OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2602/1

Pure Mathematics 2

Wednesday **18 JANUARY 2006** Afternoon 1 hour 20 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Question **3**.
- You are permitted to use only a scientific calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

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[Turn over

1 **(a)** The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = \sqrt{x}$ for $0 \leqslant x \leqslant 1$.

Fig. 1 shows the graph of $y = \mathrm{f}(x)$.

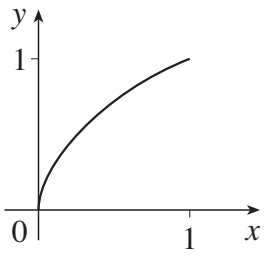

Fig. 1

Sketch the graphs of

(i) $y = 2\mathrm{f}(x)$,

(ii) $y = \mathrm{f}(x + 1)$. [4]

(b) Differentiate $\sqrt{1+2x}$.

Hence show that if $y = x\sqrt{1+2x}$,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1+3x}{\sqrt{1+2x}}.$$ [6]

(c) Given that $\mathrm{f}(x) = x^2$ and $\mathrm{g}(x) = \log(1 - x)$, where $-1 < x < 1$, show that

$$\mathrm{g}(x) + \mathrm{g}(-x) = \mathrm{gf}(x).$$ [4]

[Total 14]

2 A sequence u_r is defined for $r = 1, 2, 3, \ldots$ by

$$u_1 = a, \qquad u_{r+1} = bu_r + c,$$

where a, b and c are constants.

(i) In the case where $a = 3$, $b = -1$ and $c = 8$, write down the values of u_1, u_2, u_3 and u_4.

State what type of sequence this is. [4]

(ii) Find the values of a, b and c which produce the sequence $1, 3, 5, 7, \ldots$.

State what type of sequence this is, and show that the sum of the first n terms of the sequence is n^2. [6]

(iii) In the case where a and b are non-zero and $c = 0$, write down u_1, u_2 and u_3 in terms of a and b. State what type of sequence is produced. Given that the sum to infinity of this sequence is $3u_1$, find the value of b. [5]

[Total 15]

3 Answer part (i) of this question on the insert provided.

Vijay is interested in how the prices of used cars are related to their ages. He wishes to test a model of the form $V = V_0 b^t$, where £V is the value of a car that is t years old and V_0 and b are constants.

He looks in a motoring magazine and finds that, for a certain type of car, the value £V after t years is given by the following table.

Age t (years)	Value V (£)
2	7800
4	5800
6	4200
8	3000

(i) Explain how plotting values of $\ln V$ against t can be used to test the appropriateness of the model. Use the insert provided to plot $\ln V$ against t. Hence estimate the values of V_0 and b for this type of car. [8]

(ii) Use these values to estimate the value of this type of car after 20 years. Comment on the validity of this result. [3]

Another type of car costs £12 000 when new, and $b = 0.8$.

(iii) Use the model $V = V_0 b^t$ to calculate how long it takes for this type of car to lose half of its initial value. [4]

(iv) Describe the significance of the different values of b for the two types of car. [1]

[Total 16]

[Turn over

4 Fig. 4 shows the graph of $y = f(x)$, where $f(x) = \dfrac{e^x - 1}{e^x + 1}$.

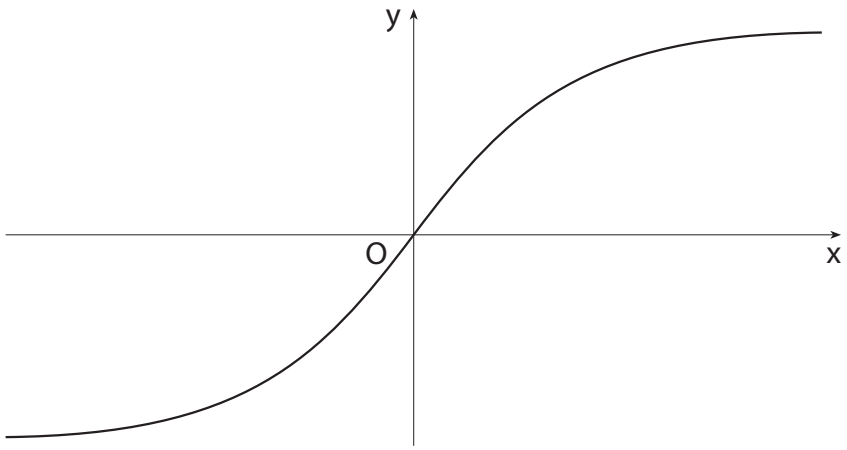

Fig. 4

(i) Prove that $f(x)$ is an odd function. State how this relates to the shape of the graph of $y = f(x)$. [4]

(ii) Find the gradient of the curve $y = f(x)$ at the origin. [5]

(iii) Find $\int \dfrac{e^x}{e^x + 1}\,dx$. [2]

(iv) Verify that $f(x) = \dfrac{2e^x}{e^x + 1} - 1$.

Hence show that the area of the region enclosed by the curve $y = f(x)$, the x-axis and the line $x = 1$ is given by $2\ln\left(\dfrac{e+1}{2}\right) - 1$. [4]

[Total 15]

Candidate Name	Centre Number	Candidate Number

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INSERT

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INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **3.**
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

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[Turn over

3 (i)

Age t (years)	2	4	6	8
Value V (£)	7800	5800	4200	3000
ln V (to 3 sig. fig.)	8.96