OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2622

Decision and Discrete Computation

Monday **23 JANUARY 2006** Afternoon 2 hours 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Question **2** parts **(ii)** and **(iii)**.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages, 3 blank pages and an insert.

1 The Chancellor of the Exchequer in Mathland wishes to achieve a steady rate of inflation of 3% per annum in the Mathland economy. He has a Budget each autumn in which he puts in place measures which are intended to achieve this. Unfortunately he does not realise that his measures only take effect after 2 years.

In autumn 2004 the percentage rate of inflation in Mathland was 3.5, and the Chancellor announced measures to reduce the percentage rate of inflation by 0.5. However these measures will not take effect until autumn 2006. In autumn 2005 earlier actions produced a percentage rate of inflation of 2.8. The Chancellor therefore put in place measures which will increase it by 0.2, not realising that this increase will not take effect until autumn 2007.

(i) Show that the percentage inflation rate in autumn 2006 will be 2.3. [1]

(ii) Given that the Chancellor continues to pursue the same policy, find the percentage inflation rates for autumn 2007, 2008, 2009, 2010, 2011 and 2012.

Describe what will happen subsequently. [3]

Let u_n be the percentage inflation rate in the autumn of year $2005 + n$, so that $u_0 = 2.8$, $u_1 = 2.3$, etc. Then a recurrence relation describing the effect of the Chancellor's policy is

$$u_{n+2} = u_{n+1} + (3 - u_n).$$

(iii) Construct a spreadsheet in which column A contains the numbers $i = 0, 1, 2, 3, \ldots, 40$, and in which column B contains the values of u_i, $i = 0, 1, 2, 3, \ldots, 40$. [1]

A group of the Chancellor's advisors say that his policies will fail because he is not sufficiently bold. They advise him to aim for an adjustment in each budget which is 25% greater than the difference between the target of 3 and the actual percentage inflation rate. Thus the recurrence relation would be

$$u_{n+2} = u_{n+1} + 1.25(3 - u_n).$$

(iv) Add column C to your spreadsheet showing the percentage inflation rates that would be produced by following this policy for 40 years, again starting with $u_0 = 2.8$ and $u_1 = 2.3$. Negative rates of inflation are never seen in Mathland, and any negative values should be replaced by 0 as they are produced.

Show the formula that you used for an entry in column C. [3]

Another group of advisors say that the Chancellor is being too heavy-handed, and that he should aim for an adjustment which is 75% of the difference between the target of 3 and the actual percentage inflation rate.

(v) Add column D to your spreadsheet showing the percentage inflation rates that would be produced by following this policy for 40 years, again starting with $u_0 = 2.8$ and $u_1 = 2.3$.

Produce a print-out of your spreadsheet, showing columns A, B, C and D. [2]

The Chancellor is so convinced by the second group's argument that he decides to aim for an adjustment which is only 9% of the difference between the target of 3 and the actual percentage rate of inflation. Thus the rate of inflation follows the recurrence relation $u_{n+2} = u_{n+1} + 0.09(3 - u_n)$, with $u_0 = 2.8$ and $u_1 = 2.3$.

(vi) Solve this recurrence relation to find an expression for u_n in terms of n. [8]

(vii) Describe the consequences of following this policy. [2]

2 There is an insert for use in parts (ii) and (iii) of this question.

Fig. 2.1 represents a directed network of connected pipes together with weights representing their capacities.

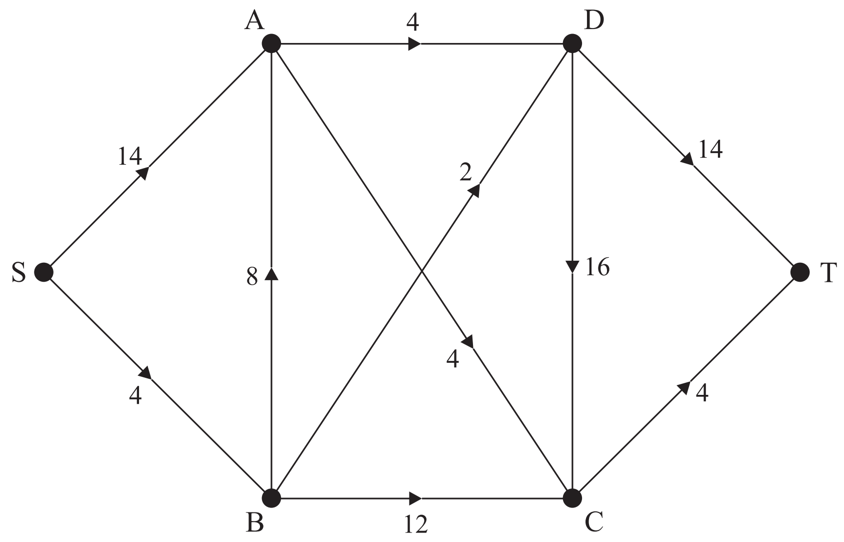

Fig. 2.1

(i) The maximum flow from S to T is 10 units. Give a cut with this capacity. [2]

A flow of 4 units is established along SBADT and a flow of 4 units along SACT.

(ii) Label these flows, together with potential flows and potential backflows, on Fig. 2.2 on the insert. [2]

(iii) Give a single flow-augmenting path which will achieve a total flow of 10 units. Mark the resulting flow along each pipe on Fig. 2.3 on the insert. [2]

(iv) Construct a linear programming model to find the maximum flow through the network, using variables such as SA to represent the flow along the pipe from vertex S to vertex A.

Use your linear programming package to solve the problem and include a copy of the printout. [4]

(v) The network also models another problem in which the weights now represent distances in the indicated directions. Change your LP formulation so that it finds the shortest distance from S to T.

Run your LP, include a copy of the printout, and interpret the solution. [6]

(vi) The arc AD is now changed to be undirected with a weight of 4.

Why does this not affect your answer to part **(iv)**?

Change your LP in part **(v)** and use it to find the new shortest route from S to T. [4]

3 An island has three electricity power stations. Their maximum power outputs are 3, 5 and 8 MW (megawatts) respectively. Each can be operated at any power output up to its maximum. Their respective hourly costs are 0.75, 0.70 and 0.73 monetary units per MW.

(i) Explain why the following linear program will find the best way of fulfilling an hourly demand for 6.5 MW. [3]

minimise $2.25x_1 + 3.5x_2 + 5.84x_3$

subject to

$$\text{Demand} = 6.5$$
$$3x_1 + 5x_2 + 8x_3 - \text{Demand} = 0$$
$$0 \leqslant x_1 \leqslant 1$$
$$0 \leqslant x_2 \leqslant 1$$
$$0 \leqslant x_3 \leqslant 1$$

(ii) Use your linear programming package to solve the problem. Include a printout of the solution and interpret that solution. [3]

(iii) Find the best solution to satisfy an hourly demand of 4.5 MW. [2]

(iv) A larger island has 10 power stations, with maximum power outputs and hourly costs per MW as follows.

Station number	1	2	3	4	5	6	7	8	9	10
Max power (MW)	3	5	8	6	4	3	8	4.5	6.2	9
Hourly costs per MW	0.75	0.70	0.73	0.72	0.76	0.69	0.77	0.74	0.76	0.73

Use your linear programming package to find the cheapest way to satisfy an hourly demand for 38.2 MW. [4]

(v) A new power station is to be constructed on the larger island. This will cost 4.7 units per hour to run, plus an hourly cost of 0.24 units per MW. Its capacity will be 10 MW.

Incorporate this power station into your electricity supply model for the larger island and find the best solution for an hourly demand of 38.2 MW. [6]

(vi) By using your LP model or otherwise, find the minimum hourly demand for power for which it is worth using the new station. You are given that this minimum number of MW is an integer. [2]

4 During lunchtimes the intervals between cars arriving at a roadside restaurant have the distribution given in Table 4.1.

Table 4.1

Interval (minutes)	1	2	3	4	5	10
Probability	0.6	0.2	0.1	0.05	0.03	0.02

(i) Construct a spreadsheet in which column A contains 100 uniformly distributed random numbers between 0 and 1, and column B contains the associated time intervals between cars arriving at the restaurant. (The first such time interval is to represent the time of arrival of the first car.)

Add column C to show the simulated arrival times of cars.

Give the formulae you used in columns A, B and C. (If you used a 'lookup' formula in column B, then produce a copy of your lookup table as well.) [5]

Table 4.2 gives the distribution of the number of people per car.

Table 4.2

Number of people	1	2	3	4
Probability	0.3	0.4	0.2	0.1

(ii) Add column(s) to your spreadsheet to simulate the number of people arriving in each car.

Give the formulae which you use. [4]

(iii) Use the 'IF' function to produce a final column in which an entry contains the number of people in the corresponding car if the arrival time of the car is 120 minutes or less, and 0 otherwise.

Place a formula giving the sum of the numbers in this column into a convenient cell.

Give the formulae which you use. [4]

(iv) Your spreadsheet simulates the number of people arriving at the restaurant in the two-hour lunchtime period. Run your simulation 10 times and produce a table of results.

Compute the mean number of people per simulation run. [3]

(v) From your results calculate an estimate of the standard error of the mean number of people arriving over the two-hour period.

Estimate the number of simulation runs that are required for you to be confident that the mean number of people given by the simulation is correct to within 1 person. [4]

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Candidate Name	Centre Number	Candidate Number

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INSERT

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- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

[Turn over

2 (ii)

key

capacity	forward potential
flow	backward potential

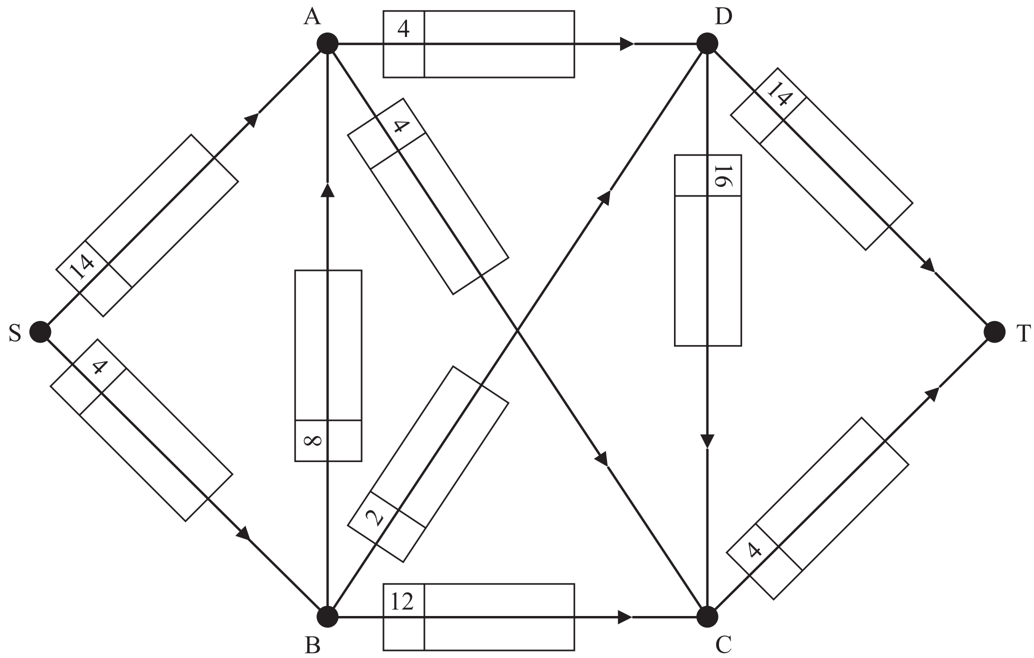

Fig. 2.2

(iii)

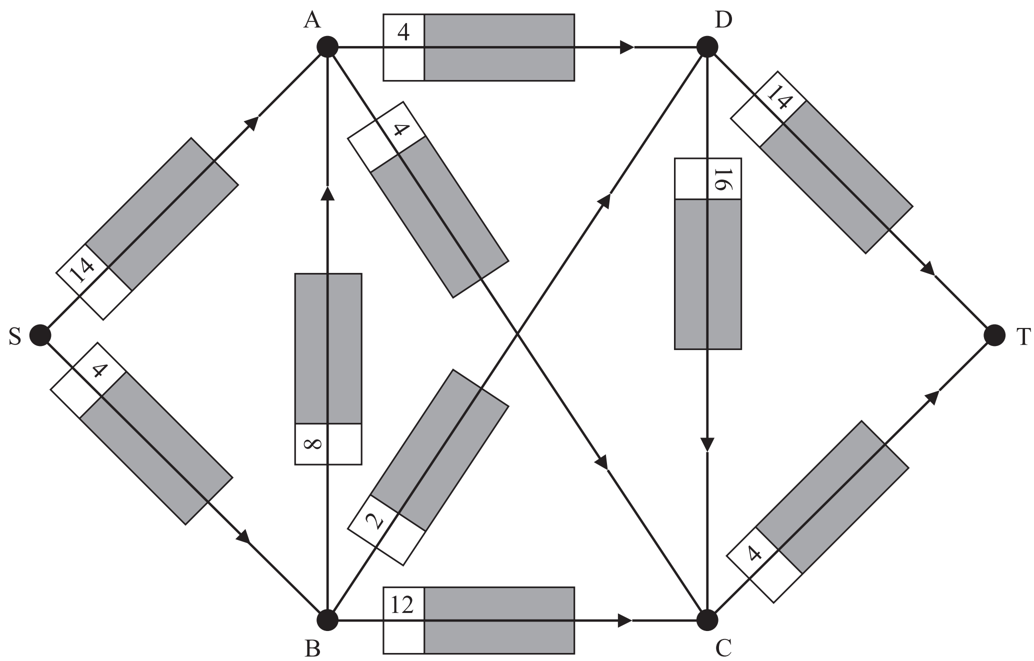

Fig. 2.3