OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2621/1

Decision and Discrete Mathematics 2

Friday **27 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Question **4** part **(iii)**.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages and an insert.

Registered Charity Number: 1066969
[Turn over

1 **(i)** Produce a truth table for $[((\sim a) \wedge (\sim b)) \Rightarrow c] \wedge [(\sim c) \wedge (\sim b)]$.

If this is true, what are the truth values of a, b and c? [8]

(ii) You are told that if neither Jones nor Smith did it then it must have been Bunter. You know that Bunter and Smith are innocent. What can you deduce and why? [2]

2 An investor has to decide whether to sell her holding of 500 shares today, when they are worth £10 each, or to keep them for another day. If she keeps them for another day then she believes that there is a 60% chance that they will increase in value on average by £0.26 per share, and a 40% chance that they will decrease in value on average by £0.19 per share.

(i) Draw a decision tree for the investor. Give her best course of action and its EMV. [3]

The investor can seek a consultation with her financial adviser. He boasts that, whether he advises to sell or not to sell, he gets it right 90% of the time. She calculates that he must therefore advise to sell on 3 days out of 8 and not to sell on 5 days out of 8.

(ii) Draw an extended decision tree for the investor.

What is the value of the advice? Assuming that the adviser charges less than this for the consultation, what is the investor's best course of action? [7]

3 In a manufacturing process three products are made, A, B and C. Each kg of product, A, B or C, gives a profit of £8. Making A, B and C uses four resources. The resource requirements are shown in Table 3.

Table 3

	A	B	C	Availability
Resource 1	4	4	2	80
Resource 2	2	10	10	176
Resource 3	10	2	10	176
Resource 4	4	4	4	80

(i) Formulate a linear programming problem to maximise profit given the resource constraints. [6]

(ii) Explain how the following initial tableau models the LP problem. Your explanation should indicate the meanings of the rows and the meanings of the columns. [4]

P	*a*	*b*	*c*	*s1*	*s2*	*s3*	*s4*	RHS
1	−8	−8	−8	0	0	0	0	0
0	4	4	2	1	0	0	0	80
0	2	10	10	0	1	0	0	176
0	10	2	10	0	0	1	0	176
0	4	4	4	0	0	0	1	80

(iii) Produce the first iteration of the simplex method. Iterate on the '*a*' column. [6]

The final tableau for this problem is as follows.

P	*a*	*b*	*c*	*s1*	*s2*	*s3*	*s4*	RHS
1	0	0	0	0	0	0	2	160
0	0	1	0	0	0	$-\frac{1}{8}$	$\frac{5}{16}$	3
0	0	0	0	4	1	1	−7	112
0	1	0	0	0.5	0	$\frac{1}{8}$	$-\frac{9}{16}$	17
0	0	0	1	−0.5	0	0	0.5	0

(iv) Interpret this tableau. What do the two shaded zeros tell you? [3]

Iterating on the *s1* column in the final tableau gives the alternative solution $a = 3$, $b = 3$, $c = 14$, and iterating on the *s3* column in the final tableau gives the alternative solution $a = 3$, $b = 17$, $c = 0$.

(v) Give a geometrical description of the set of all solutions to the problem. [1]

4 There is an insert provided for part (iii) of this question.

Floyd's algorithm is applied to the four-node network shown in Fig. 4, in which the weights on the arcs represent distances.

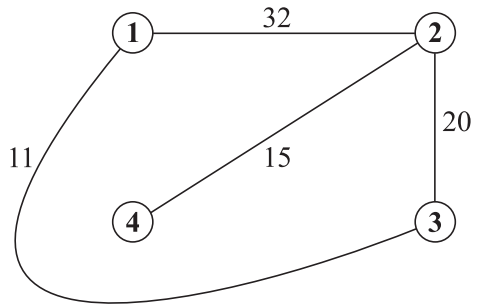

Fig. 4

The output from the algorithm consists of the following two matrices.

Matrix 1

	1	2	3	4
1	22	31	11	46
2	31	30	20	15
3	11	20	22	35
4	46	15	35	30

Matrix 2

	1	2	3	4
1	3	3	3	3
2	3	4	3	4
3	1	2	1	2
4	2	2	2	2

(i) Explain what information is contained within the matrices, giving a clear example. [4]

A fifth node is connected to the network by an arc of weight 8, connecting it to node 1, and an arc of weight 5, connecting it to node 4.

(ii) Explain why it is not possible to produce final matrices for the extended network by doing just one more iteration of Floyd's algorithm. Illustrate your answer with an example of a possible route which would not be considered by one extra iteration. [2]

(iii) Without using Floyd's algorithm produce final matrices for the extended network. You may use the blank tables given in the insert. [5]

(iv) By excluding node 5 and its arcs from the extended network, find a lower bound for the (practical) travelling salesperson problem on the extended network. (You are not required to use a minimum connector algorithm in this part of the question.) [4]

(v) Use the nearest neighbour algorithm starting at node 5 on the complete network represented by your matrices from part (iii) to find a Hamilton cycle. [2]

(vi) Interpret your Hamilton cycle from part (v) in terms of the extended network, and give bounds between which the length of the optimal (practical) travelling salesperson's route must lie. [3]