OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2623/1**

Numerical Methods

Wednesday **25 JANUARY 2006** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

 [Turn over

1 **(a)** **(i)** Show carefully that the equation

$$x^3 + x - 1 = 0$$

has only one root, and that this root lies between 0 and 1. [3]

(ii) Carry out three steps of the bisection method to obtain an estimate of the root. State the maximum possible error in this estimate.

Determine how many further steps of the bisection method would be needed to reduce the maximum possible error to less than 0.005. [8]

(b) **(i)** The number A is an approximation to the number a. It is known that A is correct to 2 decimal places. State the maximum possible error in A. [1]

(ii) The number B is an approximation to the number b. The maximum possible error in B is known to be 0.005. Give an example of possible values for B and b such that B is not correct to 2 decimal places.

Give a further example to show that B may not be correct to the nearest integer. [3]

2 In certain computer applications, a rough approximation is required to $\sqrt{x}$ where $0.25 \leqslant x \leqslant 1$. A formula sometimes used is

$$\sqrt{x} \approx \tfrac{2}{3}x + 0.36. \qquad (*)$$

(i) Find the two values of x for which there is zero error in this approximation.

[Hint: form a quadratic equation in t where $t = \sqrt{x}$.] [4]

(ii) Find the absolute and relative errors when the approximation is used for $x = 0.25$ and $x = 0.64$. [4]

If s is the approximation to $\sqrt{x}$ given by (*), then an improved approximation is given by

$$\frac{s^2 + x}{2s}.$$

(iii) Find the relative error in the improved approximation when $x = 0.25$. [3]

(iv) Suppose that s overestimates $\sqrt{x}$ with a relative error of 1%. Write down an equation for s in terms of $\sqrt{x}$. Hence show that $\frac{s^2 + x}{2s}$ is very nearly $1.000\,05\sqrt{x}$. State the relative error in the improved approximation. [4]

3 A function $f(x)$ has values as shown in the table.

x	0	1	2	3	4	5
$f(x)$	0.2	1.8	6.1	14.2	27.3	46.6

(i) Show, by means of a difference table, that $f(x)$ appears to be approximated well by a cubic polynomial. [4]

(ii) Extend the difference table to give an estimate of $f(6)$. Discuss briefly the reliability of this estimate. [4]

(iii) Use Newton's interpolation formula to estimate $f(2.5)$. Comment on the reliability of this estimate. [5]

(iv) Explain, without doing any further calculations, what you would regard as the best method for estimating the gradient of $f(x)$ at $x = 2.5$. [2]

4 **(i)** Find, in terms of a, the value of the integral $I = \int_0^a x^2 \, dx$.

Hence find the error in each of the following approximations to I:

(A) T_1, the value given by the trapezium rule with one strip,

(B) M_1, the value given by the mid-point rule with one strip.

Find the value of S, where $S = \frac{1}{3}(T_1 + 2M_1)$, and comment. [7]

(ii) For the integral $J = \int_{0.2}^{1} \frac{1}{\sqrt{1 + x^3}} \, dx$, find the values of T_1 and M_1; hence find an improved estimate, S.

Find also the values of the estimates T_2 and M_2, given by the trapezium and mid-point rules with two strips, and S^*, where $S^* = \frac{1}{3}(T_2 + 2M_2)$. Hence give the value of J to the accuracy that is justified by your working. [8]

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