OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Section A

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this section is 72.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

HN/3

[Turn over

Section A (36 marks)

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence find the range of the function $f(\theta)$, where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

Write down the greatest possible value of $\dfrac{1}{7 + 3\cos\theta + 4\sin\theta}$. [6]

2 Find the first 4 terms in the binomial expansion of $\sqrt{4 + 2x}$. State the range of values of x for which the expansion is valid. [6]

3 Solve the equation

$$\sec^2\theta = 4, \quad 0 \leqslant \theta \leqslant \pi,$$

giving your answers in terms of π. [4]

4 Fig. 4 shows a sketch of the region enclosed by the curve $\sqrt{1 + e^{-2x}}$, the x-axis, the y-axis and the line $x = 1$.

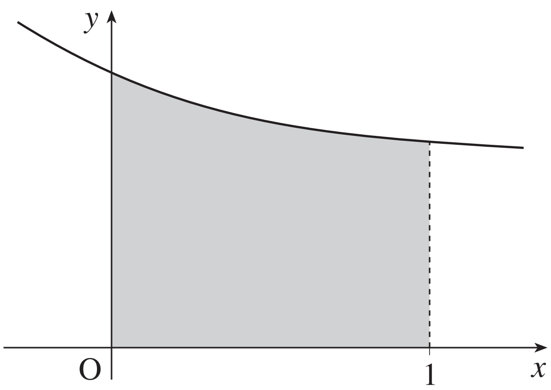

Fig. 4

Find the volume of the solid generated when this region is rotated through 360° about the x-axis. Give your answer in an exact form. [5]

5 Solve the equation $2\cos 2x = 1 + \cos x$, for $0° \leqslant x < 360°$. [7]

6 A curve has cartesian equation $y^2 - x^2 = 4$.

(i) Verify that

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t},$$

are parametric equations of the curve. [2]

(ii) Show that $\frac{dy}{dx} = \frac{(t-1)(t+1)}{t^2+1}$. Hence find the coordinates of the stationary points of the curve. [6]

Section B (36 marks)

7 In a chemical process, the mass M grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{dM}{dt} = \frac{M}{t(1+t^2)}.$$

(i) Find $\int \frac{t}{1+t^2}\, dt$. [3]

(ii) Find constants A, B and C such that

$$\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt + C}{1+t^2}.$$ [5]

(iii) Use integration, together with your results in parts **(i)** and **(ii)**, to show that

$$M = \frac{Kt}{\sqrt{1+t^2}},$$

where K is a constant. [6]

(iv) When $t = 1, M = 25$. Calculate K.

What is the mass of the chemical in the long term? [4]

 [Turn over

8 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the line of the hole.

A 20 cm × 30 cm × 30 cm cuboid is to be cut and drilled. The cuboid is positioned relative to x-, y- and z-axes as shown in Fig. 8.1.

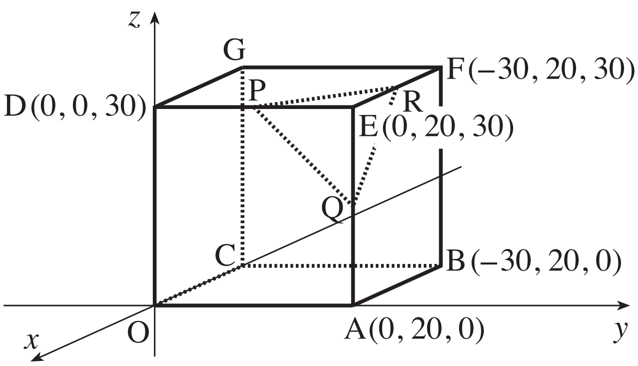

Fig. 8.1

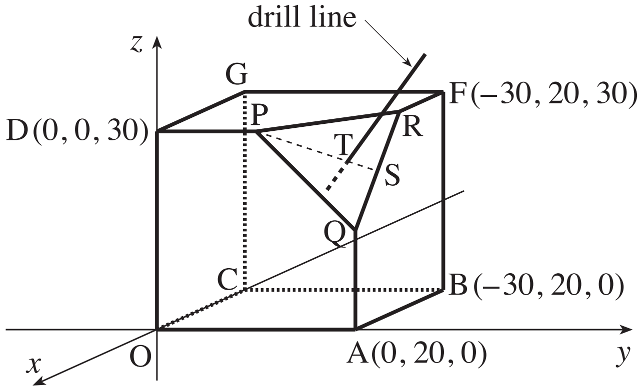

Fig. 8.2

First, a plane cut is made to remove the corner at E. The cut goes through the points P, Q and R, which are the midpoints of the sides ED, EA and EF respectively.

(i) Write down the coordinates of P, Q and R.

Hence show that $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$. [4]

(ii) Show that the vector $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ is perpendicular to the plane through P, Q and R.

Hence find the cartesian equation of this plane. [5]

A hole is then drilled perpendicular to triangle PQR, as shown in Fig. 8.2. The hole passes through the triangle at the point T which divides the line PS in the ratio 2:1, where S is the midpoint of QR.

(iii) Write down the coordinates of S, and show that the point T has coordinates $\left(-5, 16\tfrac{2}{3}, 25\right)$. [4]

(iv) Write down a vector equation of the line of the drill hole.

Hence determine whether or not this line passes through C. [5]

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