RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2611**

Mechanics 5

Thursday **12 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8\text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 A particle P of mass m, moving in the x-y plane, is subject to a force $-6mat\mathbf{j}$, where t is time and a is a positive constant. Initially the particle is at the origin and has velocity $2a\mathbf{i}$.

(i) Find the velocity of P at time t and show that the position vector of P at time t is $\mathbf{r} = a(2t\mathbf{i} - t^3\mathbf{j})$. [7]

(ii) Find the cartesian equation of the path of P and sketch the path for $t \geqslant 0$. [5]

(iii) Find the power of the force at time t. [2]

(iv) Hence find by integration the work done in the interval $0 \leqslant t \leqslant 2$. Verify that this is equal to the change in kinetic energy of P. [6]

2 An aircraft flies with a constant speed relative to the air (air speed) of $250\,\mathrm{km\,h^{-1}}$. On a particular day, the wind blows at $20\,\mathrm{km\,h^{-1}}$ from the south-west. The aircraft takes off at noon and flies from airport A to another airport B which is 200 km due south of A.

(i) Draw a relative velocity diagram for the aircraft and hence find the speed of the aircraft relative to the ground (i.e. its ground speed). [7]

A second identical aircraft also takes off at noon and flies at the same air speed from airport B to another airport C due east of B. The wind still blows at $20\,\mathrm{km\,h^{-1}}$ from the south-west.

(ii) Draw a relative velocity diagram for the second aircraft and hence find the ground speed in this case. [6]

(iii) Find the shortest distance between the two aircraft and the time taken to reach this position. [7]

3 A particle is moving in a plane. Unit vectors in the radial and transverse directions are $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ respectively.

(i) Using the results $\dfrac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}t} = \dot{\theta}\hat{\boldsymbol{\theta}}$ and $\dfrac{\mathrm{d}\hat{\boldsymbol{\theta}}}{\mathrm{d}t} = -\dot{\theta}\hat{\mathbf{r}}$, derive an expression for the velocity of the particle and show that the acceleration is $(\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})\hat{\boldsymbol{\theta}}$. [7]

The particle is subject to a force $-mk\mathbf{r}$ where k is a positive constant.

(ii) Show that $r^2\dot{\theta}$ is constant. [3]

(iii) Denoting the constant value of $r^2\dot{\theta}$ by h, find $\ddot{r}$ in terms of r, k and h and hence show that $\dot{r}^2 = -kr^2 - \dfrac{h^2}{r^2} + A$, where A is an arbitrary constant. [6]

(iv) Hence find the speed of the particle in terms of r, k and A. [4]

4 A solid cylinder of radius a, length a and mass M has density ρ which varies with radius r according to the formula $\rho = \rho_0\left(1 - \frac{r^2}{a^2}\right)$, where ρ_0 is a constant.

(i) Show that $M = \frac{1}{2}\pi\rho_0 a^3$. [6]

(ii) Find the moment of inertia of the cylinder about its axis of symmetry in terms of M and a. [6]

A uniform disc has radius a and mass m.

(iii) Assuming that the moment of inertia of this disc about the axis perpendicular to its plane through its centre is $\frac{1}{2}ma^2$, deduce its moment of inertia about any diameter.

When the disc is rotated about a diameter with angular speed ω, its kinetic energy is E. When the cylinder is rotated about its axis of symmetry with angular speed 3ω, its kinetic energy is $2E$. Find m in terms of M. [8]

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