OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2604**

Pure Mathematics 4

Wednesday **18 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 A curve has equation $y = \dfrac{1-x}{(x-2)(x-10)}$.

(i) Write down the equations of the three asymptotes. [2]

(ii) Find $\dfrac{dy}{dx}$. Hence find the coordinates of the stationary points. [5]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{1-x}{(x-2)(x-10)} < \dfrac{1}{20}$. [5]

(v) On a separate diagram, sketch the curve with equation $y^2 = \dfrac{1-x}{(x-2)(x-10)}$.

Give the coordinates of the points on this curve where the tangent is parallel to the x-axis, and the point where the tangent is parallel to the y-axis. [5]

2 **(a)** Find the sum of the series

$$(1 \times 2) + (3 \times 4) + (5 \times 6) + \ldots + (2n-1)(2n),$$

giving your answer in a fully factorised form. [5]

(b) Find $\displaystyle\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+5)}$. [7]

(c) Prove by induction that $\displaystyle\sum_{r=1}^{n} \frac{(r^2+1)2^r}{r(r+1)} = \frac{n2^{n+1}}{n+1}$. [8]

3 **(a)** The equation $z^2 + 4z + 9 = 0$ has complex roots α and β, where α is the root with a positive imaginary part.

(i) Find α and β in the form $a + b\mathrm{j}$, giving the exact values of a and b. [3]

(ii) Find the modulus and argument of each of the complex numbers

$$\alpha, \ \beta, \ \frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha},$$

giving the arguments in radians between $-\pi$ and π, correct to 2 decimal places.

Illustrate these four complex numbers on an Argand diagram. [8]

(iii) Describe in words the locus in the Argand diagram of points representing complex numbers z which satisfy

$$|z - \alpha| = |z|.$$

Draw this locus on your diagram. [3]

(b) The complex numbers z and w satisfy the simultaneous equations

$$(1 + \mathrm{j})z + 2w = 3 + 7\mathrm{j},$$
$$3z - (1 + \mathrm{j})w = 7 + 20\mathrm{j}.$$

Find z and w, giving your answers in the form $a + b\mathrm{j}$. [6]

4 **(a)** The matrix $\begin{pmatrix} 4 & 2 \\ 12 & -1 \end{pmatrix}$ defines a transformation in the (x, y)-plane.

Find the two values of m for which $y = mx$ is an invariant line of the transformation. [6]

(b) **(i)** Find the vector product $(4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}) \times (10\mathbf{i} - 4\mathbf{j} - 5\mathbf{k})$. [2]

(ii) Find the equation of the line of intersection of the two planes

$$4x + 2y + 7z = 2,$$
$$10x - 4y - 5z = 50.$$

[3]

You are now given the matrix equation $\begin{pmatrix} 4 & 2 & 7 \\ 10 & -4 & -5 \\ k & 3 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 50 \\ a \end{pmatrix}$.

Using your answer to part **(ii)** or otherwise,

(iii) when $k = 4$, express x, y and z in terms of a, [5]

(iv) when $k = 3$, find the value of a for which there are solutions, and give the general solution in this case. [4]

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