OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Monday **23 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension**.

This question paper consists of 4 printed pages.

1 **(a)** Find the first three non-zero terms of the binomial expansion of $\frac{1}{\sqrt{4-x^2}}$, given that $|x| < 2$. [4]

(b) A curve is defined parametrically by the equations

$$x = \sin^2\theta, \qquad y = 1 - \cos\theta.$$

Show that $\frac{dy}{dx} = \frac{1}{2}\sec\theta$. [5]

(c) Express $5\sin\theta + 12\cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants to be determined, and $0° < \alpha < 90°$.

Hence solve the equation $5\sin\theta + 12\cos\theta = 13$, where $0° \leqslant \theta \leqslant 360°$. [5]

[Total 14]

2 Fig. 2 shows a tetrahedron ABCD with vertices A(–2, 4, 1), B(2, 3, 4), C(4, 8, 3) and D(2, –3, –11).

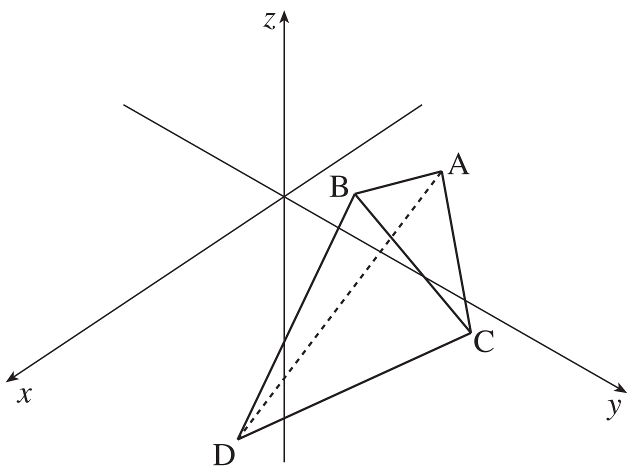

Fig. 2

(i) By calculating a suitable scalar product, show that angle ABC is a right angle.

Hence calculate the area of triangle ABC. [5]

(ii) Verify that $\mathbf{n} = \begin{pmatrix} 7 \\ -5 \\ -11 \end{pmatrix}$ is normal to the plane ABC. Hence find the cartesian equation of this plane. [4]

(iii) Write down a vector equation of the line passing through D and perpendicular to the plane ABC. Find the point of intersection of this line and the plane ABC. [6]

[Total 15]

3 In a game of rugby, a kick is to be taken from a point P (see Fig. 3). P is a perpendicular distance y metres from the line TOA. Other distances and angles are as shown.

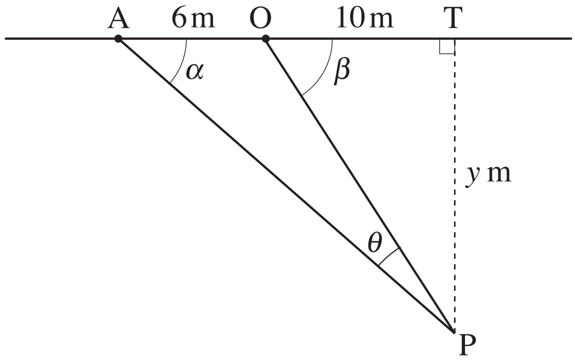

Fig. 3

(i) Show that $\theta = \beta - \alpha$, and hence that $\tan\theta = \dfrac{6y}{160 + y^2}$.

Calculate the angle θ when $y = 6$. [7]

(ii) By differentiating implicitly, show that

$$\frac{d\theta}{dy} = \frac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta.$$ [5]

(iii) Use this result to find the value of y that maximises the angle θ. Calculate this maximum value of θ. [You need not verify that this value is indeed a maximum.] [4]

[Total 16]

[Turn over

4 Some years ago an island was populated by red squirrels and there were no grey squirrels. Then grey squirrels were introduced.

The population x, in thousands, of red squirrels is modelled by the equation

$$x = \frac{a}{1 + kt},$$

where t is the time in years, and a and k are constants. When $t = 0$, $x = 2.5$.

(i) Given that the initial population of 2.5 thousand red squirrels reduces to 1.6 thousand after one year, calculate a and k. [3]

(ii) What is the long-term population of red squirrels predicted by this model? [1]

The population y, in thousands, of grey squirrels is modelled by the differential equation

$$\frac{dy}{dt} = 2y - y^2.$$

When $t = 0$, $y = 1$.

(iii) Express $\frac{1}{2y - y^2}$ in partial fractions. [3]

(iv) Hence show by integration that $\ln\left(\frac{y}{2 - y}\right) = 2t$.

Show that $y = \frac{2}{1 + e^{-2t}}$. [7]

(v) What is the long-term population of grey squirrels predicted by this model? [1]

[Total 15]