OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2605

Pure Mathematics 5

Monday **16 JANUARY 2006** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 **(a)** The equation $x^3 + 5x^2 - 8 = 0$ has roots α, β and γ. Find a cubic equation with integer coefficients which has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. [4]

(b) You are given the polynomial $f(x) = kx^7 + mx^4 + 18x^2 - 125x + 380$, where k and m are constants.
When $f(x)$ is divided by $(x - 2)$, the remainder is 26.
When $f(x)$ is divided by $(x + 2)$, the remainder is 14.

(i) Find k and m, and show that $f'(2) = -21$. [7]

(ii) When $f(x)$ is divided by $(x^2 - 4)$, the quotient is $g(x)$ and the remainder is $ax + b$, so that

$$f(x) = (x^2 - 4)g(x) + ax + b.$$

Find a and b. [4]

(iii) Find the remainder when $f(x)$ is divided by $(x - 2)^2$. [5]

2 **(i)** Prove that $\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right)$. [5]

(ii) Find $\int_0^4 \frac{1}{\sqrt{3x^2 + 16}}\,dx$, giving your answer in logarithmic form. [5]

(iii) Find the exact value of $\int_0^4 \frac{1}{3x^2 + 16}\,dx$. [4]

(iv) Use the substitution $x\sqrt{3} = 4\tan\theta$ to show that $\int_0^4 \frac{1}{(3x^2 + 16)^{\frac{3}{2}}}\,dx = \frac{1}{32}$. [6]

3 **(i)** Express $e^{-\frac{1}{2}j\theta} + e^{\frac{1}{2}j\theta}$ in trigonometric form, and show that $\left(1 + e^{j\theta}\right)^2 = 4e^{j\theta}\cos^2\frac{1}{2}\theta$. [6]

(ii) For a positive integer n, series C and S are given by

$$C = 1 + \binom{2n}{1}\cos\theta + \binom{2n}{2}\cos 2\theta + \binom{2n}{3}\cos 3\theta + \ldots + \cos 2n\theta,$$

$$S = \binom{2n}{1}\sin\theta + \binom{2n}{2}\sin 2\theta + \binom{2n}{3}\sin 3\theta + \ldots + \sin 2n\theta.$$

Show that $C = 4^n \cos n\theta \cos^{2n}\frac{1}{2}\theta$, and find a similar expression for S. [9]

(iii) Given that $w = e^{j\phi}$ is a cube root of 1, state the three possible values of ϕ with $-\pi < \phi < \pi$, and find the possible values of $(1 + w)^6$. [5]

4 **(a)** A parabola has parametric equations $x = at^2$, $y = 2at$.

(i) Show that the chord joining the points $P_1\ (at_1^2, 2at_1)$ and $P_2\ (at_2^2, 2at_2)$ on the parabola has equation

$$(t_1 + t_2)y = 2x + 2at_1t_2.$$ [4]

(ii) Hence or otherwise find the equation of the tangent to the parabola at a general point $(at^2, 2at)$. [3]

The tangents to the parabola at P_1 and P_2 meet at the point (p, q).

(iii) Show that $t_1t_2 = \frac{p}{a}$, and find an expression for $t_1 + t_2$. [4]

(iv) Show that P_1P_2 crosses the x-axis at the point $(-p, 0)$. [3]

(b) A conic has polar equation $\frac{7}{r} = 3 + 4\cos\theta$.

(i) Find the eccentricity, and state which type of conic the equation represents. [2]

(ii) Sketch the conic, using a continuous line for sections where $r > 0$ and a broken line for sections where $r < 0$. [4]

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