OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2617

Statistics 5

Tuesday **10 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 Independent trials, on each of which the probability of a 'success' is p $(0 < p < 1)$, are being carried out. The random variable X counts the number of trials up to and including that on which the first 'success' is obtained.

(i) Write down an expression for $\mathrm{P}(X = x)$ for $x = 1, 2, \ldots$ and show that the probability generating function of X is

$$\mathrm{G}(t) = \frac{pt}{1 - qt}$$

where $q = 1 - p$. [6]

(ii) Use $\mathrm{G}(t)$ to find the mean and variance of X. [11]

(iii) The random variable Y counts the number of trials up to and including that on which the kth 'success' is obtained. Write down, in terms of p, q and t, an expression for the probability generating function of Y. [3]

2 The discrete random variable X takes the values -1, 0 and 1, each with probability $\frac{1}{3}$.

(i) Write down the values of μ, the mean, and σ^2, the variance, of X. [2]

(ii) Find the moment generating function of X. [2]

(iii) Let Z denote the standardised mean for a random sample of n independent observations on X, i.e.

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where $\overline{X}$ is the sample mean. Stating carefully any general results about moment generating functions that you use, show that the moment generating function of Z is given by $\mathrm{M}(\theta)$ where

$$\mathrm{M}(\theta) = \left\{\frac{1}{3}\left(1 + \mathrm{e}^{\frac{\theta\sqrt{3}}{\sqrt{2n}}} + \mathrm{e}^{-\frac{\theta\sqrt{3}}{\sqrt{2n}}}\right)\right\}^n.$$ [7]

(iv) By expanding the exponential functions in $\mathrm{M}(\theta)$, show that, for large n,

$$\mathrm{M}(\theta) \approx \left(1 + \frac{\frac{1}{2}\theta^2}{n}\right)^n.$$ [5]

(v) Use the result $\mathrm{e}^y = \lim_{n\to\infty}\left(1 + \frac{y}{n}\right)^n$ to show that $\mathrm{M}(\theta)$ tends to $\mathrm{e}^{\theta^2/2}$ as n tends to ∞.

Deduce the approximate distribution of Z for large n. [4]

3 A railway line and a road intersect at a busy level crossing. The lengths of time that the crossing has to be closed to the road vary from one closure to another, but it has been found that 44% of such closures last more than one minute.

New signalling is installed on the railway in the hope of reducing closure times. Subsequently, it is found that, out of 50 randomly chosen closures, 15 lasted more than one minute.

(i) Set up appropriate null and alternative hypotheses concerning a probability p that you should define. Hence test whether the new signalling has been successful, using a 5% significance level. [12]

(ii) Calculate an approximate two-sided 95% confidence interval for p. [8]

4 An orchestra performs a weekly series of concerts. Some of these concerts consist wholly or mainly of works that are established as part of the normal concert repertoire; these are referred to as 'traditional' concerts. The other concerts consist mainly of unfamiliar works; these are referred to as 'innovative' concerts. The orchestra managers accept that on the whole the audiences for the 'innovative' concerts may be smaller than those for the 'traditional' ones, but are concerned about the variability in the audience sizes for the two types of concerts.

The next 18 concerts consist of 8 'innovative' and 10 'traditional'. Regard these as random samples. For the 8 'innovative' concerts, the usual unbiased estimate of the population variance of audience size is found to be 758. For the 10 'traditional' concerts, the corresponding estimate is found to be 384.

(i) Test at the 5% level of significance whether the population variances of audience size for the two types of concerts may be assumed equal. [8]

(ii) Provide a two-sided 80% confidence interval for the population variance of audience size for the 'traditional' concerts. [6]

(iii) Denoting the lower and upper limits of this confidence interval by l and u respectively, a manager states that the probability that the population variance is between l and u is 80%. Explain why this interpretation is incorrect. Give the correct interpretation. [6]

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