	2
(ii)	PCLM ↑ + ve $0.4\times 32 - 0.6u = 0.4v_P + 0.6\times 4$ NEL ↑ +ve $\frac{4 - v_P}{-u - 32} = -0.1$ Solving $u = 18$ $v_P = -1$ so 1 m s^{-1} downwards	M1 A1 M1 A1 E1 A1 A1	Use of PCLM Any form Use of NEL. Allow sign errors. Any form Must be obtained from a pair of correct equations. If given u = 18 used then v_P = -1 must be obtained from 1 equation and both values tested in the second equation cao. Accept use of given u = 18 cao	7
(iii)	Considering the momenta involved $0.5\begin{pmatrix}-3.6\\5.2\end{pmatrix} = 0.2\begin{pmatrix}3\\4\end{pmatrix} + 0.3\mathbf{v}_D$ $\mathbf{v}_D = \begin{pmatrix}-8\\6\end{pmatrix}$ so $a = -8$ and $b = 6$ Gradients of the lines are $\frac{4}{3}$ and $\frac{6}{-8}$ Since $\frac{4}{3}\times\frac{6}{-8} = -1$, they are at 90°	M1 B1 B1 A1 A1 A1 M1 E1	PCLM applied. May be implied. LHS momentum of C correct Complete equation. Accept sign error. cao cao Any method for the angle Clearly shown	8
				17

Q 2		mark		Sub
(i)	Moments about C $240 \times 2 = 3R_D$ $R_D = 160$ so 160 N Resolve vertically $R_C + R_D = 240$ $R_C = 80$ so 80 N	M1 A1 M1 F1	Moments about C or equivalent. Allow 1 force omitted Resolve vertically or moments about D or equivalent. All forces present. FT from **their** R_D only	4
(ii) (A)	Moments about D $240 \times 1 = 4T \sin 40$ $T = 93.343\ldots$ so 93.3 N (3 s. f.)	M1 M1 A1 A1	Moments about D or equivalent Attempt at resolution for RHS RHS correct	4
(ii) (B)	In equilibrium so horizontal force needed to balance cpt of T. This must be friction and cannot be at C.		Need reference to horizontal force that must come from friction at D.	1
(iii) (A)	Moments about B $3 \times 240 \times \cos 30 = 6P$ $P = 60\sqrt{3}$ (103.92…..) P inclined at 30° to vertical Resolve horizontally. Friction force F $F = P \sin 30$ so $F = 30\sqrt{3}$ (51.961…)	M1 E1 B1 M1 A1	All terms present, no extras. Any resolution required attempted. Accept decimal equivalent Seen or equivalent or implied in (iii) (A) or (B). Resolve horizontally. Any resolution required attempted Any form	5

(iii) (B)	Resolve vertically. Normal reaction R $P\cos 30 + R = 240$	M1 A1	Resolve vertically. All terms present.and resolution attempted	
	Using $F = \mu R$ $\mu = \dfrac{30\sqrt{3}}{240 - 60\sqrt{3} \times \dfrac{\sqrt{3}}{2}}$	M1 A1	Substitute **their expressions** for F and R	
	$= \dfrac{30\sqrt{3}}{240-90} = \dfrac{\sqrt{3}}{5}$ = 0.34641 so 0.346 (3 s. f.)	A1	cao. Any form. Accept 2 s. f. or better	5
				19

Q 3		mark		Sub
(a)(i)	$80\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 48\begin{pmatrix}6\\2\end{pmatrix} + 12\begin{pmatrix}1\\-3\end{pmatrix} + 20\begin{pmatrix}11\\9\end{pmatrix}$	M1	Correct method for c.m.	
	$80\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}520\\240\end{pmatrix}$	B1	Total mass correct	
		B1	One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$\bar{x} = 6.5$	E1		
	$\bar{y} = 3$	A1	cao	5
(ii)	Consider x coordinate $520 = 76 \times 6.4 + 4x$	M1	Using additive principle o. e. on x cpts	
		B1	Areas correct. Allow FT from masses from (i)	
	so $x = 8.4$	A1	cao	3
(iii)	y coordinate is 1 so we need	B1	Position of centre of square	
	$240 = 76\bar{y} + 4 \times 1$ and $\bar{y} = 3.10526...$	M1		
	so 3.11 (3 s. f.)	A1	cao	3
(b)(i)	Moments about C $4R = 120 \times 3 + 120 \times 2$	M1	Moments equation. All terms present	
	so 4 R = 600 and R = 150	E1		2
(ii)		B1		
	A↑ $150 + T_{AE}\cos 30 = 0$	M1	Equilibrium at a pin-joint	
	$T_{AE} = -100\sqrt{3}$ so $100\sqrt{3}$ N (C)	A1	Any form. Sign correct. Neglect (C)	
	E↓ $120 + T_{AE}\cos 30 + T_{EB}\cos 30 = 0$	M1	Equilibrium at E, all terms present	
	$T_{EB} = 20\sqrt{3}$ so $20\sqrt{3}$ N (T)	F1	Any form. Sign follows working. Neglect (T).	
		F1	T/C consistent with answers	6
(iii)	Consider → at E, using (ii) gives ED as thrust	E1	Clearly explained. Accept 'thrust' correctly deduced from wrong answers to (ii).	1
				20

Q 4		mark		Sub
(i)	$\dfrac{0.5\times 20\times 8^2 - 0.5\times 20\times 5^2 + 510}{6}$	M1	Use of P = WD/t	
		B1	ΔKE. Accept ± 390 soi	
		A1	All correct including signs	
	= 150 W	A1		4
(ii) (A)	$20g\times\frac{3}{5}x - 5gx$	M1	Use of mgh on both terms	
		B1	Either term (neglecting signs)	
	$7gx$ ($68.6x$) gain	A1	$\pm 7gx$ in any form.	
		A1	cao	4
(B)	$11gx$	B1		1
(C)	$0.5\times 25\times 4^2 = 7gx + 11gx = 18gx$	M1	Use of work-energy equation. Allow 1 RHS term omitted.	
		B1	KE term correct	
	x = 1.13378… so 1.13 m (3 s. f.)	A1	cao. Except follow wrong sign for $7gx$ only.	3
(iii)	**either**			
	$0.5\times 35\times v^2 - 0.5\times 35\times 16$	M1	Use of work-energy. KE, GPE and WD against friction terms present.	
	$= 15g\times 0.5 - 11g\times 0.5 - 12g\times 0.5$	B1	ΔGPE correct inc sign ($1.5g$ J loss)	
		A1	All correct	
	$v^2 = 13.76$ so v = 3.70944… so 3.71 m s^{-1} (3 s. f.)	A1	cao	
	or			
	$15g - T = 15a \qquad T - 12g - 11g = 20a$	M1	N2L in 1 or 2 equations. All terms present	
	so $a = -2.24$	A1	cao	
	$v^2 = 4^2 + 2\times(-2.24)\times 0.5$	M1	Use of appropriate (sequence of) *uvast*	
	so 3.71 m s^{-1} (3 s. f.)	A1	cao	4
				16

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1(a)(i)	$\mathrm{M\,L\,T^{-2}}$	B1 **1**	Allow $\mathrm{kg\,m\,s^{-2}}$
(ii)	$(\mathrm{T}) = (\mathrm{M\,L\,T^{-2}})^{\alpha}\,(\mathrm{L})^{\beta}\,(\mathrm{M\,L^{-1}})^{\gamma}$ Powers of M: $\alpha+\gamma=0$ of L: $\alpha+\beta-\gamma=0$ of T: $-2\alpha=1$ $\alpha=-\frac{1}{2}$, $\beta=1$, $\gamma=\frac{1}{2}$	B1 M1 M2 A2 **6**	For $\mathrm{M\,L^{-1}}$ For three equations Give M1 for one equation Give A1 for one correct
(iii)	$kF_1^{\ \alpha}l_1^{\ \beta}\sigma^{\gamma} = kF_2^{\ \alpha}l_2^{\ \beta}\sigma^{\gamma}$ $F_1^{-\frac{1}{2}}\,l_1 = F_2^{-\frac{1}{2}}\,l_2$	M1 A1	Equation relating F_1, F_2, l_1, l_2
	OR $F^{\alpha}l^{\beta}$ is constant M1 F is proportional to l^2 A1		or equivalent
	$F_2 = 90\times\dfrac{2.0^2}{1.2^2}$ $= 250$ (N)	M1 A1 **4**	
(b)(i)	$\dfrac{2\pi}{\omega}=0.01$ $\omega=200\pi$ Maximum speed is $A\omega = 0.018\times 200\pi$ $= 11.3$ $(\mathrm{m\,s^{-1}})$	B1 M1 A1 **3**	 Accept 3.6π
(ii)	Using $v^2=\omega^2(A^2-x^2)$ $8^2=(200\pi)^2(0.018^2-x^2)$ $x=0.0127$ (m)	M1 M1 A1 A1 **4**	 Substituting values
	OR $v=3.6\pi\cos(200\pi t)=8$ M1 when $200\pi t=0.785$ A1 $(t=0.001249)$ $x=0.018\sin(200\pi t)=0.018\sin(0.785)$ M1 $=0.0127$ A1		*Condone the use of degrees in this part*

2 (a)	$\omega = \dfrac{2\pi}{2.4\times10^6}$ $(=2.618\times10^{-6})$ Acceleration $a = r\omega^2$ (or $\dfrac{v^2}{r}$) $= 2.604\times10^{-3}$ Force is $ma = 7.5\times10^{22}\times2.604\times10^{-3}$ $=1.95\times10^{20}$ (N)	B1 M1 M1 A1 **4**	or $v = \dfrac{2\pi\times3.8\times10^8}{2.4\times10^6}$ $(=994.8)$ M0 for $F - mg = ma$ etc Accept 1.9×10^{20} or 2.0×10^{20}
(b)(i)	Change in PE is $mg(3.5-4\sin\theta)$ By conservation of energy $\frac{1}{2}mv^2 = mg(3.5-4\sin\theta)$ $v^2 = 68.6 - 78.4\sin\theta$	B1 M1 A1 **3**	or as separate terms Accept $7g - 8g\sin\theta$
(ii)	$0.2\times9.8\sin\theta - R = 0.2\times\dfrac{v^2}{4}$ $1.96\sin\theta - R = 0.05(68.6-78.4\sin\theta)$ $R = 5.88\sin\theta - 3.43$	M1 M1 A1 E1 **4**	Radial equation of motion (3 terms) Substituting from part (i) Correctly obtained
(iii)	When $\theta = 40°$, $v^2 = 18.21$ Radial acceleration is $\dfrac{v^2}{4} = 4.55$ (m s^{-2}) Tangential acceleration is $9.8\cos40$ $=7.51$ (m s^{-2})	M1 A1 M1 A1 **4**	or $0.2g\sin40 - R = ma$ Accept 4.5 or 4.6 M0 for $a = mg\cos40$ etc
(iv)	Leaves surface when $R=0$ $\sin\theta = \dfrac{3.43}{5.88}$ $\theta = 35.7°$	M1 M1 A1 cao **3**	Accept $36°$, 0.62 rad

3 (i)	$\frac{\lambda}{15} \times 0.8 = 12 \times 9.8$ $\lambda = 2205$ (N)	M1 E1 **2**	
(ii)	$\frac{2205}{15} \times 5 - 12 \times 9.8 = 12a$ $a = 51.45$ $(\mathrm{m\,s^{-2}})$	M1 A1 A1 **3**	Equation of motion including tension Accept 51 or 52
(iii)	Loss of EE is $\frac{1}{2} \times \frac{2205}{15} \times 5^2$ $(= 1837.5)$	M1 A1	Calculating elastic energy
	By conservation of energy $12 \times 9.8 \times h = 1837.5$ $h = 15.625$ $\mathrm{OA} = 20 - h = 4.375$ (m)	M1 F1 A1 **5**	Equation involving EE and PE
	OR $12 \times 9.8 \times 5 + \frac{1}{2} \times 12 \times v^2 = 1837.5$ M1 $v^2 = 208.25$ $0 = 208.25 - 2 \times 9.8 \times H$ F1 $H = 10.625$ $\mathrm{OA} = 15 - H = 4.375$ (m) A1		Equation involving EE, PE and KE
(iv)	$T = \frac{2205}{15}(0.8 + x)$ $12 \times 9.8 - \frac{2205}{15}(0.8 + x) = 12\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -12.25x$	B1 M1 A1 E1 **4**	or $T = \frac{\lambda}{l}(x_0 + x)$ Equation of motion with three terms or $mg - \frac{\lambda}{l}(x_0 + x) = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ provided that $mg = \frac{\lambda}{l}x_0$ appears somewhere Correctly obtained *No marks for just writing* $-\frac{2205}{15}x = 12\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ *or just using the formula* $\omega^2 = \frac{\lambda}{ml}$ *If x is clearly measured upwards, treat as a mis-read*
(v)	$x = 4.2\cos(3.5t)$ Rope becomes slack when $x = -0.8$ $4.2\cos(3.5t) = -0.8$ $t = 0.504$ (s)	M1 A1 M1 A1 **4**	For $\cos(\sqrt{12.25}\,t)$ or $\sin(\sqrt{12.25}\,t)$ Accept 0.50 or 0.51

4 (i)	$\int y\,dx = \int_0^2 (4-x^2)\,dx \quad = \left[4x - \frac{1}{3}x^3\right]_0^2 \quad (=\frac{16}{3})$	B1	
	$\int xy\,dx = \int_0^2 x(4-x^2)\,dx$	M1	
	$= \left[2x^2 - \frac{1}{4}x^4\right]_0^2 \quad (=4)$	A1	
	$\bar{x} = \frac{4}{\frac{16}{3}}$	M1	
	$= 0.75$	E1	Correctly obtained
	$\int \frac{1}{2}y^2\,dx = \int_0^2 \frac{1}{2}(16 - 8x^2 + x^4)\,dx$	M1	
	$= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5\right]_0^2 \quad (=\frac{128}{15})$	A1	
	OR $\int yx\,dy = \int_0^4 y\sqrt{4-y}\,dy$ M1		Valid method of integration
	$= \left[-\frac{2}{3}y(4-y)^{\frac{3}{2}} - \frac{4}{15}(4-y)^{\frac{5}{2}}\right]_0^4$ A1		or $\left[-\frac{8}{3}(4-y)^{\frac{3}{2}} + \frac{2}{5}(4-y)^{\frac{5}{2}}\right]_0^4$
	$\bar{y} = \frac{\frac{128}{15}}{\frac{16}{3}}$	M1	
	$= 1.6$	E1 **9**	Correctly obtained *SR* If $\frac{1}{2}$ is omitted, marks for $\bar{y}$ are M1A0M0E0
(ii)	$\bar{x} = \frac{12\times 0 + 6.5\times 0.75 + 6.5\times 2.75}{12 + 6.5 + 6.5}$	M1 M1	For $6.5\times 0.75 + 6.5\times 2.75$ Using $\left(\sum m\right)\bar{x} = \sum mx$
	$= \frac{22.75}{25} = 0.91$	A1	
	$\bar{y} = \frac{12\times 0 + 6.5\times 1.6 + 6.5\times 1.6}{25}$	M1	Using $\left(\sum m\right)\bar{y} = \sum my$
	$= \frac{20.8}{25} = 0.832$	A1 **5**	
(iii)		M1 M1	For CM vertically below A For trig in a triangle containing θ, or finding the gradient of AG
	$\tan\theta = \frac{2-0.91}{4-0.832} \quad (=\frac{1.09}{3.168})$	A1	Correct expression for $\tan\theta$ or $\tan(90-\theta)$
	$\theta = 19.0°$	A1 **4**	Accept 0.33 rad

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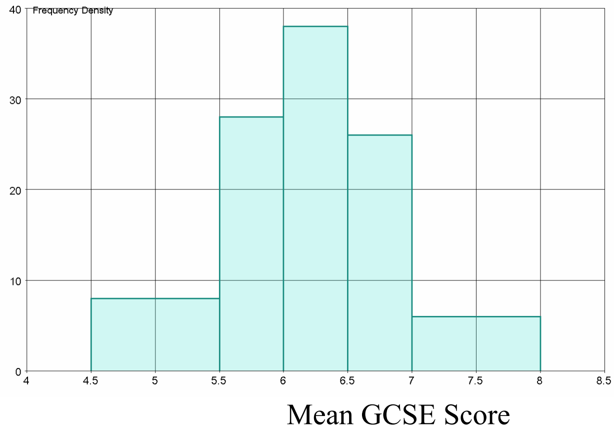
Q 1 **(i)**	The range = 55 – 15 = 40 The interquartile range = 35 – 26 = 9	B1 CAO B1 CAO	**2**
(ii)	35 + 1.5 x 9 = 48.5 26 – 1.5 x 9 = 12.5 Any value > 48.5 is an outlier (so 55 will be an outlier),	M1 for 48.5 oe M1 for 12.5 oe A1 (FT their IQR in (i))	**3**
(iii)	One valid comment such as eg: Positively skewed Middle 50% of data is closely bunched	E1	**1**
		TOTAL	**6**
2 **(i)**	Impossible because if 3 letters are correct, the fourth must be also.	E1	**1**
(ii)	There is only one way to place letters correctly. There are 4! = 24 ways to arrange 4 letters. OR: $\frac{1}{4}$ x $\frac{1}{3}$ x $\frac{1}{2}$ NOTE: **ANSWER GIVEN**	E1 E1 B1 for $\frac{1}{4}$ x $\frac{1}{3}$ B1 for x $\frac{1}{2}$	**2**
(iii)	E(X) = 1 x $\frac{1}{3}$ + 2 x $\frac{1}{4}$ + 4 x $\frac{1}{24}$ = 1 E(X^2) = 1 x $\frac{1}{3}$ + 4 x $\frac{1}{4}$ + 16 x $\frac{1}{24}$ = 2 So Var(X) = 2 – 1^2 = 1	M1 For $\sum xp$ (at least 2 non-zero terms correct) A1 CAO M1 for $\sum \boldsymbol{x}^2 \boldsymbol{p}$ (at least 2 non-zero terms correct) M1*dep* for – their E(X)² A1 FT their E(X) provided Var(X) > 0	**5**
		TOTAL	**8**

3 **(i)**	$X \sim B(10,0.2)$ $P(X < 4) = P(X \le 3) = 0.8791$ OR attempt to sum $P(X = 0,1,2,3)$ using $X \sim B(10,0.2)$ can score M1, A1	M1 for $X \le 3$ A1	**2**
(ii)	Let p = the probability that a bowl is imperfect $H_0 : p = 0.2 \quad H_1 : p < 0.2$ $X \sim B(20,0.2)$ $P(X \le 3) = 0.2061$ $0.2061 > 5\%$ Cannot reject H_0 and so insufficient evidence to claim a reduction. OR using critical region method: CR is {0} B1, 2 not in CR M1, A1 as above	B1 Definition of p B1, B1 B1 for 0.2061 seen M1 for this comparison A1 *dep* for comment <u>in context</u>	**3** **3**
		TOTAL	**8**
4 **(i)**	The company could increase the mean weight. The company could decrease the standard deviation.	B1 CAO B1	**2**
(ii)	Sample mean = 11409/25 = 456.36 $S_{xx} = 5206937 - \frac{11409^2}{25} = 325.76$ Sample s.d = $\sqrt{\frac{325.76}{24}}$ =3.68	B1 M1 for S_{xx} A1	**3**
		TOTAL	**5**
5 **(i)**	P(A ∩ B) = 0.4	B1 CAO	**1**
(ii)	P(C U D) = 0.6	B1 CAO	**1**
(iii)	Events B and C are mutually exclusive.	B1 CAO	**1**
(iv)	P(B) = 0.6, P(D) = 0.4 and P(B ∩ D) = 0.2 0.6 x 0.4 $\neq$ 0.2 (so B and D not independent)	B1 for P(B ∩ D) = 0.2 soi E1	**2**
		TOTAL	**5**
6 **(i)**	Number of selections = $\binom{12}{7}$ = 792	M1 for $\binom{12}{7}$ A1 CAO	**2**
(ii)	Number of arrangements = 7! = 5040	M1 for 7!, A1 CAO	**2**
		TOTAL	**4**

<table>
<tr><td>7
(i)</td><td>Mean score = (2x8 + 3x7 + 4x6 + 5 + 4)/11 =

6.36</td><td>M1 for $\sum fx/11$
A1 CAO</td><td>2</td></tr>
<tr><td>(ii)</td><td>
</td><td>G1 Linear sensible scales

G1 fds of 8, 28, 38, 26, 6 or 4k, 14k, 19k, 13k, 3k for sensible values of k either on script or on graph.

G1 (dep on reasonable attempt at fd) Appropriate label for vertical scale eg ‘Frequency density’, ‘frequency per ½ unit’, ‘students per mean GCSE score’. (allow Key)</td><td>3</td></tr>
<tr><td>(iii)</td><td><table>
<tr><td>Mid point, x</td><td>f</td><td>fx</td><td>fx²</td></tr>
<tr><td>5</td><td>8</td><td>40</td><td>200</td></tr>
<tr><td>5.75</td><td>14</td><td>80.5</td><td>462.875</td></tr>
<tr><td>6.25</td><td>19</td><td>118.75</td><td>742.1875</td></tr>
<tr><td>6.75</td><td>13</td><td>87.75</td><td>592.3125</td></tr>
<tr><td>7.5</td><td>6</td><td>45</td><td>337.5</td></tr>
<tr><td></td><td>60</td><td>372</td><td>2334.875</td></tr>
</table>
Sample mean = 372/60 = 6.2

$S_{xx} = 2334.875 - \frac{372^2}{60} = 28.475$

Sample s.d = $\sqrt{\frac{28.475}{59}} = 0.695$</td><td>B1 mid points

B1FT $\sum fx$ and $\sum fx^2$

B1 CAO

M1 for their S_{xx}

A1 CAO</td><td>5</td></tr>
<tr><td>(iv)</td><td>Prediction of score = 13 x 7.4 – 46 = 50.2
So predicted AS grade would be B</td><td>M1 For 13 x 7.4 – 46
A1 dep on 50.2 (or 50) seen</td><td>2</td></tr>
<tr><td>(v)</td><td>Prediction of score = 13 x 5.5 – 46 = 25.5

So predicted grade would be D/E
(allow D or E)
Because score roughly halfway from 20 to 30,
OR (for D) closer to D than E
OR (for E) past E but not up to D boundary</td><td>M1 For 13 x 5.5 – 46

A1 dep on 25.5 (or 26 or 25) seen
E1 For explanation of conversion – logical statement/argument that supports their choice.</td><td>3</td></tr>
<tr><td>(vi)</td><td>Mean = 13 x 6.2 – 46 = 34.6
Standard deviation = 13 x 0.695 = 9.035</td><td>B1 FT their 6.2
M1 for 13 x their 0.695
A1 FT</td><td>3</td></tr>
<tr><td></td><td></td><td>TOTAL</td><td>18</td></tr>
</table>

8 **(i)**	P(all jam) $= \frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}$ $= \frac{1}{22} = 0.04545$	M1 5 x 4 x 3 or $\binom{5}{3}$ in numerator M1 12 x 11 x 10 or $\binom{12}{3}$ in denominator A1 CAO	**3**
(ii)	P(all same) $= \frac{5}{12}\times\frac{4}{11}\times\frac{3}{10} + \frac{4}{12}\times\frac{3}{11}\times\frac{2}{10} + \frac{3}{12}\times\frac{2}{11}\times\frac{1}{10}$ $= \frac{1}{22}+\frac{1}{55}+\frac{1}{220} = \frac{3}{44} = 0.06818$	M1 Sum of 3 reasonable triples or combinations M1 Triples or combinations correct A1 CAO	**3**
(iii)	P(all different) $= 6\times\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}$ $= \frac{3}{11} = 0.2727$	M1 5,4,3 M1 $6\times$ three fractions or $\binom{12}{3}$ denom. A1 CAO	**3**
(iv)	P(all jam given all same) = $\frac{1}{22} \Big/ \frac{3}{44} = \frac{2}{3}$	M1 Their (i) in numerator M1 Their (ii) in denominator A1 CAO	**3**
(v)	P(all jam exactly twice) $= \binom{5}{2}\times\left(\frac{1}{22}\right)^2\times\left(\frac{21}{22}\right)^3 = 0.01797$	M1 for $\binom{5}{2}$ x … M1 for their $p^2\ q^3$ A1 CAO	**3**
(vi)	P(all jam at least once) $= 1-\left(\frac{21}{22}\right)^5 = 0.2075$	M1 for their q^5 M1 indep for 1 – 5^{th} power A1 CAO	**3**
		TOTAL	**18**

Mark Scheme 4767
January 2006

Question 1

(i)	Faults are detected randomly and independently Uniform (mean) rate of occurrence	B1 B1	**2**
(ii)	(A) $\mathrm{P}(X = 0) = \mathrm{e}^{-0.15}\dfrac{0.15^0}{0!} = 0.8607$ (B) $\mathrm{P}(X \geq 2) = 1 - 0.8607 - \mathrm{e}^{-0.15}\dfrac{0.15^1}{1!}$ $= 1 - 0.8607 - 0.1291 = 0.0102$	M1 for probability calc. M0 for tables unless interpolated A1 M1 A1	**4**
(iii)	$\lambda = 30 \times 0.15 = 4.5$ Using tables: $\mathrm{P}(X \leq 3) = 0.3423$	B1 for mean (SOI) M1 attempt to find $\mathrm{P}(X \leq 3)$ A1	**3**
(iv)	Poisson distribution with $\lambda = 10 \times (0.15 + 0.05) = 2$ $\mathrm{P}(X = 5) = \mathrm{e}^{-2}\dfrac{2^5}{5!} = 0.0361$ (3 s.f.) or from tables $= 0.9834 - 0.9473 = 0.0361$	B1 for Poisson stated B1 for $\lambda = 2$ M1 for calculation or use of tables A1 FT	**4**
(v)	Mean no. of items in 200 days = $200 \times 0.2 = 40$ Using Normal approx. to the Poisson, $X \sim \mathrm{N}(40,40)$: $\mathrm{P}(X \geq 50) = \mathrm{P}\left(Z > \dfrac{49.5 - 40}{\sqrt{40}}\right)$ $= \mathrm{P}(Z > 1.502) = 1 - \Phi(1.502) = 1 - 0.9334$ $= 0.0666$ (3 s.f.)	B1 for Normal approx. (SOI) B1 for both parameters B1 for continuity corr. M1 for probability using correct tail A1 **cao, (**but FT wrong or omitted CC)	**5**
			18

Question 2

(i) **(*A*)**	$X \sim N(42,3^2)$ $P(X > 50.0) = P\left(Z > \frac{50.0-42.0}{3.0}\right)$ $= P(Z > 2.667)$ $= 1 - \Phi(2.667) = 1 - 0.9962$ $= 0.0038$	M1 for standardizing M1 for prob. calc. with correct tail A1 **NB answer given**	**3**
(i) **(*B*)**	P(not positive) = 0.9962 P(At least one is out of 7 is positive) $= 1 - 0.9962^7 = 1 - 0.9737$ $= 0.0263$	B1 for use of 0.9962 in binomial expression M1 for correct method A1 CAO	**3**
(i) **(*C*)**	If an innocent athlete is tested 7 times in a year there is a reasonable possibility (1 in 40 chance) of testing positive. Thus it is likely that a number of innocent athletes may come under suspicion and suffer a suspension so the penalty could be regarded as unfair. *Or* this is a necessary evil in the fight against performance enhancing drugs in sport.	E1 comment on their probability in (i) B E1 for sensible contextual conclusion consistent with first comment	**2**
(ii) **(*A*)**	B(1000, 0.0038)	B1 for B(,) or Binomial B1 *dep* for both parameters	**2**
(ii) **(*B*)**	A suitable approximating distribution is Poisson(3.8) P(at least 10 positive tests) $= P(X \geq 10) = 1 - P(X \leq 9)$ $= 1 - 0.9942$ $= 0.0058$ *NB Do not allow use of Normal approximation.*	B1 for Poisson soi B1FT *dep* for $\lambda = 3.8$ M1 for attempt to use 1 $- P(X \leq 9)$ A1 FT	**4**
(iii)	P(not testing positive) = 0.995 From tables $z = \Phi^{-1}(0.995) = 2.576$ $\frac{h-48.0}{2.0} = 2.576$ $h = 48.0 + 2.576 \times 2.0 = 53.15$	B1 for 0.995 seen (or implied by 2.576) B1 for 2.576 (FT their 0.995) M1 for equation in *h* and positive z-value A1 CAO	**4**
			18

Question 3

<table>
<tr>
<td>(i)</td>
<td>
<table>
<tr><td>Rank x</td><td>1</td><td>5</td><td>4</td><td>7</td><td>6</td><td>8</td><td>10</td><td>3</td><td>9</td><td>2</td></tr>
<tr><td>Rank y</td><td>2</td><td>4</td><td>5</td><td>8</td><td>9</td><td>7</td><td>10</td><td>6</td><td>3</td><td>1</td></tr>
<tr><td>d</td><td>-1</td><td>1</td><td>-1</td><td>-1</td><td>-3</td><td>1</td><td>0</td><td>-3</td><td>6</td><td>1</td></tr>
<tr><td>d²</td><td>1</td><td>1</td><td>1</td><td>1</td><td>9</td><td>1</td><td>0</td><td>9</td><td>36</td><td>1</td></tr>
</table>

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 60}{10 \times 99}$$

= 0.636 (to 3 s.f.) [*allow* 0.64 to 2 s.f.]
</td>
<td>M1 for ranking (allow all ranks reversed)

M1 for d^2

A1 CAO for Σd^2

M1 for structure of r_s using their Σd^2
A1 f.t. for $|r_s| < 1$
NB No ranking scores zero</td>
<td>5</td>
</tr>
<tr>
<td>(ii)</td>
<td>H_0: no association between x and y
H_1: positive association between x and y
Looking for positive association (one-tail test):
Critical value at 5% level is 0.5636

Since 0.636 > 0.5636, there is sufficient evidence to reject H_0,
i.e. conclude that there appears to be positive association between temperature and nitrous oxide level.</td>
<td>B1 for H_0
B1 for H_1
NB H_0 H_1 <u>not</u> ito rho
B1 for $\pm$ 0.5636
(FT their H_1)
M1 for comparison with c.v., provided $|r_s| < 1$
A1 for conclusion in words f.t. their r_s and sensible cv</td>
<td>5</td>
</tr>
<tr>
<td>(iii)</td>
<td>Underlying distribution must be bivariate normal.
If the distribution is bivariate normal then the scatter diagram will have an elliptical shape.
This scatter diagram is not elliptical and so a PMCC test would not be valid.
(Allow comment indicating that the sample is too small to draw a firm conclusion on ellipticity and so on validity)</td>
<td>B1 CAO for bivariate normal
B1 indep for elliptical shape
E1 dep for conclusion</td>
<td>3</td>
</tr>
<tr>
<td>(iv)</td>
<td>n=60, PMCC critical value is r = 0.2997

So the critical region is $r \geq 0.2997$</td>
<td>B1

B1 FT their sensible c.v.</td>
<td>2</td>
</tr>
<tr>
<td>(v)</td>
<td>Any three of the following:
• Correlation does not imply causation;
• There could be a third factor (causing the correlation between temperature and ozone level);
• the claim could be true;
• increased ozone could cause higher temperatures.</td>
<td>E1

E1

E1</td>
<td>3</td>
</tr>
<tr><td></td><td></td><td></td><td>18</td></tr>
</table>

Question 4

(i)	H_0: no association between method of travel and type of school; H_1: some association between method of travel and type of school;..	B1 for both	**1**
(ii)	Expected frequency = 120/200 × 70 = 42 Contribution = $(21 – 42)^2$ / 42 = 10.5	M1 A1 M1 for valid attempt at $(O\text{-}E)^2/E$ A1 FT their 42 provided O = 21 (at least 1 dp)	**4**
(iii)	$X^2 = 42.64$ Refer to $\Xi_2^{\,2}$ Critical value at 5% level = 5.991 Result is significant There appears to be some association between method of travel and year group. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final E1	B1 for 2 deg of f(seen) B1 CAO for cv B1 for significant (FT their c.v. provided consistent with their d.o.f. E1	**4**
(iv)	H_0: $\mu = 18.3$; H_1: $\mu \neq 18.3$ Where μ denotes the mean travel time by car for the whole population. Test statistic $z = \dfrac{22.4 - 18.3}{8.0/\sqrt{20}} = \dfrac{4.1}{1.789}$ = 2.292 10% level 2 tailed critical value of z is 1.645 2.292 > 1.645 so significant. There is evidence to reject H_0 It is reasonable to conclude that the mean travel time by car is different from that by bus.	B1 for both correct B1 for definition of μ M1 (standardizing sample mean) A1 for test statistic B1 for 1.645 M1 for comparison leading to a conclusion A1 for conclusion in words and context	**7**
(v)	The test suggests that students who travel by bus get to school more quickly. This may be due to their journeys being over a shorter distance. It may be due to bus lanes allowing buses to avoid congestion. It is possible that the test result was incorrect (ie implication of a Type I error). More investigation is needed before any firm conclusion can be reached.	E1, E1 for any two valid comments	**2**
			18

Question 4 chi squared calculations

H_0: no association between method of travel and type of school;
H_1: some association between method of travel and type of school;

Observed		Type of school Year 6	Year 11	**Row totals**
Method of travel	Bus	21	49	**70**
	Car	65	15	**80**
	Cycle/Walk	34	16	**50**
Column totals		**120**	**80**	**200**

Expected		Type of school Year 6	Year 11	**Row totals**
Method of travel	Bus	42	28	**70**
	Car	48	32	**80**
	Cycle/Walk	30	20	**50**
Column totals		**120**	**80**	**200**

Chi Squared Contribution		Type of school Year 6	Year 11	**Row totals**
Method of travel	Bus	10.50	15.75	**26.25**
	Car	6.02	9.03	**15.05**
	Cycle/Walk	0.53	0.80	**1.33**
Column totals		**17.05**	**25.58**	**42.64**

Mark Scheme 4768
January 2006

Q1	$F(t) = 1 - e^{-t/3}$ (t>0)			
(i)	For median m, $\frac{1}{2} = 1 - e^{-m/3}$ $\therefore\ e^{-m/3} = \frac{1}{2} \Rightarrow -\frac{m}{3} = \ln\frac{1}{2} = -0.6931$ $\Rightarrow m = 2.079$ For 90th percentile p, $0.9 = 1 - e^{-p/3}$ $\therefore\ e^{-p/3} = 0.1 \Rightarrow -\frac{p}{3} = \ln 0.1 = -2.3026$ $\Rightarrow\ p = 6.908$	M1 M1 A1 M1 A1	attempt to solve, here or for 90th percentile. Depends on previous M mark.	5
(ii)	$f(t) = \frac{d}{dt}F(t)$ $= \frac{1}{3}e^{-t/3}$ $\mu = \int_0^\infty \frac{1}{3}te^{-t/3}dt$ $= \frac{1}{3}\left\{\left[\frac{te^{-t/3}}{-1/3}\right]_0^\infty + 3\int_0^\infty e^{-t/3}dt\right\}$ $= [0-0] + \left[\frac{e^{-t/3}}{-1/3}\right]_0^\infty = 3$	M1 A1 M1 M1 A1	(for t>0, but condone absence of this) Quoting standard result gets 0/3 for the mean. attempt to integrate by parts	5
(iii)	$P(T > \mu) =$ [from cdf] $e^{-\mu/3} = e^{-1}$ $=0.3679$	M1 A1	[or via pdf] ft c's mean (>0)	2
(iv)	$\bar{T} \sim$ (approx) $N\left(3, \frac{9}{30} = 0.3\right)$	B1 B1 B1	N ft c's mean (>0) 0.3	3
(v)	**<u>EITHER</u>** can argue that 4.2 is more than 2 SDs from μ ($3 + 2\sqrt{0.3} = 4.095$; <u>must</u> refer to SD ($\bar{T}$), not SD(T)) i.e. outlier $\Rightarrow$ doubt **<u>OR</u>** formal significance test: $\frac{4.2-3}{3/\sqrt{30}} = 2.191$, refer to N(0,1), sig at (eg) 5% $\Rightarrow$ doubt	M1 M1 A1 M1 M1 A1	Depends on first M, but could imply it.	3
				18

Q2	$X \sim \text{N}(180,\ \sigma = 12)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$\text{P}(X < 170) = \text{P}(Z < \frac{170-180}{12} = -0.8333)$ $= 1 - 0.7976 = 0.2024$	M1 A1 A1	For standardising. Award once, here or elsewhere.	3
(ii)	$X_1 + X_2 + X_3 + X_4 + X_5 \sim \text{N}(900, \sigma^2 = 720[\sigma = 26.8328]$ $\text{P}(\text{this} < 840) = \text{P}(Z < \frac{840-900}{26.8328} = -2.236)$ $= 1 - 0.9873 = 0.0127$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iii)	$Y \sim \text{N}(50, \sigma = 6)$ $X + Y \sim \text{N}(230, \sigma^2 = 180[\sigma = 13.4164])$ $\text{P}(\text{this} > 240) = \text{P}(Z > \frac{240-230}{13.4164} = 0.7454)$ $= 1 - 0.7720 = 0.2280$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iv)	$\frac{1}{4}X \sim N\left(45, \sigma^2 = \frac{1}{16} \times 144 = 9[\sigma = 3]\right)$ Require t such that $0.9 = \text{P}(\text{this} < t) = \text{P}\left(Z < \frac{t-45}{3}\right) = \text{P}(Z < 1.282)$ $\therefore t - 45 = 3 \times 1.282 \Rightarrow t = 48.85\ (48.846)$	B1 M1 B1 A1	Variance. Accept sd. FT incorrect mean. Formulation of requirement. 1.282 ft only for incorrect mean	4
(v)	$I = 45 + T$ where $T \sim \text{N}(120,\ \sigma = 10)$ $\therefore I \sim \text{N}(165, \sigma = 10)$ $\text{P}(I < 150) = \text{P}(Z < \frac{150-165}{10} = -1.5)$ $= 1 - 0.9332 = 0.0668$	B1 A1	for unchanged σ (candidates might work with P (T<105)) c.a.o.	2
(vi)	$J = 30 + \frac{3}{5}T$ where $T \sim \text{N}(120,\ \sigma = 10)$ $\therefore J \sim \text{N}\left(102, \sigma^2 = \frac{9}{25} \times 100 = 36[\sigma = 6]\right)$ $\text{P}(J < 105) = \text{P}(Z < \frac{105-102}{6} = 0.5) = 0.6915$	B1 B1 A1	Cands might work with $\text{P}(\frac{3}{5}T < 75)$. $\frac{3}{5}T \sim N(72,36)$ Mean. Variance. Accept sd. c.a.o.	3
				18

Q3				
(a)	H$_0$: $\mu_D = 0$ (or $\mu_A = \mu_B$) H$_1$: $\mu_D > 0$ (or $\mu_B > \mu_A$) where μ_D is "mean for B – mean for A"	B1 B1 B1	Hypotheses in words only must include "population". Or "<" for $A - B$. For adequate verbal definition. Allow absence of "population" if correct notation μ is used, but do NOT allow "$\overline{X}_A = \overline{X}_B$" or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean.	
	Normality of <u>differences</u> is required <u>MUST</u> be PAIRED COMPARISON t test. Differences are: 2.1 1.0 0.8 0.6 0.4 -1.0 -0.3 0.8 0.9 1.1	B1		
	$\bar{d} = 0.64$ $s_{n-1} = 0.8316$	B1	$s_n = 0.7889$ but do <u>NOT</u> allow this here or in construction of test statistic, but FT from there.	
	Test statistic is $\dfrac{0.64 - 0}{\frac{0.8316}{\sqrt{10}}}$	M1	Allow c's $\bar{d}$ and/or s_{n-1}. Allow alternative: 0 + (c's 1.833) × $\frac{0.8316}{\sqrt{10}}$ (= 0.4821) for subsequent comparison with $\bar{d}$. (Or $\bar{d}$ – (c's 1.833) × $\frac{0.8316}{\sqrt{10}}$ (= 0.1579) for comparison with 0.)	
	=2.43(37).	A1	c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{d}$ scores M1A0, but ft.	
	Refer to t_9. Single-tailed 5% point is 1.833. Significant. Seems mean amount delivered by B is greater that that by A	M1 A1 E1 E1	No ft from here if wrong. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. Special case: (t_{10} and 1.812) can score 1 of these last 2 marks if either form of conclusion is given.	11
(b)	We now require Normality for the amounts delivered by machine A. For machine A, $\bar{x} = 250.19$ $s_{n-1} = 3.8527$	B1 B1	 $s_n = 3.6549(83)$ but do NOT allow this here or in construction of CI.	
	CI is given by $250.19 \pm 2.262\dfrac{3.8527}{\sqrt{10}}$	M1 B1 M1	ft c's $\bar{x}$ ±. 2.262 ft c's s_{n-1}.	
	$= 250.19 \pm 2.75(6) = (247.43(4), 252.94(6))$	A1	c.a.o. Must be expressed as an interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_9 is OK.	
	250 is in the CI, so would accept H$_0$: $\mu = 250$, so no evidence that machine is not working correctly in this respect.	E1		7
				18

Q4

(i)

	$\underbrace{1 \qquad 30}_{31}$	62	70	$\underbrace{34 \qquad 3}_{37}$
e_i	$\underbrace{1.49 \qquad 37.85}_{39.34}$	55.62	58.32	$\underbrace{44.62 \qquad 2.10}_{46.72}$

(i)		M1	for grouping	
	$X^2 = 1.7681 + 0.7318 + 2.3392 + 2.0222$	M1	Allow the M1 for correct method from wrongly grouped or ungrouped table.	
	$= 6.86$	A1		
	Refer to χ_1^2.	M1	Allow correct df (= cells – 3) from wrongly grouped or ungrouped table, and FT. Otherwise, no FT if wrong.	
	Upper 5% point is 3.84	A1	No ft from here if wrong.	
	Significant	E1	ft only c's test statistic.	
	Suggests Normal model does not fit	E1	ft only c's test statistic.	7
(ii) (A)	t test unwise …	E1		
	… because underlying population appears non-Normal	E1	FT from result of candidate's work in (i)	2

(B)

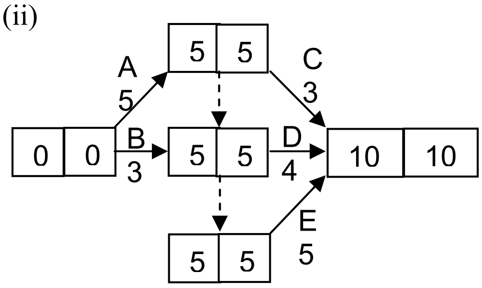
Data	Median 301	Difference	Rank of |diff|
301.3		0.3	3
301.4		0.4	4
299.6		- 1.4	8
302.2		1.2	7
300.3		- 0.7	5
303.2		2.2	10
302.6		1.6	9
301.8		0.8	6
300.9		- 0.1	1
300.8		- 0.2	2

(B)		M1	for differences. ZERO in this section if differences not used.	
		M1	for ranks.	
		A1	FT if ranks wrong.	
	$T = 1 + 2 + 5 + 8 = 16$ (or $3+4+6+7+9+10 = 39$)	B1		
	Refer to tables of Wilcoxon single sample (/paired) statistic	M1		
	Lower (or upper if 39 used) 5% tail is needed	M1		
	Value for $n = 10$ is 10 (or 45 if 39 used)	A1		
	Result is not significant	E1		
	No evidence against median being 301	E1		9
				18

Mark Scheme 4771
January 2006

1.

(i) & (ii)



(i) & (ii)	B1 C OK B1 D OK B1 E OK M1 early and late A1 times
Critical: A, E	B1 critical
(iii) A, E and D 6 days	B1 B1

2.

(i)

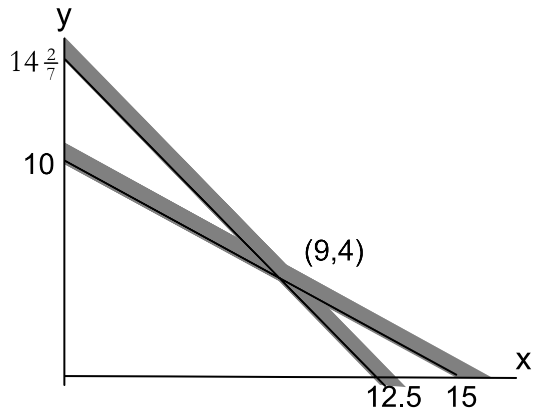
Step number	List 1	List 2	A	B	List 3
	2, 34, 35, 56	13, 22, 34, 81, 90, 92			
1	34, 35, 56	22, 34, 81, 90, 92	2	13	
3	35, 56	22, 34, 81, 90, 92	34	13	2
4	35, 56	34, 81, 90, 92	34	22	2, 13
4	35, 56	81, 90, 92	34	34	2, 13, 22
3	56	81, 90, 92	35	34	2, 13, 22, 34
4	56	90, 92	35	81	2, 13, 22, 34, 34
3		90, 92	56	81	2, 13, 22, 34, 34, 35
3		90, 92	56	81	2, 13, 22, 34, 34, 35, 56, 81, 90, 92

	M1 sca A1 to first step 3 inc. A1 to second step 3 A1 rest
(ii) Merges ordered lists to give an ordered list	B1
(iii) 7	B1
(iv) Max = x + y – 1 Min = min (x, y)	B1 B1

3.

(i)	Ins and outs	M1
	One more out than in at D. Vice-versa at A.	A1
	Start at D and end at A	B1
(ii)	Existence – A B D C A	B1
	Uniqueness – Only alternative is A B C …!!!	M1 A1
	Extra arc – New possibility A D C B … !!!	A1
(iii)	B D C A B	B1

4.

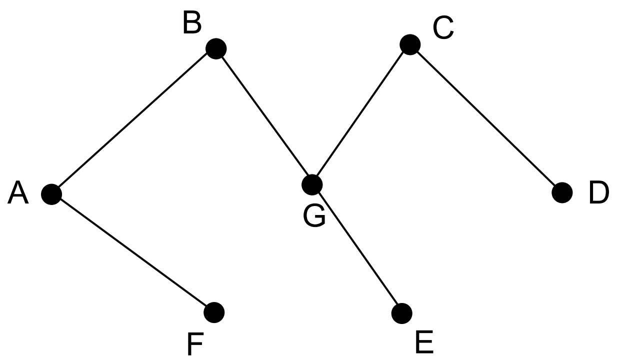
(i)	12.5 kg 250 g (of butter)	B1 B1
	10 kg 3 kg (of sugar)	B1 B1
(ii)	Identification of variables e.g. Let x = kg of toffee made Let y = kg of fudge made	
	Max x + y	B1
	st $100x + 150y \le 1500$	B1
	$800x + 700y \le 10000$	B1
		B1 axes labelled and scaled B1 butter line B1 sugar line B1 shading
	Make 9 kg toffee and 4 kg fudge	B1 max x+y + solution
(iii)	12.5 kg of toffee and no fudge – either by comparing 68.75 with 67.50 with 45, or by a gradient argument	M1 A1
	Toffee price must <u>decrease</u> by £0.36, or to £5.14.	B1 B1

5.

(i)

	1	2	5	6	4	7	3
	A	B	C	D	E	F	G
~~A~~	~~–~~	~~10~~	~~–~~	~~–~~	~~–~~	~~12~~	~~15~~
~~B~~	10	~~–~~	~~15~~	~~20~~	~~–~~	~~–~~	~~8~~
~~C~~	~~–~~	~~15~~	~~–~~	~~7~~	~~–~~	~~–~~	11
~~D~~	~~–~~	~~20~~	7	~~–~~	~~20~~	~~–~~	~~13~~
~~E~~	~~–~~	~~–~~		~~20~~		~~17~~	9
F	12	–	–	–	17	–	13
~~G~~	~~15~~	8	~~11~~	~~13~~	~~9~~	~~13~~	~~–~~

M1
A1 selections
A1 order of selecting
A1 deletions

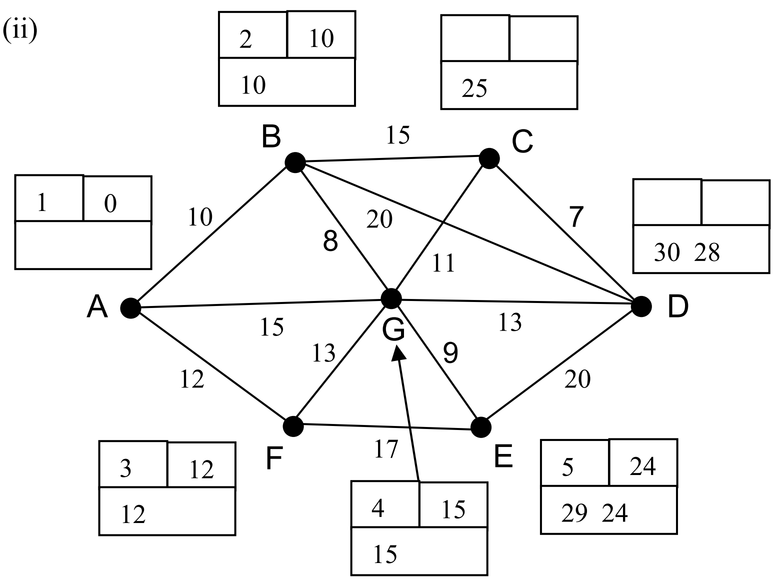


B1

Total length = 57 miles — B1

Might be used to determine where to lay pipes or cables to connect the towns. — B1

(ii)



M1 sca Dijkstra
A1 labels
A1 order of labelling
A1 working values

Shortest route: AGE — B1

Length: 24 — B1

(iii) Shortens mst to 53 miles (√ by 4) — B1

New shortest route ABGE – 23 miles (√ by 1) — B1 B1

6.

(i) e.g. 0 – 6 petrol 7 – 9 other	B1
(ii) e.g. 0 – 2 1 min 3 – 6 1.5 mins 7 – 8 2 mins 9 2.5 mins	M1 A1
(iii) e.g. 00 – 13 1 min 14 – 41 1.5 mins 42 – 69 2 mins 70 – 83 2.5 mins 84 – 97 3 mins 98, 99 reject	M1 some rejected A1 2 rejected A1
Two digits – fewer rejects	B1

(iv)

Customer number	Inter-arrival time	Arrival time	Type of customer	Arrival at till	Time at till	Departure time	Queuing + paying
1	1	1	F	1	1	2	1
2	0.5	1.5	N	2	2	4	2.5
3	3.5	5	N	5	1.5	6.5	1.5
4	3	8	F	8	1.5	9.5	1.5
5	1	9	F	9.5	1	10.5	1.5
6	0.5	9.5	F	10.5	1	11.5	2
7	1.5	11	F	11.5	2.5	14	3
8	2	13	N	14	2.5	16.5	3.5
9	2	15	F	16.5	2	18.5	3.5
10	0.5	15.5	F	18.5	1.5	20	4.5

	B1 arrival times M1 types M1 service start M1 service duration M1 service end M1 time in shop A1
(v) 24.5/10 = 2.45 mins	M1 A1

Mark Scheme 4776
January 2006

MEI Numerical Methods 4776

1 Use binomial expansion of $(1 + r)^{-1}$ or sum of GP, or $1 - r^2$
with r^2 taken to be zero to obtain given result. **[M1A1]**
Relative error in reciprocal is of same magnitude but opposite sign **[E1E1]**
E.g. 10 is approx 2% greater than 9.8
1/10 = 0.1 is approx 2% less than 1/9.8 = 0.10204 **[M1A1]**
[TOTAL 6]

2(i) $x_{r+1} = 1/\sin(x_r)$ **[M1]**

r	0	1	2	3	10	11	12	
x_r	1	1.188395	1.077852	1.135147	1.113855	1.114323	1.114067	**[M1A1]**

root is 1.11 to 3 sf. **[A1]**
[subtotal 4]

(ii)

x	2.7725	2.7735		
1/x - sin(x)	-8.4E-05	0.000719	change of sign, so 2.773 correct to 3 dp	**[M1A1A1]**

[subtotal 3]
[TOTAL 7]

3

h	M	T
2	2.60242	2.44866
1	2.56982	

T2 = (M1 + T1)/2 = 2.52554 **[M1A1]**
T4 = (M2 + T2)/2 = 2.54768 **[A1]**
S1 = (2M1 + T1)/3 = 2.55117 **[M1A1]**
S2 = (2M2 + T2)/3 = 2.55506 **[A1]**

I = 2.56 (or 2.555) is justified **[A1]**
[TOTAL 7]

4

x	1	2	3	4	5
f(x)	3	4.5	5.4	6.2	6.7

	(i)	**(ii)**		
h	f '(3)	min	max	**(i)** **[M1A1A1]**
2	0.925	0.9	0.95	**(ii)**
1	0.85	0.8	0.9	**[M1A1A1]**

Larger h gives smaller interval (or 0.9 is the only common value) **[E1]**
f '(3) = 0.9 is the value that seems justified. **[E1]**
(Or 0.8 seems to be the limit the process is tending to. [E1E1])
[TOTAL 8]

5

x	1	3	4
$g(x)$	4	1	11

L(2) = 4(2-3)(2-4)/(1-3)(1-4) + 1(2-1)(2-4)/(3-1)(3-4) + 11(2-1)(2-3)/(4-1)(4-3) **[M1A1A1A1]**

= $-1^1/_3$ **[A1]**

x	g(x)	Δg(x)	Δ^2g(x)		
1	4	-5.33333	7.666666		
2	-1.33333	2.333333	7.666667	2nd differences constant	
3	1	10		so correct for a quadratic	**[M1A1E1]**
4	11				

[TOTAL 8]

---

6 (i) y ' = $10x^9 - 10 = 0$ only when x = 1, hence at most one turning point **[M1A1]**

tenth degree polynomial is positive as x tends to plus or minus infinity **[E1]**

(hence exactly one turning point) (or other methods)

x	0	1	2	
f(x)	1	-8	1005	**[M1A1]**

changes of sign so roots in [0,1] and [1,2] **[E1]**

since only one turning point cannot be any more roots **[E1]**

[subtotal 7]

(ii) NR: $x_{r+1} = x_r - (x_r^{10} - 10x_r + 1)/(10x_r^9 - 10)$ **[M1A1A1]**

r	0	1	2	3	4	5	
x_r	1.2	1.315589	1.284353	1.280004	1.279928	1.279928	**[M1A1A1]**
						1.2799	***[subtotal 6]***

(iii) $x_{r+1} = (x_r^{10} + 1)/10$ **[M1A1]**

If $x_0 = 0.1$ then $x_1 = (0.1^{10} + 1)/10 = 0.1^{11} + 0.1$ **[M1A1]**

This is so close to 0.1 that further iterations are unnecessary **[E1]**

[subtotal 5]

[TOTAL 18]

7 (i) Mid-point rule with h=1 and 4 strips to obtain given result. **[M1A1]**

$\log_e(5)$ =	1.609438			
Mid-pt =	1.574603	error is (-)	0.034835	**[M1A1A1]**
				[subtotal 5]

(ii)

N	10	20	40	80	
$\log_e(N)$	2.302585	2.995732	3.688879	4.382027	**[M1A1A1]**
					[subtotal 3]

(iii)(iv)

N	Mid-pt	ln(N)	est k	diffs	ratio of diffs		
10	2.266511	2.302585	0.036074				**estimates** **[M1A1A1]**
20	2.959346	2.995732	0.036386	0.000312			***[subtotal 3]***
40	3.652416	3.688879	0.036463	7.72E-05	0.247231	**diffs**	**[M1A1]**
80	4.345543	4.382027	0.036484	2.02E-05	0.261472	**ratio**	**[M1A1]**
					(approx 0.25)		

extrapolating:	0.036489	5.05E-06	
(or equivalent)	0.036490	1.26E-06	
	0.036490	3.15E-07	**[M1A1]**

0.03649(0) seems secure **[A1]**

[subtotal 7]

[TOTAL 18]

Report on the Units
January 2006

4751: Introduction to Advanced Mathematics (C1) (Written Examination)

General Comments

The candidature was a little smaller than last January, with fewer year 13 students this year compared with those transferring to the new specification then.

There were many excellent scripts, but also a long tail of very weak candidates. It appears that some centres are using this first examination as a ‘wake-up call’ to weak candidates as to the standards expected during their A level course, as well as hoping that their better candidates gain a good mark on C1 and can concentrate confidently on C2 during the rest of year 12.

A calculator is not allowed in this paper and, as last year, some candidates found this a considerable disadvantage, making errors in basic arithmetic, in particular with negative numbers and fractions. Lack of skill in factorising was apparent, with many candidates resorting to the quadratic formula in questions 9 and 10, for instance.

Using graph paper to draw graphs when a sketch graph has been requested remains an issue. Some candidates not only waste time in drawing such graphs, but also tend to use too large a vertical scale and too few points to achieve a good shape. It is acknowledged that the present listing of graph paper as additional materials on the front of the paper has not helped this situation.

In general, time was not an issue. A few candidates petered out towards the end of question 12, but there were very few for whom this appeared to be because they had run out of time.

Comments on Individual Questions

Section A

1) This proved to be a testing starter. Most candidates used a few different values, often some odd and some even – several commented that they had thereby proved the result by exhaustion! In the better solutions there was a realisation that generalisation was required, with good arguments being produced, the $n^2 + n$ being more popular than the $n(n + 1)$ form. A few good candidates substituted $2k$ and $2k + 1$.

2) In part (i), many did not realise that the word ‘translation’ was needed to describe the transformation and simply wrote ‘moves by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$’ or equivalent. In part (ii) most had the correct answer, but the distractors were also used.

3) Many candidates used the binomial theorem successfully, with usually just the weaker candidates not knowing how to proceed. Very few tried to multiply out the brackets, but those who did were often successful.

4) The majority of candidates scored at least 3 of the 4 marks for solving this inequality. The steps which caused the most difficulty were multiplying by 4, and subtracting 3 from −24. The mark scheme allowed for follow-through from wrong steps.

5) Better students had no problems in changing the subject of this formula, but many candidates had no idea how to proceed after $PC + 4P = C$, whilst many did not get this far and appeared to have no strategy at all.

6) Use of the remainder theorem nearly always led to full marks. In some centres, long division

was popular, with many candidates unable to cope with the first subtraction required and gaining no marks, although there were also quite a few candidates who used this method successfully.

7) There was a lot of difference between centres in candidates' responses here. A good number of candidates gained full marks, but there were also many arithmetical errors, either in simplifying a correct equation, or in proceeding from $\frac{1}{4}x = 3\frac{1}{2}$ via $x = \frac{3\frac{1}{2}}{\frac{1}{4}}$. Many missed out the last part or used $x = 0$ instead of $y = 0$. Some of the weaker candidates had no idea about the condition $m_1 m_2 = -1$ for perpendicular lines.

8) Better candidates had no problem with the first part, but errors in manipulating the surds here were frequent, with common errors being work such as $4\sqrt{50} = 4 + 5\sqrt{2} = 9\sqrt{2}$ or $5\sqrt{8} = 5\sqrt{4 \times 2} = 20\sqrt{2}$. Weaker candidates often had little idea how to proceed here. Most knew the method for rationalising the denominator in the second part, but many stopped at $\frac{3 + 6\sqrt{3}}{33}$ or made errors after this in obtaining the required form.

9) This was only completed successfully by the best candidates, with quite a few of the better ones getting as far as $k < 25/4$ but omitting the equality. Weaker candidates perhaps knew the answer had something to do with $b^2 - 4ac$ and gained a method mark or simply tried a number of values of k or omitted the question. In part (ii), disappointingly few immediately recognised the perfect square. Those who attempted the quadratic formula often made errors in working out $4 \times 4 \times 25$ or were bemused when the discriminant was zero, though many used it correctly.

Section B

10 (i) This was mostly well answered. A few became muddled and thought the centre was (3, 6) or (6, 3); a small number stated the radius as 45.

(ii) A majority of the candidates understood how to eliminate a variable from the equation of the circle in order to produce a quadratic equation in one variable. This was usually done accurately and led to the correct answer. A few errors did occur by making the wrong substitution (e.g. $y = x - 3$), or by expanding $(3 - x)^2$ incorrectly. Once they had reached the correct quadratic equation, the correct solution generally followed. A larger number of candidates used the formula to solve the resulting quadratic, rather than factorising – with mixed results, depending on how well they remembered the formula - many incorrect versions were quoted and used. Candidates often worked with the harder equation $2x^2 - 6x - 36 = 0$ (or its equivalent in y) instead of first dividing through by 2, which would have simplified both factorising and use of the formula.

A few candidates combined the two equations without eliminating a variable e.g. $x^2 + y^2 - 45 = x + y - 3$ and were therefore unable to proceed any further.

Those who achieved the correct solution rarely failed to calculate the distance AB correctly.

11 (i) Although the best candidates did this correctly there was evidence to suggest that many of them are unable to calculate $(^7/_2)^2$. So although they understood what to do they were let down by their poor arithmetical ability. Some candidates added an =' to the expression, taking the 6 'across ', subtracting it from 3.5^2 and then 'taking it back'. Those who used this approach often forgot to negate their result and so lost out – the examiners do not recommend this method.

(ii) Many candidates realised that there was a connection between this part and the first and generally their answers did follow through, though there were occasional errors in sign. Other candidates started all over again, sometimes using calculus. Many of these attempts were unsuccessful, particularly in calculating the y-coordinate.

(iii) Most managed to find at least one of the intercepts, with a good number finding all three. The graph sketching was not done particularly well. The curves were often not very smooth, nor were they symmetrical.. Many candidates plotted the 'intercept' points first and tried to fit a curve to them, which is not an easy task. This is one reason why the use of graph paper may not have been their best option.

(iv) Most candidates were able to make a good attempt at this part and were usually successful, even if they had failed to pick up many marks in the earlier parts. However, after a correct first step, a significant minority made errors in simplifying and solving their equation.

12 (i) The sketches were again disappointing but some examiners reported that they were more successful that in the previous question. The shape was more often correct this time. but often not both through (0, 0) and tangential to (3, 0). A few candidates drew parabolas or negative cubic shapes. Weaker candidates did not see the usefulness of the given form of the equation, expanding and working out coordinates rather than seeing the roots by inspection.

(ii) Most candidates were able to gain the marks here, with some having already done part of the work for this in part (i) and just a few making sign or other errors as they attempted to reach the given answer.

(iii) Most candidates used the factor theorem to show that x = 2 was a root of the equation – a small number of candidates incorrectly tried f(−2). Other candidates relied on the long division process which they used in the next part of the question. The division was generally done correctly – the source of any errors generally came from errors in dealing with negative numbers. Those who used inspection were usually successful. Having found the quadratic factor most candidates made a reasonable attempt at deriving the other two solutions. Full credit was given for getting as far as $\frac{4 \pm \sqrt{12}}{2}$; candidates benefited from this since various arithmetic slips occurred in their attempts to simplify it.

The final mark for showing the location of these roots on their sketch was very rarely earned. The *x*-values were sometimes marked on the *x*-axis rather than on the curve; occasionally a further cubic was drawn, translated by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (which was not what candidates were asked to do).

4752: C2 Concepts for Advanced Mathematics

General Comments

The paper was well received and no-one seemed to be short of time. All questions were accessible to at least half of the candidates. There was the usual number of excellent scripts and rather fewer ones scoring less than 15 marks.

Comments on Individual Questions

Section A

1) Most candidates knew how to convert degrees to radians. The request to find the value of k confused many and they submitted the answers $^{7\pi}/_9$ or 2.44. The word "exact" in the question warns against using calculators, k = 0.778 was penalised. Weak candidates simply solved our "equation" 140 = kπ to get 44.6.

2) Successful candidates made a simple interpretation of the sigma notation; many spoiled their efforts by including 6 cubed. They must have thought that summing to 5 implied 5 terms. Weaker candidates quoted inappropriate formulae from work with APs or GPs. Interestingly many candidates studying for higher papers used the formula for the sum of cubes, and they used it correctly except that not one of them noticed that it did not start at 1, they all got 225.

3) The Examiners did want to see a diagram for this work, and most candidates provided one. We wanted to see Pythagoras's Theorem applied to the half triangle and the relevant figures identified on their diagram. Just a few argued with the cosine rule (no diagram needed) to get cos 60 = ½, $\sin^2 60$ = ¾ and hence the result.

4) Candidates worked well here and, avoiding all the opportunities to make a slip, successfully found the required area. Many did make slips of course and some got the strip width wrong and some failed to use the two sets of brackets correctly. Most correctly identified the overestimate with more or less convincing reasons. Some made it clear on their diagrams that their trapezia were in fact rectangles, this was penalized.

5) (i) The sketches were often excellent.

(ii) Those who separated the trig. functions and the numbers were usually successful. Unfortunately a very large fraction of the entry divided LHS by RHS to get $^4/_3 \tan x$. If they had left the RHS as 1 they had a chance, but it was invariably left blank or 0. The candidates were stuck with $^4/_3$ and they had no good ideas as to how to handle it.

6) Most correctly found y" and showed that y"(3) = 0. Some also showed that $y'(3) = 0$ and so three of the four marks were earned. The only completely successful attempts showed a before and after test on y'. No-one said that y" was 0 and was changing sign.

7) (i) This was usually correct.

(ii) The three main methods used were (a) cosine rule on triangle OAB (b) calculate the base angles of OAB and use the sine rule (c) bisect the triangle and use 2 x 5 sin 0.6.

8) Apart from very weak candidates who differentiated or integrated x^3 in situ this was a good source of marks for most. The fifth mark was for showing the arbitrary constant somewhere.

9) The dependent variable being log y confused many candidates; they assumed that all the given coordinates should also be logged. There were two marks for arriving at $0.5x + 3$ whatever they called it. Many good candidates coped very well and had no problem converting their part (i) equation to the other form. The commonest error was to convert $\log y = 0.5x + 3$ to $y = 10^{0.5x} + 10^{3}$.

Section B

10) The whole of this question was very well done by many and full marks were common.
In (i) there was a penalty for those who did not use calculus, they were asked to. Some completed the square and even "sum of the roots = -b/a" was seen.
Part (ii) was often fully correct and in part (iii) many could see a rectangle of area 48 if they had produced a decent sketch in part (i).

11) (i) This part was usually correct.

(ii) Part (ii) was often not attempted. Many of those who did work here suffered from very poor arithmetic and algebra; those who got as far as $x^2 = 2$ then said $x = \sqrt{2}$ or ± 2 or ± 1. Good candidates had no trouble finding the required range.

(iii) In part (iii) many knew how to find the equation of the tangent but again suffered from carelessness. Just a few thought that the gradient at (-1,7) was $3x^2 - 6$ and many more than those converted their gradient of -3 to 1/3, just because "it's always tested". Solving the simultaneous equations usually scored a mark for the method but few battled through to the correct coordinates.

12) The candidates were well versed in the methods used in APs and GPs and many scored the first 8 marks.
In (ii)B the main problem was distinguishing between the n^{th} term and the sum of the terms up to it. Those who used the latter got nowhere. The common error in the work using $5 \times (1.1)^{n-1}$ was to log it as $\log 5 \times (n-1)\log 1.1$.
Most good candidates chose the correct formula and scored the 4 marks.
Nearly all the candidates solved the given inequality and arrived at $n > 25.16$ but about a third left that as their answer and another third rounded to 25.

4753: (C3) Methods for Advanced Mathematics (Written Examination)

General Comments

This paper was perhaps a little tougher than last summer's, and attracted a wide range of responses from candidates. Weaker candidates found it a tough test, and showed evidence of a lack of maturity in the understanding of the concepts and in problem solving and algebraic ability – it should be born in mind that this paper, unlike P2, is of full A2 standard. However, average candidates found plenty to do, and there were many excellent scripts achieving over 60 marks.

Although most candidates managed to attempt all the questions, there was some evidence of them rushing in question 8. This was usually caused by inefficient methods, or repeating the same question a number of times.

The general level of algebra remains a source of concern. In particular, many candidates are omitting brackets, and this led to marks being lost through algebraic errors with negative signs – see the comments on questions 7(ii), (iv) and 8(ii). A large number of candidates also failed to give their answers exactly in questions 7 and 8.

Comments on Individual Questions

Section A

1 The chain rule was generally well known, although there were occasional errors with the derivative of $u^{1/3}$. Weaker candidates failed to show the result was $1/y^2$; stronger ones cubed both sides and used implicit differentiation, which proves the result very efficiently.

2 Many candidates misinterpreted the units of P and used 8 000 000 and 6 000 000 instead of 8 and 6. We allowed generous follow through for this, so that they could achieve 5 marks out of 6 with this error. There were the usual errors in taking logs to solve for b, but in general this was quite well done. The final part was quite often omitted.

3 Although able candidates sailed through this test, solving a logarithmic equation like this seemed to be a fairly unfamiliar context to many candidates. The answer 2 ln $3x$ was a common error in part (i), and part (ii) suffered accordingly. However, there were 2 easy marks in (iii) for solving the quadratic equation, preferably by factorisation. We allowed follow through on negative roots for the final part, in which candidates had to phrase their answers carefully: for example, 'logarithms cannot be negative' was not allowed here.

4 This question proved to be the most demanding of the section A questions. In particular, the derivation of the volume of the cone result was done successfully by only the best candidates. Although only awarded 2 marks, this may have put off weaker candidates, although most wrote down the $\mathrm{d}V/\mathrm{d}t$ for a mark in part (i), and recognised the chain rule in part (iii).

5 Having the result of the implicit differentiation proved helpful to candidates here, and most either achieved 4 or 5 marks or 1 or 0. The derivative of the xy term stumped some candidates, and although $\mathrm{d}x/\mathrm{d}y = 1/(\mathrm{d}y/\mathrm{d}x)$ was well known, we wanted to see the resulting fraction inverted for the final mark.

6 The first three marks proved accessible to most candidates, provided they understood the notation and concept of inverse trigonometric functions. Candidates who started $x = 1 + 2 \sin x$ were penalised, however, and the domain was poorly answered. The use of radians in degrees in the coordinates of A and C was quite a common error, and quite a few candidates spent a lot of time trying to establish the coordinates, for example using calculus.

Section B

7 Plenty of candidates scored well on this question. However, the area in part (iv) lost three marks to virtually all and sundry.

(i) Most set y to zero, but many failed to solve the subsequent equation – again, equations, even simple ones, involving logarithms appeared unfamiliar to many candidates.

(ii) Most knew the derivative of ln x, and could apply a product rule to x ln x. However, omitting the bracket led quite a few to a derivative of
$2 - 1 + \ln x = 1 + \ln x$, and subsequent errors in the coordinates of the turning point. Many candidates worked approximately here, rather than expressing their answers in terms of e.

(iii) These 3 marks relied upon part (ii) being correct, although we allowed 1 follow through method mark on their derivative in part (ii). The product of the gradients result was well known.

(iv) The integration by parts was generally well done, although some took u and v the wrong way round, and we required to see the integral term simplified to ½ x before being integrated. However, a large majority then used the resultant integrand for the area, rather than integrating $2x - x \ln x$! This lost three marks to virtually all − the final answer was very rarely seen.

8 Some candidates were clearly pushed for time to answer this question.

(i) The results $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$ were not well known, and so answers to this part were weak. Many thought the function was even; but an equal number or more guessed it was odd without being able to prove it, and completed the graph correctly.

(ii) The quotient rule was well known, although some made errors in the derivative. Missing brackets out in the numerator often led to errors in showing the result in the paper, for example using $\cos^2 x - \sin^2 x = 1$. Many candidates found $x = 60$ rather than $\pi/3$, and few found the exact value of the y−coordinate of the turning point, or the range.

(iii) Most good candidates got four marks here quite easily. Weaker candidates failed to use substitution correctly in this trigonometric context.

(iv) We followed through their graphs in part (i), and as a result many achieved this mark – the stretch was well known, although some used scale factor 2 rather than ½.

(v) Again, we allowed generous follow through on answers to part (iii) – if this was halved, they got the marks. Some saw the point about the stretch, many re-started the integral (and often ran out of time!).

4754: Applications of Advanced Mathematics (C4)

General Comments

This was the first January session for this paper. The legacy 2603 was available for the last time for re-sit candidates and so the composition of the entry was different to other years.
The standard of work, like that of 2603 in January in previous years, was pleasingly high although there were also some weaker candidates.
Candidates generally scored proportionately similar marks on both Sections A and B. The exception being the weaker candidates who tended to perform better on the Comprehension.
There were some questions that candidates found more difficult than others. In particular, the trigonometric equation in question 4 was not well answered. It was also particularly disappointing to see that so few candidates included a constant of integration in question 8. Candidates should be advised to remember to give full reasoning when showing given results.

Comments on Individual Questions

1) This question was well answered. Candidates almost all had a good understanding of the required method. Errors tended to be sign errors such as
$-4x-2x= -8x$.

2) Candidates generally had a good understanding of the method and scored well. Some candidates tried, without success, to eliminate t from the two equations before differentiating. Another quite common error, which was disappointingly very similar to an error in the June 2005 paper, involved incorrectly inverting fractions term by term,
$$\frac{dx}{dt}=1-\frac{1}{t}\Rightarrow\frac{dt}{dx}=1-t$$
was often seen. Thus $\frac{dy}{dx}=\left(1+\frac{1}{t}\right)\times\left(1-t\right)$ was incorrectly given as the differential.

3) Full marks were often obtained in this question. Some candidates, however, incorrectly used the position vectors of the points instead of the directions between points when trying to find the scalar product. The evaluation of the scalar product was usually shown. The area was almost always correct although it was occasionally omitted.

4) This was the least successful question in Section A. There were two different approaches.
Those using substitution with double angle formulae had difficulty unless they chose to use $\cos 2\theta = 1- 2\sin^2\theta$ initially or used other substitutions in order to eliminate the constant term. Without an appropriate substitution they were unable to factorise the expression. There were also many inaccurate forms of the double angle formulae used.
For those that did use a correct substitution to form either $4\sin\theta\cos\theta - 2\sin^2\theta=0$ or $4\tan\theta - 2\tan^2\theta= 0$,or equivalent, many then factorised but cancelled out the term $\sin\theta=0$ or $\tan\theta=0$ losing the two solutions 0° and 180°.
Some candidates used the approach from $R\sin(2\theta+\alpha)$ with success.

5) Many candidates obtained full marks in this question. In part (i) there were some that did not start with $x-y+2z= c$, trying to use vector forms rather than the required cartesian equation. They then only tended to obtain one mark for $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}7+\lambda\\12+\lambda\\9+2\lambda\end{pmatrix}$ in part (ii).

6) (i) Candidates were generally successful when factorising out the term ½. Most errors in (i) came from numerical errors when simplifying the terms, including using $-\frac{x^2}{4}$ without the negative sign. A few thought that $\sqrt{(4-x^2)} = 2-x$.

(ii) Their expression was usually integrated correctly. However, some substituted values without first integrating.

(iii) Although there were many completely correct solutions, others omitted this part or gave the answer 30°.

7) (i) Most candidates established that *θ=β-α* although the reasoning was disappointingly imprecise. Some failed to use this result in the next part and did not realise that the use of the compound angle formula was required. There were, however, many completely correct solutions. The majority of the candidates correctly found the angle *θ*.

(ii) The use of the quotient rule for the differentiation of $\frac{6y}{160+y^2}$ was often successful. The implicit differentiation tended to be muddled although good candidates gave clear and complete solutions including, in some cases, the use of $\frac{dy}{d\theta}$ on the right hand side.

(iii) Although candidates found this relatively easy, a surprising number gave the value of *y* that gave the maximum but did not continue and find the value of *θ*.

8) (i) The first part of this question was successful for good candidates. Many others made errors. Much depended upon the approach. For those using the chain rule the common error was to differentiate $1/(1+kt)$ as $-1/(1+kt)^2$ and forget to multiply by k. Another error involved not realising how to change $-ak/(1+kt)^2$ to $-kx^2/a$. For those using the quotient rule the error involved not differentiating the constant *a* as 0.

(ii) This was usually successful.

(iii) Some candidates omitted this but the answer was usually correct.

(iv) The partial fractions almost always achieved full marks.

(v) Few candidates showed the separation of variables. Most attempted the integration of the partial fractions but many made mistakes either, for instance, by thinking that $\int \frac{1}{2y} dy = \ln 2y$ and missing out the $\frac{1}{2}$ or by missing the minus sign in $\frac{1}{2}\int \frac{1}{2-y} dy = -\frac{1}{2}\ln(2-y)$ or changing the sign of the partial fraction in (iv) to fit the different sign of the logarithm in the given solution.

Even for those that integrated this part correctly, **very** few candidates included a constant of integration and thus they could not establish it was zero, or equivalent, and complete the solution.

In the final part, the anti-logging was usually correct. *y* was often isolated correctly but the final form was not always achieved.

(vi) This was often correct but sometimes omitted. There was some confusion between $y \to 2$ (2000 squirrels) and 2 squirrels.

Section B The Comprehension

1) Usually correct but did not always refer to the **largest** number.

2) There were many completely correct solutions. In part (ii) there were several numerical errors but the main error involved dividing the previous number in the table by 2 or 3 instead of the original number.

3) Some did not give the value of *a* merely quoting $5.6 < a \leq 11.2$.

4) The table was almost always correct. Some candidates mistakenly thought that the end-point of an interval was only the lower (or the higher) limit. Others failed to support their argument with the required example from the table.

5) (i) Usually understood but not always well explained.

(ii) Only rarely successful. Many tried to simplify the inequalities as one expression which was possible but difficult. For those that treated it as two separate parts there were some successful solutions from able candidates.

(iii) Few explained why the number of unused votes could be zero but had to be less than a.

4755: Further Concepts for Advanced Mathematics (FP1)

General Comments

Most of the candidates for this paper were extremely good, as might be expected for a January sitting of FP1, and consequently it was high scoring.

A small proportion of candidates were not well prepared for a paper of this nature.

The overall standard of scripts was pleasing, although even among the best scripts there was some evidence of immaturity in the written mathematics – missing brackets, imprecise explanations and poor use of notation.

All of the questions worked well, including the slightly out-of-the-ordinary question 9 which involved a singular transformation.

A small number of the weaker candidates either missed out question 9, or made only a very poor attempt, which may indicate they had run out of time.

Comments on Individual Questions

1) **Matrices**

This question was well answered. The most common mistakes resulted from careless arithmetic but a significant number of candidates thought that a 3×2 matrix (on the left) could not be multiplied by a 2×2 matrix (on the right). Another common error was to calculate $2\mathbf{B}$ as $\mathbf{B}^2$.

In part (ii) a large majority of candidates knew the meaning of commutativity but a few muddled it with associativity and some failed to conclude their argument by stating that they had shown, for their chosen **X** and **Y**, that $\mathbf{XY} \neq \mathbf{YX}$.

2) **Complex numbers**

Many candidates got this question fully right but a significant minority did not know the meaning of $|z|$.

Many answers showed poor use of brackets. Many answers were not very logically presented. The most lucid found zz^* and $|z|^2$, then showed the subtraction to give 0.

3) **Series summation**

This question required the use of the standard results for $\sum r^2$ and $\sum 1$. It attracted many correct answers. Nearly all candidates knew how to approach the question but many wrote $\sum 1 = 1$ instead of $\sum 1 = n$. Another common mistake was to fail in the step where both expressions had to be written over a common denominator.

4) **Use of matrices to solve simultaneous equations**

In part (i) candidates had to write a matrix equation as a pair of simultaneous equations and nearly everyone was successful in this.

In part (ii) candidates were required to show that the determinant of the matrix was zero and this was mainly done successfully. They were then asked to interpret this in the context of the equations; only a minority of candidates gave the full answer, that there were either no solutions or infinitely many. A significant minority of candidates attempted to answer this last part in terms of geometrical transformations even though this was not what was asked for.

5) **Roots of a cubic equation**

This question was well answered, with most candidates knowing what to do. However, quite a number of candidates made sign errors.

Part (ii) required candidates to construct a related equation. This could be done either by substitution or by manipulating the roots algebraically to find the coefficients of the new equation. Most, but not all, candidates chose the latter method. A number of candidates gave a polynomial expression rather than equation, missing out the "= 0" and this cost them one mark.

6) **Proof by induction**

Many candidates knew just what to do for this question and scored full marks.

A significant minority, however, did not give the argument explicitly; some used their own phraseology to reduce the amount of writing but in so doing bypassed the essential logic. In extreme cases, candidates scored all the marks for the algebra but none of those for presenting the argument.

The standard of algebra displayed in this question by most candidates was pleasingly high, though many were guilty of poor use of brackets in their working.

A handful of candidates attempted to prove the statement by using methods other than induction and they were given no marks as the question explicitly required proof by induction.

7) **Graph**

This question was on the whole well answered but many candidates dropped a few marks as they went through it and most failed to earn full marks for part (v).

- (i) The vast majority of candidates answered this correctly.
- (ii) This asked for the equations of the asymptotes and a significant minority of candidates gave the horizontal asymptote as $y = 0$ instead of $y = -1$.
- (iii) Candidates were asked to describe the behaviour of the curve for large positive and negative values of x and to justify their answers. Many candidates merely gave the horizontal asymptote, which had already been asked for in the previous part; they were, of course, expected to show, with justification, whether the curve approached this asymptote from above or below.
- (iv) Most candidates earned all three marks. However, a significant minority failed to show the asymptotes clearly or give the intercept.
- (v) This was about an inequality and candidates were expected to see from their graphs that the solution involved two intervals. However, many made the mistake of giving only one interval. Only the very best candidates got this fully correct. The most successful method was to solve $\dfrac{3+x^2}{4-x^2} = -2$ and use the solutions, with the sketch, to identify the regions.

8) **Complex numbers**

This question, about a cubic equation with complex roots, was well answered and many candidates got it fully right.

(i) Candidates were asked to justify given values for two of the coefficients of a cubic equation and this caused some difficulty to a minority who failed to equate real and imaginary parts to 0.

(ii) This asked for all the roots of the cubic equation. While most candidates got this right, some used very inefficient methods to do so. The easiest method was to identify the second root as the conjugate of the one that had been given, and then to use either the sum or product of the roots to find the third one. Most candidates used the complex roots to find factors, multiplied to get a quadratic, then divided this into the cubic to get the third factor and hence the root. Many did this correctly, but it was a long-winded method.

(iii) Most candidates got the Argand diagram in part (iii) correct, but there were a few surprising errors with points plotted in quite the wrong positions.

9) **Singular transformation**

Although a few candidates made little or no progress with this question, the majority were able to follow it through part by part and there were many high scores. For those who were on the whole successful the greatest difficulties occurred in part (v), where they had to recognise that any other line with gradient 2 would be translated onto *l*, and in the explanation at the end of part (vii). There were also quite a number of candidates who did not know how to find the matrix in part (vi).

This question demanded some understanding and so it was pleasing to see how many candidates achieved success.

(i) The vast majority of candidates answered this correctly.

(ii) A large proportion got this right although $(2x, y)$ and $\left(\frac{x}{2}, y\right)$ were common errors.

(iii) Most got this right, but many omitted to answer and y = 2x was a common error.

(iv) Most got this right, but many incorrectly gave lines parallel to the y-axis, rather than the x-axis.

(v) This required some deeper thinking and most candidates got this part wrong.

(vi) A large proportion of candidates did not know how to find the transformation matrix.

(vii) Most who had found a matrix in part (vi) could calculate its determinant and knew the meaning of ‘singular’. Only the very best candidates earned all the explanation marks. The best explanation was to state that the transformation was many-to-one.

4756: Further Methods for Advanced Mathematics (FP2)

General Comments

There was quite a range of performance on this paper. There were some really good scripts, with about 10% of candidates scoring more than 60 marks out of 72. On the other hand, about 20% of candidates scored less than 30 marks; many were clearly not ready to take the paper and found it to be a severe challenge. Some candidates appeared to run out of time, but this was usually a consequence of using very long and complicated methods in the integration questions.
In Section A, the work on the matrices topic (question 3) was of a much higher standard than that on calculus and complex numbers.
In Section B, almost every candidate chose the question on hyperbolic functions.

Comments on Individual Questions

1) This question, on polar equations and integration, was found to be quite difficult, especially part (c), The average mark was about 10 out of 18.
In part (a)(i), most candidates drew a curve of the correct shape with three loops, but the use of continuous and broken lines was usually incorrect. A common error was to use broken lines in the third and fourth quadrants, which corresponds to the domain $0 \le \theta \le \pi$ instead of the given $-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi$.
In part (a)(ii), the calculation of the area was generally well understood, although the limits of integration were quite often incorrect. Most candidates realised that the integration of $\cos^2 3\theta$ required the use of a double angle formula, but the details were not always correct.
In part (b), most candidates recognised that this integral involved arcsin, and some were able to write down a completely correct result with little difficulty. The factor $\frac{1}{2}$ was often omitted, and $\frac{4x}{3}$ sometimes appeared instead of $\frac{2x}{\sqrt{3}}$.
In part (c), very many candidates did not make a tan substitution, and so were unable to make any progress. Some used the correct substitution and obtained an integral involving $\frac{\sec^2\theta}{\sec^3\theta}$ but failed to simplify this to $\cos\theta$ and complete the integration. Only a few obtained the correct answer.

2) This question, on complex numbers, was the worst answered, with an average mark of about 9 out of 18.
Some candidates sailed through part (i), but the majority made at least one slip, particularly with the arguments; quite a few gave the modulus of jw as $\frac{1}{2}\mathrm{j}$. Many appeared to be very uncertain about what was required, possibly because there was not a precise value of θ to work with.
The proof in part (ii) was handled well, and usually scored full marks. Those who started by writing $w^* = \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta}$ had an easier time than those who went straight to $w^* = \frac{1}{2}(\cos 3\theta - \mathrm{j}\sin 3\theta)$.
In part (iii), most candidates knew that they should consider $C + \mathrm{j}S$, but many seemed to be unfamiliar with the methods required to progress beyond this, so the marks in this part were often low. Some who obtained the correct sum of the infinite series were unable to convert it into a form with a real denominator. However, there were some confident and efficient solutions from candidates who recognised the connection with part (ii) and then kept the numerator in exponential form.

3) This question, on matrices, was by far the best answered, with an average mark of about 14 out of 18. Most candidates displayed good algebraic and numerical skills.
In part (i) the characteristic equation was usually obtained correctly, by a great variety of methods. There was even some use of elementary row operations.
In part (ii), almost all candidates found the eigenvalues accurately.
Part (iii) was often answered well, although some candidates solved $(\mathbf{M}+\mathbf{I})\mathbf{x}=-\mathbf{x}$ or $(\mathbf{M}-\mathbf{I})\mathbf{x}=\mathbf{0}$ instead of $(\mathbf{M}+\mathbf{I})\mathbf{x}=\mathbf{0}$.
Most candidates were successful in part (iv). The simplest method was to transform the given vectors and recognise the images as multiples of the original vectors, but some used much longer methods, deriving the eigenvectors in the same way as in part (iii).
In part (v), most candidates knew that **P** was the matrix of eigenvectors, but many gave **D** as the diagonal matrix of eigenvalues instead of their cubes.
In part (vi), the Cayley-Hamilton theorem and its application were generally well understood. Sometimes **I** was omitted from the equations.

Section B

4) The average mark for this question, on hyperbolic functions, was about 10 out of 18.
In part (a), most candidates converted the equation to a quadratic in exponential form, with a substantial number obtaining the correct answers.
In part (b), those who wrote $\sinh x$ in exponential form were usually successful, although there were a few sign errors. Very many attempted to use integration by parts, which is not an appropriate method here.
In part (c), the general form of the derivative of $\operatorname{arsinh}\frac{2}{3}x$ was usually correct, although many had an incorrect numerical factor. Integration by parts was often applied correctly, but very few managed to produce a completely convincing derivation of the given answer. The first difficulty was the integration of x times their answer to part (i); many stopped at this point, and others obtained an incorrect numerical factor. The next problem was the derivation of the $2\ln 3$ term; $\operatorname{arsinh}\frac{4}{3}=\ln 3$ was often stated without any explanation.

5) There were fewer than ten attempts at this question on the investigation of graphs. There was some competent work in parts (i) to (iv), but no candidate scored any marks for the improved sketches in part (v).

4758: Differential Equations (Written Examination)

General Comments

The standard of work was generally good. Questions 1 and 4 were attempted by almost all of the candidates. Most then chose question 2 rather than question 3. Candidates often produced accurate work; however errors in integration were common.

Comments on Individual Questions

1) (i) This was often completely correct.

(ii) Many correct solutions were seen, but some candidates could not state the correct complementary function associated with a repeated root of the auxiliary equation. When considering the behaviour as t tends to infinity, it was recognised that candidates may not know the behaviour of the $t\,\mathrm{e}^{-3t}$ term, and so credit was given on the basis of how they dealt with the other term.

(iii) When considering the complementary function in the general case, many candidates omitted to consider complex roots of the auxiliary equation. When considering the real roots, candidates often did not explain why both roots must be negative.

2) (i) This was often done well, except for slips in the integration. However a minority of students ignored the suggestion to separate variables and when using the integrating factor method found the resulting integral difficult.

(ii) The integrating factor method was usually applied and understood, but errors were common, in particular with the integration by parts and either omitting the constant or failing to divide it by the integrating factor when expressing y in terms of x.

(iii) The Euler calculation was often done well but some worked in degrees and others produced unrecognisable figures with no indication of method.

3) (i) Most candidates were able to use Newton's second law to obtain the differential equation, but explanations of the signs were often vague.

(ii) Candidates tried various methods to solve the differential equation, and it was common for separation of variables to be ignored. Even those who used the correct method often made errors in integration.

(iii) This differential equation also caused problems for candidates. Many did not identify the correct method, and even among those who did, correct solutions were extremely rare.

(iv) Many were able to deduce the terminal velocity. The standard of graph sketching was very variable. Some did not consider the entire motion, some ignored the initial conditions, but some produced good sketches. Even some of those who had been unable to solve the differential equations gained full credit here by deducing the key features of the graph from the given information.

4) (i) The elimination of y was often done well, although a few differentiated the first equation with respect to x rather than t.

(ii) The solution was often done well, although minor slips were common.

(iii) Many candidates correctly used the first equation. Some tried to set up and solve a differential equation for y; such attempts never tried to relate the arbitrary constants in the two solutions.

(iv) The limiting expressions were usually well done, but many candidates did not make clear conclusions in either case.

4761: Mechanics 1 (Written Examination)

General Comments

Most of the candidates seemed to find a lot that they could do on this paper. There were few candidates with very low scores and many who achieved full marks or nearly full marks.

It was pleasing that fewer candidates than in the past made elementary mistakes with the inappropriate use of constant acceleration results, and the determination of forces in a coupling and of a normal reaction; it was also saddening that there are still so many who seem unaware of the correct methods.

On the whole, poor presentation of scripts seemed less of a problem than in some recent sessions but many candidates manage to confuse themselves even if they cannot quite manage to confuse the examiner.

Comments on Individual Questions

Section A

1) The use of an acceleration-time graph.

(i) This was usually done correctly but some candidates inverted the fraction and others omitted the negative sign.

(ii) Answered correctly by the majority of the candidates.

(iii) This was one of the least well done parts of the paper. Quite a few candidates did not seem to realize that the displacement from $t = 4$ to $t = 9$ was negative. Many thought that the particle started getting further away from A when the *gradient* of the graph became positive and others assumed that the displacement at $t = 12$ was required. Of course, many others efficiently found the required closest approach.

2) Equilibrium of an object involving a smooth pulley.

(i) Many correct answers were seen. The most common errors were to think that the string being light and/or the system being in equilibrium were enough to ensure the same tension throughout.

(ii) Almost everyone found the tension correctly as the weight of the object.

(iii) There were many correct solutions to this. The most common error was to equate the tension in the string round the pulley to the component of tension in *only one* of BC and BD. A few candidates did not resolve at all and quite a few attempted to resolve in the direction of BC or BD obtaining $T = 39.2 \cos 20°$.

3) The magnitude and direction of a vector. Vectors in the same direction.

(i) This was usually done well. The most common mistake was not to give the direction as a bearing.

(ii) This was usually done well.

(iii) The majority of the candidate knew how to do this but many failed to carry through their plan accurately. The most common errors were to invert one of the ratios, to 'drop' the negative sign in front of 18 or to think that $-18 + q = 72$ gives $q = 54$.

4) Newton's second law applied to a car and trailer accelerating on a horizontal road.

There were many very good, concise responses to this question but the usual misunderstandings about the application of Newton's second law to connected particles were seen.

(i) Many of those candidates who first considered only the car omitted the tension and so did not see the need for a second equation. Quite a few elementary arithmetic mistakes were seen.

(ii) Often done better than part (i). Quite a few candidates used wrong exotic methods to find the tension.

5) **The dynamics and kinematics of a particle with acceleration given in vector form.**

Many candidates scored marks only in parts (i) and (ii) as they wrongly applied the constant acceleration results in part (iii)

(i) In this part and part (ii), some candidates 'lost' the vector notation.

(ii) Most candidates knew they should apply Newton's second law; the chief error was to give only the magnitude of the force and this was not penalised.

(iii) The use of constant acceleration results was particularly disappointing as part (i) had been deliberately set to draw the attention of the candidates to the acceleration having different values at different times. Quite a few candidates did not involve an arbitrary constant, others assumed it was (vector) zero and yet others gave it wrongly without working shown as –5 **i**.

6) **A kinematics problem involving constant acceleration and simultaneous equations**

Many candidates obtained full marks efficiently. A common mistake was to misread the question as saying that the given speed was achieved 7 seconds instead of 5 seconds after passing the sign.

(i) The chief common mistake was to argue that the car is travelling at 7 m s^{-1} after the first 2 seconds (i.e. assuming that the acceleration is zero); this trivialised the problem. Other errors usually involved inconsistencies in the values taken for u, v and t .

(ii) This was usually done well but many candidates missed it out – perhaps they overlooked it.

Section B

7) **Statics and kinematics of a box being pulled by a string**

Many candidates scored full marks on this question and it was pleasing to see a better standard this session on questions such as parts (iii) and (v). Some errors to very simple arithmetic were seen in the answers.

(i) Most candidates answered this correctly.

(ii) Most candidates knew what to do but some exchanged sine and cosine.

(iii) There were many good answers but, as in previous sessions, many candidates seem to believe that the normal reaction is the component of the weight acting perpendicular to the surface and so they omit the component of the tension in the string.

(iv) This was answered well by most candidates. A common error was to omit the 125 N force.

(v) Quite a few candidates continued to use the 'old' acceleration. Those who realized that the acceleration had changed usually found it correctly and went on to obtain the correct distance.

(vi) It was pleasing to see the large number of correct answers to this part. The most common errors were to fail to resolve Q or, less frequent, to exchange sine and cosine.

8) **A stone thrown over a wall**

Many candidates answered much of the question well but many found some of the parts challenging. The lack of given answers did not seem to be a problem – indeed it seems that many candidates are more comfortable getting the right answer than explaining why a given answer is right.

(i) This was generally done well except by candidates who had struggled elsewhere on the paper. Confusion of sine and cosine was less common than elsewhere in the paper, perhaps because the results were so familiar. The most common error was to omit the +1 in the expression for the vertical height above the ground.

(ii)

(A) Most candidates knew what to do and did it well. The most common errors were: to find the time for the stone to return to its height when projected and not halve this value: to assume that the greatest height reached by the stone is 6 m.

(B) Again, there were many correct answers (or, at least, answers that followed from (A)). Common mistakes were to start again and now get the time wrong or to incorrectly apply the formula for the range.

(C) Again, most candidates knew what to do. Errors usually came from starting again and this time forgetting the 1 m above the ground or from using values t that were insufficiently accurate to establish the value of 2.5 to 2 significant figures.

(iii) Many candidates clearly did not know what this meant and some of these left out the part. Some thought it was a request to write the equation in terms of t in a vector form.

(iv) Many candidates knew that they should equate an expression for y to 6. Most chose an expression for y in terms of t but some (sensibly) used their trajectory equation. Errors often came because candidates 'started again' and this time omitted the 1 m above the ground. Quite a few candidates made errors in the solution of their quadratic equation and others gave the solutions to insufficient accuracy. At the end, many candidates gave the new distance to the wall (13.66 m) instead of the distance moved (13.66 – 8.66 = 5 m). There was only a 1 mark penalty for this.

4762: Mechanics 2 (Written Examination)

General Comments

The majority of candidates found this paper to be quite accessible with many able to obtain at least some credit on some part of every question. There was some evidence that a few candidates found the paper long. As in previous sessions the standard of presentation was high and some excellent work was seen from a large number of candidates. However, many candidates do not seem to understand the value of a diagram both in assisting them to a solution and in clarifying working to an examiner. Questions that required candidates to show a given answer or explain an answer posed problems to a significant number of candidates with many not appreciating the need to explain fully and clearly the principles or processes involved.

Comments on Individual Questions

1 **Impulse and Momentum**

Overall, this question posed few problems to the vast majority of candidates.

i) Almost all candidates could obtain full credit for this part.

ii) Sign errors were common in this part particularly when using Newton's experimental law. Those candidates who drew a diagram and then constructed equations consistent with it were inevitably more successful than those candidates who either did not draw a diagram or who drew one and then ignored it. Candidates who did not draw a diagram usually failed to explain their sign convention when showing that $u = 18$ and also omitted the direction when describing the motion of P.

iii) It was pleasing to see many complete and accurate answers to this part of the question. The majority of candidates understood the need to look at the motion in 2 dimensions and only a minority failed to establish that the velocity of D was $-8\mathbf{i} + 6\mathbf{j}$. Showing that C and D move off at right angles to each other was less well done. Most candidates calculated the directions of the two particles as angles and showed that these added to give 90° but then failed to show clearly why the angle **between** the parts was 90°. The more able candidates used either the scalar product or showed that the product of the gradients was –1.

2 **Resolving and moments**

This question was well done by many candidates

i) Almost all the candidates obtained full marks for this part.

ii) Most candidates realised that they had to take moments and could establish the correct value for T. A small minority made the problem more complicated by taking moments about either the centre of mass or about B and were then unable to proceed further. Others assumed that the reactions at C and D were the same as in part i) and tried to calculate T by resolving only. Few candidates could explain clearly why the support at D could not be smooth. To maintain horizontal equilibrium, a force at D was required to counteract the horizontal component of T. This could only come from the friction at D since the reaction at C was zero. Many candidates merely said that the beam would slide or slip if there was no friction.

iii) Most candidates could establish $P = 60\sqrt{3}$ but could not resolve correctly to obtain the frictional force. Most realised that the normal reaction at B needed to be calculated in order to find the coefficient of friction but some creative algebra and arithmetic was seen by those candidates who were determined to show that $\mu = \tan 30$.

3 **Centres of mass**

Many candidates scored well on both parts of this question. Part a) was generally attempted more successfully than part b)

a) i) The majority of candidates scored highly in this part.

ii) This posed few problems to the majority.

iii) This part caused difficulty to a minority of candidates; these did not appreciate that the y co-ordinate of the square to be removed had to be where $y = 1$. Some tried to take the square from the part below the x axis, others misinterpreted the question and took the maximum value possible for the centre of the square to be removed.

b) i) Candidates were almost invariably successful in establishing the given result.

ii) On the whole, the standard of diagrams drawn was poor. Many were less than helpful to the candidates as they had inadequately labelled forces, omitted forces or had more than one force with the same label. The directions of the internal forces were not made clear. Some candidates drew a diagram and then ignored it when calculating the internal forces. This gave rise to sign errors and inconsistencies between equations. A very small number of candidates penalised themselves in terms of time by calculating all of the internal forces, not just those that were requested. Those candidates who only drew small and separate diagrams for the forces at each pin joint were not usually as successful as those who drew a complete diagram. The directions of the forces were not always consistent between diagrams.

iii) This part was not well done by the majority of candidates. Many did not understand the need to look at the horizontal equilibrium and the direction of the calculated forces. Others stated without justification that the system would collapse.

4 **Work and energy**

This question was not as well attempted as the previous questions. It caused difficulties for a significant minority of candidates who seemed unsure about the principles involved.

i) While most candidates understood that power was the rate of doing work, many assumed that the total work done was 510 J. A very small minority understood that work was being done by the tension in the string. However, instead of using the change in kinetic energy as a measure of this, overcomplicated matters by attempting to use Newton's second law and the constant acceleration formulae to find the acceleration and distance travelled. Very few attempting this were successful.

ii) A) Many candidates successfully completed this part but a minority forgot one or other of the two masses involved. The main errors occurred when candidates treated the mass m as if it lay on the inclined plane.

B) This caused no problem to the vast majority.

C) Again, few problems were encountered by those who had successfully completed A and B. The main error was in using the mass of only one of the objects in the calculation of the kinetic energy instead of the combined mass of 25 kg.

iii) Many of the candidates completed this part successfully. Omitting one of the kinetic energy terms or the gravitational potential energy terms was the main cause of error for others. Candidates who used Newton's second law and the constant acceleration formulae did not score as highly on the whole as those who used work-energy methods

4763: Mechanics 3

General Comments

The scripts were generally of a high standard, with half the candidates scoring 60 marks or more out of 72. The questions were usually answered confidently and accurately, and the only topics which caused significant difficulty were circular motion and simple harmonic motion.

Comments on Individual Questions

1) This question, on dimensions and simple harmonic motion, was very well answered, and the average mark was about 16 out of 18.
In part (a), almost all candidates knew the dimensions of force, and understood how to find the indices α, β and γ, although some made errors when solving the equations. The tension in the second wire was very often found correctly, although some candidates assumed that the mass, rather than the mass per unit length, remained constant.
The simple harmonic motion problem in part (b) was usually answered correctly, with most candidates using $v^2 = \omega^2(A^2 - x^2)$. Those working from $x = A\sin\omega t$ were more inclined to make arithmetic errors.

2) This question, on motion in a circle, had an average mark of about 14 out of 18.
In part (a), about two-thirds of the candidates found the force correctly. The principles required were very well known, but many candidates made errors in the calculations, especially when finding the angular velocity.
In part (b)(i), some candidates did not realise that conservation of energy was needed, but generally this was well answered apart from frequent errors in the potential energy term.
In part (b)(ii), most candidates correctly considered forces in the radial direction, the most common error being omission of the weight.
In part (b)(iii), very many candidates did not know what was required, and correct answers for the tangential acceleration were not at all common.
In the final part (b)(iv), the condition for the particle to leave the surface was well understood.

3) This question, on elasticity and simple harmonic motion, was the worst answered question, with an average mark of about 12 out of 18.
In part (i), the modulus of elasticity was correctly obtained by almost every candidate.
In part (ii), about one third of the candidates omitted the weight when calculating the acceleration.
In part (iii), some candidates made errors when calculating the elastic energy, but the majority set up the energy equation correctly. Having found the vertical distance moved by the rock, a significant number omitted the final calculation of the distance OA.
In part (iv), the correct expression $147(0.8 + x)$ for the tension usually appeared, and about half the candidates derived the differential equation correctly. There was some confusion over signs, but the main reason for failure was not writing down an equation of motion with three terms.
Part (v) was rarely answered correctly. Although the form $x = A\cos\omega t$ was well known, many had $A = 5$ instead of $A = 4.2$ or $\omega = 12.25$ instead of $\omega = 3.5$. Only the best candidates realised that the rope became slack when $x = -0.8$.

4) This question, on centres of mass, was very well answered, with an average mark of about 15 out of 18.

In part (i), the methods for finding the centre of mass of a lamina were well understood, and the integrals were evaluated accurately. Most candidates scored full marks.

In part (ii), the principles were very well understood, although the centre of mass of the second lamina was often taken to be $(1.5,\ 1.6)$ instead of $(2.75,\ 1.6)$.

In part (iii), almost all candidates realised that the centre of mass was vertically below A, but most were unable to obtain the required angle accurately. Some gave the angle between PQ and the vertical instead of the horizontal.

4766: Statistics 1

General Comments

Overall many candidates were able to make a good attempt at all of the questions, with the exception of question 5 which many found difficult. It is pleasing to note that there were very few scripts where candidates seemed to have no idea of how to tackle almost anything in the paper. The new work on expectation and variance of a discrete random variable was well answered. Centres should remind their candidates that in any question involving the binomial distribution that the definition of p should be clearly stated in the solution. Many candidates were familiar with the new formula for the sample standard deviation with the divisor of $(n - 1)$ but the examiners did equally see many candidates using a divisor of n, which in the new specification is defined as the rmsd (root mean squared deviation).

The two longer questions in section B attracted good responses with question 7 proving more popular than question 8. There was some evidence, judging by the incomplete attempts at question 8, that candidates had not divided their time sensibly across the paper. This was often linked to candidates using time consuming methods, such as calculation of multiple binomial probabilities when the answer could be found in tables, or recalculating mean and standard deviation from scratch, when coding could be used.

Candidates should also be reminded that they should show sufficient working in the calculation of the mean and standard deviation. Many preferred to state answers only (often ruthlessly rounded) and it was then impossible to award the appropriate method marks. Another source of lost marks was through premature approximation of answers which were then used in further calculations.

Comments on Individual Questions

Section A

1) **Times to complete a crossword puzzle: range, IQR, outliers and description of the distribution**
This question was generally well answered with many candidates gaining high marks and almost all gaining both marks for part (i). The formula for identifying outliers in terms of 1.5 x IQR from the relevant quartiles was well known but there was a minority of candidates who believed it was referenced from the median rather than the quartiles, or used a multiplier of 2 instead of 1.5. Part (iii) was usually answered correctly but a number of candidates used the phrase 'a positive distribution' rather than positively skewed. Negative instead of positive skew was sometimes seen.

2) **Letters in envelopes: Probability distribution. Calculation of E(X) and Var(X)**
The explanations in parts (i) and (ii) were usually convincing but many candidates gave a simplistic response to both parts by trying to justify that the sum of the probabilities was unity. Whilst this was true, it did not answer the questions posed. The calculation of E(X) and Var(X) was usually well answered with only the occasional candidate using $\sum xp^2$ or $\sum(xp)^2$ instead of the correct $\sum x^2 p$. Other occasional errors included division of the correct value of E(X) by 5, and failure to subtract $(E(X))^2$ from $E(X^2)$.

3) (i) **The binomial distribution: Imperfect bowls. Hypothesis test on p**
This was generally well answered, although a number of candidates wasted time calculating probabilities, rather than using the binomial tables. Those that did use summative probabilities often floundered by omitting $P(X = 0)$ or were found wanting through premature approximation of their answers. Too many found $P(X \leq 4)$, $P(X = 4)$ or $1 - P(X \leq 3)$ instead of the required $P(X \leq 3)$.

(ii) In the hypothesis test, although many candidates gave correct hypotheses in terms of p, few defined p explicitly in words. Centres should advise candidates that such a definition does attract credit. It was notable that from any given centre it was usually the case that either almost all candidates defined p or no candidates did so. The hypotheses themselves were usually correctly given but a number of candidates still continue to lose marks through poor notation. Candidates should be aware that $H_0 = 0.2$ is **not** an acceptable notation, nor is $H_0 : P(X=0.2)$. The standard notation is H_0: $p = 0.2$. As in previous sessions, many candidates used point probabilities, which effectively prevents any further credit being gained. Those who were successful in comparing the tail probability of 0.2061 with 0.05 often lost the final mark by not putting their conclusion in context. To simply state 'Accept H_0' on its own is not sufficient to gain credit here. A conclusion along the lines of 'There is insufficient evidence to claim that there has been a reduction' is needed to gain the mark.
An argument based on critical regions is of course perfectly acceptable, but candidates preferring to use such arguments need to be very precise. To simply state that the critical region is $\{0\}$ without a probability justification is insufficient.

4) (i) **Sugar bags: Adjustments the company could make. Mean and sample standard deviation**
Comments often made no mention of the mean and standard deviation. Vacuous remarks such as 'put more sugar in', 'use bigger bags' or 'install more accurate machinery' etc were commonplace. Those that did use the mean and standard deviation were equally non committal with statements along the lines of 'adjust the mean', 'adjust the standard deviation'. Again the reasoning needs to be clear. '**Increase** the mean' and '**decrease** the standard deviation' were the answers required.

(ii) The calculation of the sample mean and sample standard deviation were usually correct but a number of candidates calculated root mean squared deviation with a divisor of n. The specification explicitly defines standard deviation with a divisor of $n - 1$ and candidates should be aware of this. A few candidates found variance correctly but then forgot to take the square root. Some candidates used the old formula $\sqrt{(\sum x^2/n - ()^2)}$ and whilst such candidates were on this occasion given credit, candidates should be aware that this approach will only in future attract any credit at all if it leads to a fully correct answer. In the calculation of any of the relevant measures of dispersion, candidates are **strongly recommended** to first calculate the sum of squares S_{xx}. The data in this question were such that premature approximation made a very large difference to the final answer for standard deviation and it is pleasing that such approximation was relatively uncommon.

5) **Athletes team: Probability rules. Mutually exclusive events. Test for independent events**
Candidates found this question difficult. Most were unaware that the answers could easily be obtained by counting the relevant ticks in the table. Instead many resorted to inappropriate use of formulae. Many assumed the events to be independent or mutually exclusive when they were not.

(i) Many candidates simply found $P(A) \times P(B)$, assuming independence.

(ii) Many candidates found $P(C \cup D) = \frac{3}{10} + \frac{4}{10}$, often then subtracting $\frac{3}{10} \times \frac{4}{10}$ instead of the correct form $P(C \cup D) = \frac{3}{10} + \frac{4}{10} - \frac{1}{10}$.

(iii) Many candidates gained credit here for identifying the mutually exclusive events.

(iv) A number of candidates correctly used a test for independence, most popularly $P(A \cap B) = P(A) \times P(B)$, or less often a test based on conditional probability. However there were many incorrect qualitative arguments seen, often noting that some athletes took part in both events B and D.

6) **Selection of songs at a performance**

This was the best answered question on the paper with a sizeable majority scoring full marks. Part (i) was invariably correct but a variety of incorrect responses was seen in part (ii) with ${}^{12}P_7$, 7^7 and 7^2 instead of 7! being the most popular of these.

7) **GCSE and A level grades: Mean. Histogram. Sample mean and Sample standard deviation. Linear coding of data**

There were some excellent responses to this question.

(i) Most candidates scored both marks.

(ii) Most candidates were aware of how to construct a histogram correctly either via a frequency density or frequency per standard interval approach. Those who chose the latter route unfortunately often simply labelled the vertical axis as 'frequency density' rather than eg 'frequency per 0.5 GCSE points' . There were equally many erroneous constructions seen by the examiners ranging from a simple frequency plot to frequency divided by mid-points plot and even frequency × class widths plots.

(iii) Many candidates used the mid-points correctly to calculate the sample mean and sample standard deviation, but frequently rounding errors led to inaccuracies. Candidates who insist on giving answers only (with no supportive working) to this type of question must do so with great care. Often the examiners saw incorrect answers followed by the legend 'calc used'. Unfortunately no marks could be awarded for such a response. As mentioned before, there is still a minority of candidates who mistakenly calculated the rmsd instead of the sample standard deviation. Likewise some candidates did not deal correctly with the frequencies, calculating $\sum xf^2$ or $\sum(xf)^2$ instead of the correct $\sum x^2 f$.

(iv) Many candidates achieved the answer of 50.2 but some failed to follow it up with a declaration that it was equivalent to 'grade B'. A small number did the reverse, offering an answer of grade B without the required supporting evidence.

(v) Curiously 5.5 was often not substituted into the formula. Comments were often based on personal feelings such as 'predict a higher grade to encourage them' rather than interpreting the value of 25.5 in the context of the table given in the question.

(vi) The coded mean was usually correct but many applied the '-46' to the sample standard deviation or simply said 'it would not change'. A substantial number of candidates wasted an inordinate amount of time by recalculating a 'new' set of data using the given formula, often making errors along the way.

8) **Probability methods applied to selecting doughnuts. Conditional probability. Binomial distribution calculations.**

Many candidates scored well in this question with some gaining full or near to full marks.

(i)(ii) Many scored full marks in both parts. However a minority did the whole question based on 'with replacement' for which some allowance was made. Another error occasionally seen was an attempt to use a binomial distribution with $n = 12$. The use of fractions rather than decimals is strongly advisable for a question such as this. Candidates using decimals usually had rounding errors that got worse as the question went on.

(iii) Multiplication by 3 rather than 3! was common and equally often no multiplier at all was seen. Another common error here was to believe that the answer was 1 − answer (ii).

(iv) The conditional probability was usually answered correctly, again with a fairly generous follow through based on the earlier answers. However some candidates applied the conditional probability formula and then wrote down a denominator of $P(A) \times P(B)$ which is of course only correct if A and B are independent. This is extremely unlikely to be the case in a conditional probability question.

(v)(vi) Both parts proved difficult for many candidates. Although often candidates realised that a binomial distribution was appropriate many of them used the wrong parameters, often using $p = {}^{5}/_{12}$, or in some cases omitted the combination factor. Others did not recognise that they should apply a binomial model. Those who used correct methods often had rounding errors at this point. The attempts at 'at least one' in part (vi) were generally successful but the examiners did occasionally see $1 - \{P(X=0) + P(X=1)\}$ or even just the calculation of $P(X=1)$ appearing in the work.

4767: Statistics 2

General Comments

The majority of candidates were well prepared for this examination with a good overall standard seen. Candidates managed to answer all questions showing a good level of understanding of the topics concerned. A high level of competence in dealing with probability calculations using Binomial, Poisson and Normal distributions was seen; however, many were often unable to assess which distribution to use in any given situation. Candidates showed less understanding of how to phrase hypotheses appropriately for the different types of hypothesis test; where parameters were needed, candidates frequently used inappropriate symbols and in other cases used parameters when not required. It appeared that most candidates had adequate time to complete the paper; even those who calculated all expected frequencies and contributions to the Chi-squared test statistic in Question 4 when asked only for the contribution from one of the six cells.

Comments on Individual Questions

Section A

1) (i) This standard request rarely produced full marks for commenting upon the required conditions - "independence (of events)" and "uniform mean rate" of occurrence. Many candidates recognised the need for independence but struggled with the second condition. Common mistakes included "n large and p small" and "mean equals variance".

(ii)A This part of the question was well done with most candidates calculating the required probability using the Poisson probability function successfully.

(ii)B Many candidates introduced rounding errors when working to different levels of accuracy on $P(X = 0)$ and $P(X = 1)$, then provided only one significant figure accuracy in their answer.

(iii) Most candidates scored full marks.

(iv) Most candidates scored full marks, or lost just one for failing to "state the distribution" that was subsequently used despite this being requested in the question.

(v) Again, well-answered with many candidates obtaining full marks. Most recognised the need for a Normal approximation although some lost marks through using N(np, npq) rather than $N(\lambda, \lambda)$. Some candidates lost a mark for omitting the necessary continuity correction or applying an incorrect one. The majority used the correct tail.

2) (i)*A* This was well answered with most candidates successfully standardising and identifying the need to use $1 - \Phi(2.667)$. As the answer was given in the question, some candidates were penalised for not indicating that $\Phi(2.667) = 0.9962$. Most candidates used Normal tables accurately, but some candidates failed to capitalise on the fact that the answer was provided, by using $1 - \Phi(2.66)$ which gives 0.0039.

(i)*B* The question tested the use of $P(X \geq 1) = 1 - P(X = 0)$, with $P(X = 0)$ being found using 0.9962^7 (although a Poisson approximation could be used to give an acceptable answer). Many candidates appeared unable to understand what was required at all; some simply found $P(X = 1)$.

(i)*C* Few fully convincing comments were provided, with many simply restating their answer to (i) *B* rather than interpreting it. Candidates could obtain full marks for commenting on the magnitude of their answer and relating this to the fairness of the drug test.

(ii)*A* Many candidates failed to recognise that the exact distribution required was B(1000, 0.0038). Those realising a binomial distribution was needed generally gained both marks; both parameters were needed for full credit.

(ii)*B* This part was well answered with most candidates correctly using a Poisson approximation with mean 3.8, of which only a few used the incorrect $P(X \geq 10) = 1 - P(X \leq 10)$; a mistake often seen in such questions in previous years. Due to values of the parameters of the binomial distribution in this question, those using a Normal approximation were given no credit.

(iii) Many scored full marks for this question. Some candidates got off to a poor start by miscalculating 1 – 0.005 as 0.95; even so, credit could still be obtained for obtaining a corresponding *z*-value and using it to obtain a value in the right-hand tail of the Normal distribution. Some candidates used -2.576 and were thus penalised for working with the wrong tail.

3) (i) Many scored full marks on this part. Those failing to rank scores scored no marks; this happened with a significant proportion of candidates. A number of candidates made errors with their ranking but otherwise applied the correct expression for calculating Spearman's rank correlation coefficient. Other candidates were penalised for omitting the "1 -" from their expression.

(ii) Most candidates scored well on this part of the question; with marks for a critical value of 0.5636 and a comparison with their r_s from part (i) gained by most. The main reasons for loss of marks in this question were a failure to provide correct, contextual hypotheses and a failure to include a contextual conclusion. Some candidates who wrote their hypotheses solely in terms of ρ were penalised; although many candidates using hypotheses in terms of ρ also stated their hypotheses in words and could gain full credit. At this level, conclusions to hypothesis tests should end with a comment relating the findings back to the original context of the question.

(iii) An increasing number of candidates now seem comfortable with the idea of the need for an underlying distribution which is bivariate normal when carrying out a test for the product moment correlation coefficient, although the majority of candidates struggle to get this across. Many candidates knew to comment on the need for an elliptical spread of points on the scatter graph and went on to make a decisive comment on the appropriateness of the test with the points provided. A large number of candidates scored no marks on this part, for answers which explained about the need for the points to lie in a straight line together with general comments about correlation.

(iv) Most candidates identified a correct critical value of 0.2997 although some used the corresponding value from the Spearman's table or the 1% two-tailed test value. Few candidates gained the second mark for identifying the critical region as being $r \geq 0.2997$; the majority quoting the critical region as $r = 0.2997$.

(v) This part was reasonably attempted with many candidates gaining marks for appreciating that "correlation does not imply causation" and that a third factor could be involved. A number of candidates obtained full marks for providing a third relevant comment such as "the claim could be true" and "it could be that increased ozone could be the cause of high temperatures". Marks were awarded for comments of a statistical nature rather than lengthy essays on global warming.

4) (i) Well answered; although candidates who avoided context were penalised, as were those using "correlation" or parameters in their hypotheses.

(ii) Many candidates gained four straightforward marks here. Some candidates clearly failed to read the question carefully; as a result, they wasted time that the question was designed to save – even so, full credit could be obtained provided that the answer of 10.5 was seen in their working. Many candidates simply worked out the expected frequency of 42 without going on to find the contribution to the value of the chi-squared test statistic.

(iii) Generally well done, but many candidates lost marks for incorrect conclusions and for failing to comment in context; simply concluding that "there appears to be an association" was not enough to be awarded the final mark.

(iv) Although many candidates scored most of the available marks, this part was not well done. Most managed the mark for the hypotheses, which needed to be expressed in terms of μ, but failed to define μ as the mean travel time by car for the whole population. Candidates with a clear understanding of the difference between population mean and sample mean generally fared better. Many failed to use the correct distribution when standardising to find the test-statistic, or when finding the critical value(s) for the sample mean; many used the distribution for car travel times and not the distribution of the mean travel time for samples of size 20. Most gained marks for identifying the critical z-value of 1.645 and comparing it with their test-statistic. A number of candidates mixed up $\mu = 18.3$ with the observed sample mean of 22.4.

(v) Many candidates struggled to make comments related to the test in part (iv) or to factors which might have affected the outcome. Popular correct answers included comments on the fact that students might not all live the same distance from school, and that more investigation is needed. No credit was given to answers speculating about buses breaking down or general, environmental comments. Centres should encourage students to comment using statistical arguments.

4768: Statistics 3

General Comments

The overall standard of the scripts seen was pleasing: many candidates appeared well prepared for this paper. However, the quality of their comments, interpretations and explanations was consistently below that of the rest of the work.
Invariably all four questions were attempted. Question 2 was found to be very accessible and most candidates scored full or nearly full marks. Questions 1, 3 and 4 were well answered, with many candidates scoring relatively high marks. There was no evidence to suggest that candidates found themselves short of time at the end.

Comments on Individual Questions

1) **Continuous random variables; exponential distribution; Central Limit Theorem; delays at a railway junction.**

(i) This part of the question was almost always answered correctly.

(ii) Most candidates found the pdf correctly, though occasional errors with the differentiation of the exponential function were seen.
By contrast quite a few candidates got into difficulty over finding the mean delay, often because they did not seem able to handle the integration by parts successfully. There were many candidates who sailed through this part with apparent ease.

(iii) By and large candidates seemed to be aware that in order to answer this part they needed to use the cdf and work out $1 - \mathrm{F}(\mu)$.

(iv) This part of the question was not answered as well as might have been expected. There were some who thought that the mean of the distribution of $\overline{T}$ would be $\overline{T}$ itself, while others multiplied the mean and variance by 30.

(v) It was expected that candidates would use the distribution of $\overline{T}$ from the previous part either to carry out a brief, informal hypothesis test or to consider the usual criterion (mean ± 2 s.d.'s) for an outlier. In practice many candidates' answers fell somewhere between the two, which was quite acceptable.

2) **Combinations of Normal distributions; the times taken to set and check exam questions.**

(i) This part was almost always answered correctly.

(ii) A few candidates worked out the variance here incorrectly as $5^2 \times 12^2$. Other than that, this part was usually correct.

(iii) Only a few candidates experienced difficulty in getting this part right.

(iv) Candidates managed to get this part wrong in one of two ways. Either they worked out the variance as $12^2 \div 4$, or they found and used the distribution of $\frac{1}{4}Y$ (from part (iii)), believing it to be $\frac{1}{4}X$. The value of $\Phi^{-1}(0.9)$ was almost always quoted and used correctly.

(v) Candidates seemed to have little difficulty with this part. Most preferred to work with $\mathrm{P}(T<105)$.

(vi) In this part most candidates preferred to work with $\mathrm{P}(3T/5<75)$, but some worked out the variance incorrectly as 60 rather than 36.

3) **The *t* distribution: paired hypothesis test for the difference between two population means; confidence interval for a population mean; production lines filling bottles of liquid soap.**

(a) The hypotheses were usually stated satisfactorily but many candidates did not define their symbol μ adequately as the mean *difference*. Nor did they manage to convey that the required assumption was that the *differences* come from a Normal population.
The test statistic was usually worked out correctly from the data. Very few candidates failed to recognise that this was a paired comparison. The correct t distribution was usually used for the test, and the only blemish was in the final conclusion: candidates did not take care to express this carefully enough, including mentioning that it is the mean that has been tested.

(b) By and large the confidence interval was found correctly, but there was a smattering of the types of errors that one might anticipate such as the wrong percentage point. Many candidates gave a satisfactory explanation about the correct working of the machine with regard to the nominal setting, although quite a few felt it necessary to quote a generic interpretation of a confidence interval. Candidates needed to take care over the required assumption here. In a question like this, including part (a) above, it is important to be clear about which population is required to be Normal.

4) **Chi-squared hypothesis test for the goodness of fit of a Normal model; Wilcoxon test for a population median; manufacture of glass tubes.**

(i) There was some uncertainty about the need for combining classes in this question. Nonetheless the correct test statistic was obtained in the majority of cases.
Many candidates stated the number of degrees of freedom incorrectly and hence looked up the wrong critical value. This was usually because they did not allow for the estimated parameters (the mean and variance) and/or for having combined classes.

(ii)

(*A*) It appeared that a lot of candidates did not read this part of the question carefully enough. They may have known that a Normal population is required for a *t* test, but they certainly did not appreciate the significance of referring back to their conclusion to part (i) for guidance.

(*B*) The Wilcoxon test was carried out very competently by many candidates. Occasionally there was some carelessness in working out the differences from the stated median and this resulted in incorrect rankings. Sometimes the final part of the conclusion, which is expected to be in context, was expressed poorly.

4771: Decision Mathematics 1

General Comments

Candidates at the lower end of the spectrum of ability performed broadly according to expectations. Conversely the proportion of candidates achieving marks of excellence was less than expected. This was largely due to question 3 and, to a lesser extent, question 2. In question 3 candidates often did not know about Hamilton cycles. Furthermore, very few candidates were able to mount convincing arguments where justification/proof was required. Question 2 was intricate rather than difficult, and many candidates lacked the discipline to be able to work through the algorithm. However, most candidates performed very well on the simulation question (question 6).

Comments on Individual Questions

1) **CPA**

(i) Most candidates were able to score well here. A common error was to have activities D and E sharing the same "i" node and the same "j" node.

(ii) Again, most candidates scored well. Some lost a mark on the backward pass by failing to take proper account of a "leaving" dummy.

(iii) Most candidates were able to answer this correctly, even if they had made earlier errors.

2) **Algorithms**

(i) A very large number of candidates tried to save themselves time by not writing down lists 1 to 3 at every step, but only when changes were made. This was a false economy since it resulted in confusion and error.
It remains a mystery why a very large number of candidates miscopied their "56" from one line to "36" on the next and subsequent lines.

(ii)(iii) Most could do part (ii). Most who completed part (i) successfully were able to do part (iii)

(iv) By design this was, arguably, the most difficult part of the paper. Not many students were expected to get it right, and not many did.

3) **Graphs**

For the most part attempts at this question were very poor. In part (i) most candidates, but certainly not all, were able to identify the start and end nodes as D and A respectively. Very few were able to marshal an argument to prove it. In part (ii) few knew what a Hamilton cycle is, and very few who did were able to consider alternatives when it came to "showing". However, most succeeded in scoring the mark in part (iii)

4) **LP**

(i) The most common answer to the first part of this part question was to compute 1500/100=15 and 10000/800=12.5, and then to conclude that 12kg of toffee was the maximum amount which could be made.
In the second part many students correctly deduced that 10kg was the maximum amount of fudge, but then computed that 4.28kg of sugar would be left over – something to do with 10/0.7–10 ?!

(ii) The majority of candidates were able to jump through most of the hoops in part (ii). However, a substantial minority identified the variables as being the number of kg of butter and the number of kg of sugar. This led to expressions such as 100x+800y and 150x+700y – and to complete subsequent confusion.
Hardly any were able to collect the maximisation mark. They could not have this without identifying the objective and applying it. Most just found where the constraint lines intersected and quoted that point as the answer.

(iii) Many candidates came to a halt after part (ii), particularly those with butter/sugar variables. Those that did proceed mostly demonstrated that they knew what to do, even if they had not done it in part (i). The final post-optimal calculation was intended to be difficult, and few candidates scored both of the marks.

5) **Networks**

(i) (Prim – tabular form)
Most candidates were very poor at showing what they were doing. Many did not show their selections, without which it cannot be inferred that Prim is being applied. Others failed to indicate their order of node selection.
Most candidates were unable to give a practical application for a minimum connector. The majority did not seem to understand what was meant by "a practical application", and many gave as their answer the order in which they had selected arcs. Of those that did understand, few of their imagined applications could be said to be "connecting" in any way.

(ii) (Dijkstra)
This was well done, although all the usual faults were to be seen. Apart from those producing solutions without clear evidence of the use of the algorithm (especially correct working values and **only** correct working values), there were many who gave the shortest route from A to **D**.

(iii) This was very well done by most candidates.

6) **Simulation**

(i)(ii) These parts were well understood and correctly answered by most candidates. Only the
(iii) rider to part (iii) gave any difficulties, with a substantial minority of candidates apparently believing that using two-digit random numbers leads to an increase in "accuracy".

(iv)(v) Excellently done – it was a pleasure to see almost everyone doing well here.

4776: Numerical Methods

General Comments

Many candidates seemed less well prepared for an examination in Numerical Methods than in recent sessions. There was evidence of whole topics being unfamiliar and of weaknesses in algebraic manipulation.

Comments on Individual Questions

1) (Error analysis)
This question proved to be beyond the grasp of the majority of candidates. Only a handful could produce any sort of algebraic argument to relate the relative errors in a number and its reciprocal. Very few were able to comment sensibly on what the given algebraic result represented. In many cases, only the arithmetic scored any marks.

2) (Solution of an equation)
The iteration leading to the first root was carried out successfully in most cases (though sometimes the accuracy was not as required). In part (ii) a substantial minority did not appreciate the need to show a change of sign in the interval (2.7725, 2.7735). Simply showing that f(2.773) is very near to zero scores no marks.

3) (Numerical integration)
Most candidates were able to get one trapezium rule estimate and one Simpson's rule estimate correct. It was much less common to find two of each correct because of uncertainty about the relationships between the methods and a lack of clarity about the value of h. The degree of accuracy in the final answer was often inappropriate.

4) (Numerical differentiation)
The point estimates and the ranges in which they lie were generally found accurately. The interpretation was a little more difficult, but there were several acceptable conclusions available for full credit. (This is one of those situations in numerical work in which there can be no definitive right answer to a question.)

5) (Lagrange's method and difference tables)
Though substantial numbers of candidates were able to score full marks here, there were also many for whom this seemed to be an unknown topic. In Lagrange's formula it was a common error to confuse the x values and the function values. In the difference table it is necessary to say something about second differences being constant for a quadratic in order to gain full marks.

6) (Location of roots, solution of an equation)
Part (i) of this question required a careful analysis of the nature of the function in order to identify the number of roots and their locations. A good many candidates showed that there were two roots in the interval (0, 2), but failed to show convincingly that there are no other roots.
In part (ii), the Newton-Raphson method was done well by most, though occasionally there was no evidence of any working whatever. Since some calculators have a solver facility, it is most unwise of a candidate to give no evidence of method.
In part (iii) most could obtain $0.1^{11} + 0.1$ as an iterate, but the argument that this value will be very close to the true root was given by almost none.

7) (Mid point rule, errors)

In part (i) most candidates offered some explanation for the mid-point rule, but a minority merely re-asserted what was given in the question. The values of $\log_e$ were sometimes wrong because the underlying laws of logarithms were not understood – though candidates often redeemed themselves by getting the values directly from their calculators in later work. Those who calmly and systematically followed instructions in parts (iii) and (iv) were generally successful.

Coursework

Administration

A significant minority of Centres failed to send the Authentication form CCS160. This form has been a requirement for a number of sessions now and replaces the original demand for the Assessor to sign each cover sheet. It contributed to an unnecessary extra burden of time on the process to have to follow these up after the receipt of coursework scripts. It also helps the process considerably to have the paperwork for the Moderator complete. This includes the filling in of the cover sheets - a few centres fail to fill in candidate numbers; in one case the name was also missing!
Most centres adhered to the deadline set by OCR very well and if the first despatch was only the MS1 then they responded rapidly to the sample request. A small minority, however, cause problems with the process by being late with the coursework despatch. We would ask that all centres heed the deadlines published by the Board and organise their own processes of assessment, internal moderation and administration to enable these deadlines to be met.

Core, C3 – 4753/02

Around 28% of centres were changed, almost exactly the same as Summer 2005 which was the first C3 session. This compares unfavourably with 2602 Pure 2 which ran consistently at around 16%. There are various reasons for this:

- C3 coursework is out of 18 instead of 15.
- Many centres have either not seen or have not heeded the awarder's reports of previous sessions, which provides important information about how the criteria are to be interpreted in practice.
- Software packages such as Autograph are used without annotation of the computer outputs, resulting in inadequate graphical illustrations.
- Incorrect notation is often given the full mark.

Many centres used the out of date cover sheet which is in the specification booklet. Some used both and we detected more than one assessor using both. This tended to make marking more erratic. More than one centre even marked their coursework out of 15!

Errors in assessment were at least as prolific as in previous sessions and were in the usual places, all of which have been detailed in previous reports.
They included the following:

- Change of sign method with no graphical illustration.
- Failures of methods being demonstrated using trivial equations.
- Failures which in fact find the root in the table of values.
- Failures because of roots being supposedly very close together in fact having no roots upon close inspection.
- Graphical illustrations not matching calculated iterates or simply being very poor.
- Lack of calculated iterates.
- Error bounds for Newton Raphson merely stated.
- Newton Raphson failures starting too far from the root.
- $g'(x)$ discussions involving general statements rather than specific comments on the gradient of the graph by comparison to the gradient of the line $y = x$.
- Comparison section too imprecise or brief.

Differential Equations - 4758/02

Only a small number of centres submitted work this session. Therefore any generalisations may be a little misleading.

Work is still being submitted containing D.E.s which are too basic and do not require techniques from this module for their solution. Effectively this tends to rule out the opportunity of marks in the later domains.

The essential function of the coursework element of this module is to test the candidate's ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Similarly, choosing 'too good' a model in the first place, e.g. flow $\alpha\ \sqrt{h}$ initially for 'Cascades', does not leave much room for improvement of the model. Consequently the marks in Domains 5 and 6 are compromised.

In Domain 4 (comparison), both a table and a graph would generally be expected. For the second mark some form of error bars would normally be expected.

Finally, for 'Aeroplane Landing', marks often seem to be automatically allocated for Domain 3 (Collection of data) when there is little discussion of the source or potential accuracy of the data.

Numerical Methods – 4776/02

There was an increased entry this year, reflecting the popularity of the module as part of the AS Further Mathematics certification. Most candidates did appropriate tasks and made good use of technology in reaching a solution. Consequently, much of the assessment was in line with national standards. Where adjustments were made, it was usually due to one or more of the following:

- If a teaching group is given a template to follow for tackling the same task, it is not appropriate to give full marks in domain 1. Some individuality is expected.
- In domain 2, candidates are expected to justify their selection of the algorithm(s) used for full marks. There is no credit for the replication of bookwork.
- In domain 3, a substantial application might be (say), T_1, T_2, T_4, T_8, T_{16}, T_{32}, T_{64}. If convergence to a high degree of accuracy is achieved, the selected integral leaves little scope for developing error analysis.
- In domain 4, an annotated printout of spreadsheet formulae is expected for the second mark, although a detailed commentary will suffice.
- In domain 5 the usual approach is to find the ratio of differences and extrapolate to an improved solution. It is not appropriate to give any credit for comparing answers with the "true" value.
- In domain 6, the last two marks might be obtained by commenting, say, on the discrepancy between the observed ratio of differences and the theoretical one.

A small number of candidates tackled inappropriate problems. This was usually on solutions of equations. Work on this syllabus area requires a development of ideas from C3, not a replication of them. In some cases a piece of software was used to generate answers - candidates are expected to demonstrate their understanding of the processes by applying the algorithms themselves.

7895-8,3895-3898 AS and A2 MEI Mathematics
January 2006 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	57	49	41	34	27	0
4752	Raw	72	56	49	42	35	28	0
4753	Raw	72	54	47	40	32	25	0
4753/02	Raw	18	14	12	10	9	8	0
4754	Raw	90	66	58	50	42	34	0
4755	Raw	72	57	50	43	36	29	0
4756	Raw	72	51	44	37	31	25	0
4758	Raw	72	58	51	44	36	28	0
4758/02	Raw	18	14	12	10	9	8	0
4761	Raw	72	59	51	43	36	29	0
4762	Raw	72	56	49	42	35	28	0
4763	Raw	72	60	52	44	36	29	0
4766	Raw	72	55	48	41	34	28	0
4767	Raw	72	57	50	43	36	29	0
4768	Raw	72	56	48	41	34	27	0
4771	Raw	72	51	44	37	31	25	0
4776	Raw	72	52	45	38	30	23	0
4776/02	Raw	18	13	11	9	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	34.6	67.3	86.5	96.2	98.1	100.0	52
3895	26.2	41.7	59.8	77.4	93.1	100.0	508
3896	25.0	40.0	60.0	75.0	80.0	100.0	20
3898	100.0	100.0	100.0	100.0	100.0	100.0	1

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