OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4773

Decision Mathematics Computation

Friday **24 JUNE 2005** Morning 2 hours 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Additional sheets, including computer print-outs, should be fastened securely to the answer booklet.
- You are permitted to use a graphical calculator in this paper.
- There is an **insert** for use in Questions **2** and **4**.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages, 2 blank pages and an insert.

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1 Jim needs to control his weight. His weight at time $n + 2$ (days) is related to his food intake from time n to time $n + 1$ by the recurrence relation

$$u_{n+2} = u_{n+1} + \tfrac{1}{144}(v_n - 32u_n), \qquad (n \text{ is a non-negative integer}),$$

where u_n is his weight in kg at time n, and where v_n is his intake of food in calories from time n to time $n + 1$.

(i) Jim weighs 80 kg. Find the intake in calories which will maintain his weight in equilibrium at 80 kg. [2]

Jim goes on holiday. At the start of his holiday (time = 0) he weighs 80kg. At the start of the next day (time = 1) he still weighs 80 kg. He consumes 3000 calories per day whilst on holiday.

(ii) Find Jim's equilibrium weight at an intake of 3000 calories per day. [2]

(iii) Solve the recurrence relation with $v_n = 3000$ (constant) to find a formula giving Jim's weight on subsequent days of his holiday. [10]

Some time after his holiday Jim decides to diet. At the start of his diet (time = 0) he weighs 90 kg. At the start of the next day (time = 1) he still weighs 90 kg. He restricts his food intake so that $v_n = 3000 - 8u_n$ whilst he is on his diet.

(iv) Build a spreadsheet to show how Jim's weight changes over time. Produce a printout showing his weight over the first two weeks of his diet, and say what happens in the long run. [3]

(v) Explain what will happen if instead of $3000 - 8u_n$, Jim restricts his daily intake to $2560 - 8u_n$ calories. [1]

[Turn over

2 Answer parts (i) to (v) of this question on the insert provided.

Bob and Lynne have a network of irrigation pipes in their garden. Pipes have not always been added in a logical fashion, and the current network is shown in Fig. 2.1. The system delivers water from a source marked S, through junctions at A, B, C and D, to locations E, F, G, H and I. The numbers on the arcs are the capacities of the pipes in litres per minute.

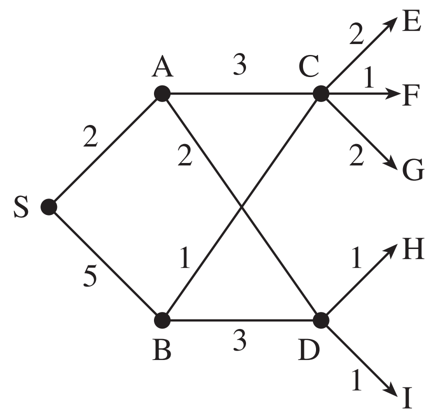

Fig. 2.1

(i) On the copy of the network in the insert, add a super sink, T, with arcs of appropriate capacities connecting it to E, F, G, H and I. [2]

Flows are established as shown in Table 2.2.

S→A	A→C	C→E	S→B	B→C	C→F	B→D	D→H	D→I
2	2	2	3	1	1	2	1	1

Table 2.2

(ii) For this flow, complete the labelling on the insert for the arc BC. [3]

(iii) Find and list a flow-augmenting path of capacity 1 litre per minute. [2]

(iv) Show on Fig. 2.3 on the insert a flow pattern giving a total flow of 6 litres per minute. [2]

(v) Give a cut of capacity 6. [2]

Bob lays a new pipe of capacity 1 from S to D. Lynne wants to know the effect of this on her olive tree at G.

(vi) Formulate an LP to find the maximum flow through this extended network. [4]

(vii) Run your LP and interpret your results. [3]

3 A railway station platform has an automatic exit barrier. When a train arrives, 100 passengers leave the train and have to pass through the barrier. The time taken for an individual to pass through the barrier is a random variable, and is specified in table 3.

Time (seconds)	1.5	2	3.5
Probability	0.15	0.75	0.1

Table 3

(i) Assume that all of the passengers arrive at the barrier at the same moment and form a single queue. Build a simulation model to find how long it takes for all of the passengers to pass through the barrier.

Include a printout of your simulation. [2]

(ii) Repeat 10 times your simulation from part **(i)** and calculate the mean time taken for all 100 passengers to pass through the barrier. From your results calculate how many repetitions are required to estimate the mean time with a maximum error of 0.5 seconds with 95% confidence.

Include a printout of your simulation. [5]

In fact passengers do not all arrive at the barrier at the same moment. The barrier is located near to the front of the train and passengers at the rear of the train take 2 minutes to walk to it.

(iii) In a column of a spreadsheet simulate 100 uniformly distributed random numbers between 0 and 120. **Copy** them and paste a fixed copy of them into a second column. To do this using EXCEL, use the **Paste Special** command from the **Edit** menu, selecting the **Paste Values** option. Now use the **Sort** command from the **Data** menu to sort them in ascending order into a third column.

This will give simulated arrival times (in seconds) of 100 passengers at the barrier. [3]

(iv) Complete a simulation of 100 passengers walking to the barrier and passing through it. Give the time taken for all 100 passengers to pass through the barrier.

Include a printout of your simulation. Show the formulae which you used for row 10 of your spreadsheet. [3]

A second identical exit barrier is installed.

(v) Using your arrival times from part **(iii)**, and assuming that a single queue is formed, simulate the 100 passengers passing through the two barriers. Give the time it takes for all 100 passengers to pass through the barriers.

Include a printout of your simulation. Show the formulae which you used for row 10 of your spreadsheet. [5]

4 Answer part (i) of this question on the insert provided.

Flyair needs to schedule a pilot for each of 12 flights between London, Berlin, Paris and Milan. Table 4.1 shows flight numbers, routes and flight times.

Flight		Departs	Arrives
101	London → Berlin	07.00	09.00
102	London → Berlin	19.00	21.00
201	Berlin → Paris	09.30	11.00
202	Berlin → Milan	10.30	11.30
203	Berlin → Paris	22.00	23.30
204	Berlin → Milan	22.30	23.30
301	Milan → Paris	12.00	13.00
302	Milan → London	15.30	17.00
303	Milan → Berlin	18.00	19.00
401	Paris → Berlin	07.00	08.30
402	Paris → Milan	12.00	13.00
403	Paris → London	14.00	15.00

Table 4.1

Fig. 4.2 represents this information diagrammatically.

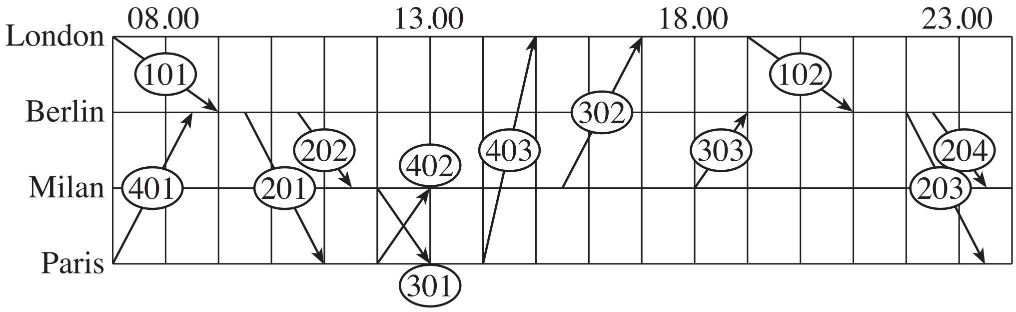

Fig. 4.2

Pilots are scheduled for up to 4 flights per day, and must end the day in their home city. Thus for a Berlin based pilot the following is a possible schedule:

$$202 \rightarrow 301 \rightarrow 403 \rightarrow 102$$

(i) Complete the table in the insert showing 20 possible schedules. [5]

(ii) Formulate an LP to choose the schedules to use so that all flights have at least one pilot, and so that the number of schedules used is minimised. Your LP should have 20 variables and 12 constraints, one for each flight. [6]

(iii) Run your LP, and interpret your solution. How many pilots are needed? [3]

(iv) Show that the optimal solution is unique. [3]

(v) Criticise the model. [1]

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Candidate Name	Centre Number	Candidate Number

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INSERT

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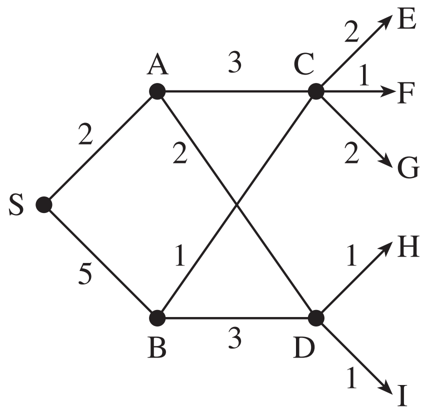
INSTRUCTIONS TO CANDIDATES

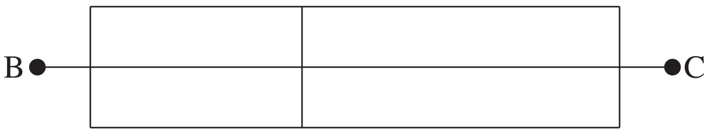
- This insert should be used in Questions **2** and **4.**
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 3 printed pages and 1 blank page.

2 **(i)**



(ii)



key

capacity	forward potential
flow	backward potential

(iii) Flow-augmenting path: ______________________

(iv)

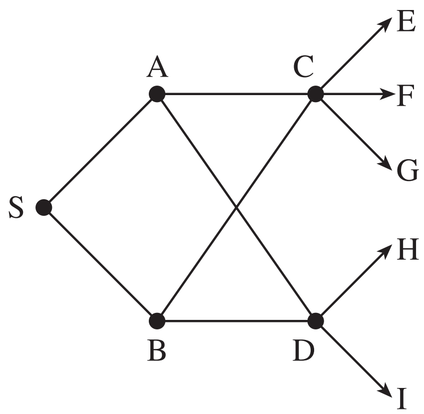


Fig. 2.3

(v) Cut: ____________________________

4

Schedule	City	Flight	City	Flight	City	Flight	City	Flight	City
S1	L	101	B	201	P	402	M	302	L
S2	L	101	B						
S3	L	101	B						
S4	L	101	B						
S5	B	201	P						
S6	B	201	P						
S7	B	201	P						
S8	B	202	M	301	P	403	L	102	B
S9	B	202	M	302	L	102	B		
S10	B	202	M						
S11	M	301	P						
S12	M	302	L						
S13	M	303	B						
S14	P	401	B						
S15	P	401	B						
S16	P	401	B						
S17	P	401	B						
S18	P	402	M						
S19	P	402	M						
S20	P	403	L						

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