OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2612**

Mechanics 6

Monday **16 JANUARY 2006** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

1 *Option 1: Rotation of a Rigid Body*

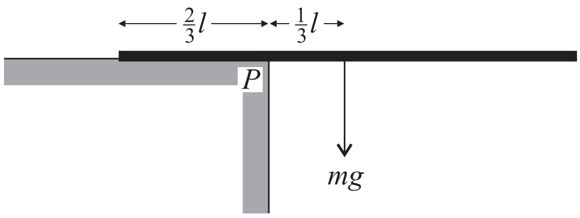

Fig. 1.1

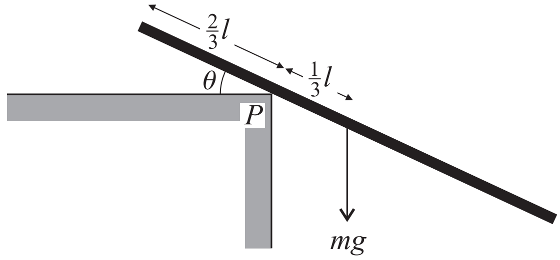

Fig. 1.2

A uniform rod of length $2l$ and mass m is held with a section $\frac{2}{3}l$ in contact with a rough horizontal table, as shown in Fig. 1.1. The rod is held perpendicular to the straight edge of the table and is allowed to fall freely from rest. Its position a short time later is shown in Fig. 1.2. During the early part of the motion, the rod does not slip at P.

The angle between the rod and the horizontal at time t is denoted by θ and the coefficient of friction between the rod and the table is $\frac{3}{4}$.

(i) Given that the moment of inertia of the rod about an axis through its centre perpendicular to its length is $\frac{1}{3}ml^2$, find its moment of inertia about P. [2]

(ii) Write down the equation of conservation of energy. Hence or otherwise find an equation giving $\ddot{\theta}$ in terms of θ, g and l. [5]

(iii) Write down the equations of motion of the centre of mass of the rod. Show that the frictional force on the rod is given by

$$F = \tfrac{3}{2}mg\sin\theta.$$

Show that the rod begins to slip when $\tan\theta = \frac{3}{8}$. [10]

(iv) If instead the rod is initially released from rest at an angle $\arctan\left(\frac{3}{8}\right)$ to the horizontal, will the rod immediately begin to slip? Explain your answer. [3]

2 *Option 2: Vectors*

Forces $\mathbf{F}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\mathbf{F}_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{F}_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ act through points with position vectors $\mathbf{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{r}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\mathbf{r}_3 = \begin{pmatrix} a \\ 1 \\ -2 \end{pmatrix}$ respectively, relative to a fixed origin O, where a is a constant.

(i) Find the resultant force $\mathbf{F}$. Find also the resultant torque, $\mathbf{C}$, about the origin in terms of a. [6]

A fourth force $\mathbf{F}_4$ is added, acting through a point with position vector $\mathbf{r}_4$. The system of forces is now in equilibrium.

(ii) Write down the values of $\mathbf{F}_4$ and $\mathbf{r}_4 \times \mathbf{F}_4$. [2]

(iii) Explain why $\mathbf{C}.\mathbf{F}_4 = 0$ and hence show that $a = \frac{8}{3}$. [4]

(iv) Given that the x-component of $\mathbf{r}_4$ is zero, calculate $\mathbf{r}_4$. [4]

(v) The force $\mathbf{F}$ is equivalent to the forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$. What is the equation of the line of action of $\mathbf{F}$? [4]

3 *Option 3: Stability and Oscillations*

A smooth circular hoop of radius a is fixed in a vertical plane. A small ring of mass m is threaded on the hoop and is joined to the highest point of the hoop by a light elastic spring of natural length l and modulus kmg where $ka > l$. At time t, the angle between the spring and the downward vertical is θ where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(i) Show that the potential energy V, relative to the highest point of the hoop, can be written in the form

$$V = \frac{2amg}{l}(ka - l)\cos^2\theta - 2kmga\cos\theta + \tfrac{1}{2}kmgl.$$ [5]

(ii) Show that $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = 2mga\sin\theta\left[k - \dfrac{2}{l}(ka - l)\cos\theta\right]$.

Given that $l < \dfrac{2ka}{k+2}$, show that there are three positions of equilibrium.

Discuss the stability of each position. [11]

(iii) In the case when $l > \dfrac{2ka}{k+2}$, show that there is just one equilibrium position and that this position is stable. [4]

4 Option 4: Variable Mass

A particle falls under gravity from rest through a stationary cloud. It picks up mass as it falls through the cloud. The rate at which the mass of the particle increases is equal to mk where m is the mass at time t and k is a constant. The velocity of the particle is v.

(i) Write down the equation of motion of the particle. [2]

(ii) Write down an expression for $\frac{dm}{dt}$. Using this and the equation of motion, show that

$$\frac{dv}{dt} + kv = g. \quad [4]$$

(iii) Solve the equation in part (ii) to show that $v = \frac{g}{k}\left(1 - e^{-kt}\right)$. Find also the distance fallen in time t. [9]

(iv) Explain what happens to the speed and the mass as t tends to infinity. [5]