MEI STRUCTURED MATHEMATICS　　2614/1

Statistics 2

Wednesday　　**25 JANUARY 2006**　　Morning　　1 hour 20 minutes

Additional materials:
　　8 page answer booklet
　　Graph paper
　　MEI Examination Formulae and Tables (MF12)

TIME　　1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.

- Answer **all** the questions.

- You are permitted to use a graphical calculator in this paper.

- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.

- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

1 A statistician, who is researching the nutritional content of tinned soups, selects a random sample of 12 tins of soup. Their sugar content x and fat content y, measured in grams per 100 ml, are recorded and illustrated in Fig. 1. These data are summarised as follows.

$$n = 12 \qquad \sum x = 30.0 \qquad \sum y = 9.6$$

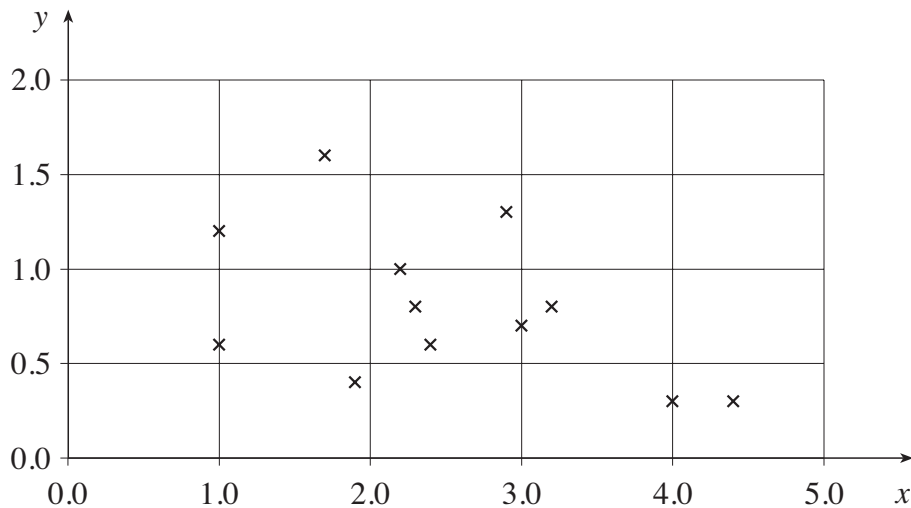$$\sum x^2 = 87.40 \qquad \sum y^2 = 9.52 \qquad \sum xy = 21.71$$

Fig. 1

(i) Show that the value of the product moment correlation coefficient, given to three decimal places, is −0.479. [4]

(ii) Carry out a hypothesis test at the 5% level to determine whether there is any correlation between sugar content and fat content in tinned soup. State your hypotheses and conclusion carefully. [5]

(iii) The researcher notes that both of the two tins of soup with the highest sugar content are a 'lo-salt' brand and decides to remove the two data items from the sample. Without any further calculation, state the effect that removing these two data items would have on the value of the correlation coefficient. Explain your answer. [3]

(iv) A student suggests that, when two variables are correlated, one must be the cause of the other. Discuss this suggestion briefly. [3]

2 A vehicle rental company has 4 vans which it hires out on a daily basis. The number of vans requested by customers on a randomly chosen day is modelled by a Poisson distribution with mean 2.8.

  (i) Find the probability that exactly 3 vans are hired out on a randomly chosen day. [2]

  (ii) Find the probability that exactly 15 vans are requested in a 5-day week. State whether or not such a demand could always be met, giving a reason for your answer. [3]

If more than 4 vans are requested on any day, then only the first 4 customers are able to hire a van that day.

(iii) Find the probability that all of the vans are hired out

  (*A*) on a randomly chosen day, [2]

  (*B*) on exactly 2 days of a 5-day week. [3]

(iv) Find the probability that the company does not have enough vans to meet the demand on a randomly chosen day. [2]

The manager of the company is considering buying an additional van. Over a long period, it would cost the company on average an additional £20 per day to run this van, taking into account the purchase cost. The daily hire charge per van is £90.

  (v) Determine whether it would be profitable for the company to buy the additional van. State one other factor apart from profit which the manager might consider when deciding whether to purchase the additional van. [3]

3 A company supplies cylinders of pressurised acetylene gas for welding. The nominal content of a cylinder is 25.00 kg. The actual content of cylinders, measured in kilograms, is Normally distributed with mean $\mu = 25.23$ and standard deviation $\sigma = 0.13$.

  (i) Find the proportion of cylinders which contain less than 25 kg. [3]

  (ii) 5% of cylinders contain more than c kg. Find the value of c. [3]

In a quality control procedure, a random sample of 20 cylinders is selected from a large batch. The batch is rejected if more than one cylinder in the sample contains less than 25 kg.

(iii) Find the probability that the batch will be accepted. [4]

The management wish to be 90% certain that a batch will be accepted. The company statistician advises that, to achieve this, the probability that a cylinder contains less than 25 kg must be reduced to 0.027.

(iv) This can be done by changing the value of μ. Find the new value of μ. [4]

  (v) State an alternative way in which a probability of 0.027 could be achieved, without changing the value of μ. (No further calculations are required.) [1]

[Turn over

4   In a school quiz, three children each win a bar of chocolate as a prize. Their prizes are randomly selected from 10 identically wrapped bars, of which 3 are plain chocolate and the rest are milk chocolate. The random variable X represents the number of plain chocolate bars which the three children win between them.

The probability distribution for X is given in the table.

r	0	1	2	3
P(X = r)	$\frac{7}{24}$	$\frac{21}{40}$	$\frac{7}{40}$	$\frac{1}{120}$

(i)   Draw a sketch to illustrate the probability distribution of X. [2]

(ii) Use a probability argument to verify that $P(X = 1) = \frac{21}{40}$. [3]

(iii) Find the expectation and variance of X. [4]

Plain chocolate bars cost 60 pence each and milk chocolate bars cost 40 pence each.

(iv) Show that the total cost of the three bars of chocolate is $(120 + 20X)$ pence. [2]

(v)   Find the expectation and standard deviation of this total cost. [4]