RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2624/1

Numerical Analysis

Friday **27 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

 [Turn over

1 This question concerns the sum of the infinite series

$$S = \frac{1}{1} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \dots .$$

You are *given that*, when the series is summed directly using a spreadsheet, the values in the following table are obtained.

number of terms	10	100	1000	10000
sum	1.995 336	2.412 874	2.549 146	2.592 376

(i) Identify two disadvantages of summing the series directly. [3]

(ii) Show, by considering the integral of the function $\frac{1}{x\sqrt{x}}$, that the sum

$$\frac{1}{a\sqrt{a}} + \frac{1}{(a+1)\sqrt{a+1}} + \frac{1}{(a+2)\sqrt{a+2}} + \dots$$

can be approximated as $\frac{2}{\sqrt{a-0.5}}$.

Using a diagram, explain carefully how the accuracy of this result depends upon the value of a. [8]

(iii) Use the result in part **(ii)**, together with the values given in the table, to find the sum S correct to 7 significant figures. [5]

(iv) Explain briefly, and without doing any calculations, how the method you have used could be adapted to sum the series

$$\sin\left(\frac{1}{1\sqrt{1}}\right) + \sin\left(\frac{1}{2\sqrt{2}}\right) + \sin\left(\frac{1}{3\sqrt{3}}\right) + \dots .$$ [4]

2 The focal length for a certain type of lens may be found by measuring two lengths, a and b, and using the formula

$$f = \frac{ab}{b-a}. \qquad (*)$$

(i) Find the partial derivatives $\frac{\partial f}{\partial a}$ and $\frac{\partial f}{\partial b}$. Hence establish the result

$$\Delta f \approx \frac{b^2 \Delta a - a^2 \Delta b}{(b-a)^2}$$

for small changes in a, b and f.

Write down an approximate equation linking the maximum possible errors in a, b and f. [8]

(ii) Given that $a = 10$ and $b = 15$, each with a maximum possible error of 0.1, calculate an estimate of f. Use (*) to calculate the range in which f must lie.

Find an approximation to this interval using the result in part **(i)**, and comment on its accuracy. [6]

(iii) Now suppose that $f = 30$ with maximum possible error 0.1, and that $a = 10$ with negligible error. Find the maximum possible error in b. [3]

(iv) Finally, suppose that $a = 10$ with maximum possible error ε, and that $b = 15$ with maximum possible error 2ε. Find the greatest value of ε such that f is known to an accuracy of 0.05. [3]

3 The differential equation

$$y' = e^x + \sqrt{y},$$

where $y = 1$ when $x = 0$, is to be solved numerically by means of Taylor series.

(i) Show carefully that the second derivative of y is given by

$$y'' = e^x + \frac{y'}{2\sqrt{y}}.$$

Find an expression for the third derivative of y. [6]

(ii) Obtain the Taylor series of order 3 centred on $x = 0$. Use this series to estimate the values of y at $x = 0.1$ and 0.2. [6]

(iii) Obtain the Taylor series of order 3 centred on $x = 0.1$. Use this series to estimate the value of y at $x = 0.2$. [5]

(iv) Which of the two estimates of y at $x = 0.2$ is likely to be more accurate and why? [3]

4 The following values of the function $f(x)$ are known correct to 1 decimal place.

x	1	2	4	5
$f(x)$	−1.3	−0.9	2.3	3.0

The value of α such that $f(\alpha) = 0$ is required.

(i) Use linear interpolation from $x = 2$ to $x = 4$ to find an initial estimate of α. [3]

(ii) Use Newton's divided difference interpolation method to obtain the quadratic that interpolates the data for $x = 1, 2, 4$.

Hence obtain a second estimate of α. [7]

(iii) You are now given that the quadratic which interpolates the data for $x = 2, 4, 5$ gives an estimate of α as 2.4 to 1 decimal place.

Explain, with the aid of a rough sketch of the data, why this estimate differs substantially from that in part (ii). [4]

(iv) Starting from the quadratic found in part (ii), add a cubic term by incorporating the data at $x = 5$. (There is no need to simplify your answer at this stage.)

Verify that the estimate of α obtained from the cubic polynomial is, to 1 decimal place, the same as that obtained in part (i). [6]