## OXFORD CAMBRIDGE AND RSA EXAMINATIONS

## Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

## 4771

Decision Mathematics 1
Friday 14 JANUARY $2005 \quad$ Morning 1 hour 30 minutes
Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- There is an insert for use in Questions 2, 4 and 5.
- You are permitted to use a graphical calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72 .


## Section A

1 The bipartite graph in Fig. 1 represents a board game for two players. At each turn a player tosses a coin and moves their counter. The graph shows which square the counter is moved to if the coin shows heads, and which square if it shows tails. Each player starts with their counter on square 1. Play continues until one player gets their counter to square 9 and wins.


Fig. 1
(i) Draw a tree to show all of the possibilities for the player's first three moves.
(ii) Show how a player can win in 3 turns.
(iii) List all squares which it is possible for a counter to occupy after 3 turns.
(iv) Show that a game can continue indefinitely.

## 2 Answer this question on the insert provided.

(i) Use Dijkstra's algorithm to find the least weight route from A to G in the network shown in Fig. 2.1. Show the order in which you label vertices, give the route and its weight.


Fig. 2.1
(ii) Fig. 2.2 shows a partially completed application of Kruskal's algorithm to find a minimum spanning tree for the network.


Fig. 2.2
Complete the algorithm and give the total weight of your minimum spanning tree.

3 The following algorithm finds the highest common factor of two positive integers. ("int (x)" stands for the integer part of $x$, e.g. int $(7.8)=7$.)


Fig. 3.1
(i) Run the algorithm with $\mathrm{A}=84$ and $\mathrm{B}=660$, showing all of your calculations.
(ii) Run the algorithm with $\mathrm{A}=660$ and $\mathrm{B}=84$, showing as many calculations as are necessary.
(iii) The algorithm is run with $\mathrm{A}=30$ and $\mathrm{B}=42$, and the result is shown in Table 3.2 below.

| A | B | Q | R 1 | R 2 |
| :---: | :---: | :---: | :---: | :---: |
| 30 | 42 | 1 | 12 |  |
| 12 | 30 | 2 |  | 6 |
|  |  |  | 6 |  |
| 6 | 12 | 2 |  | 0 |

Print 6

Table 3.2
The first line of the table shows that $12=42-1 \times 30$.
Use the second line to obtain a similar expression for 6 in terms of 30 and 12 .
Hence express 6 in the form $\mathrm{m} \times 30-\mathrm{n} \times 42$, where m and n are integers.

## Section B

## 4 Answer this question on the insert provided.

The table shows activities involved in a "perm" in a hair salon, their durations and immediate predecessors.

|  | Activity | Duration (mins) | Immediate predecessor(s) |
| :---: | :--- | :---: | :---: |
| A | shampoo | 5 | - |
| B | prepare perm lotion | 2 | - |
| C | make coffee for customer | 3 | - |
| D | trim | 5 | A |
| E | clean sink | 3 | A |
| F | put rollers in | 15 | D |
| G | clean implements | 3 | D |
| H | apply perm lotion | 20 | $\mathrm{~B}, \mathrm{~F}$ |
| I | leave to set | 3 | $\mathrm{C}, \mathrm{H}$ |
| J | clean lotion pot and spreaders | 10 | H |
| K | neutralise and rinse | 10 | $\mathrm{I}, \mathrm{E}$ |
| L | dry | 15 | K |
| M | wash up and clean up | 4 | K |
| N | style |  | G, L |

## Table 4

(i) Complete the activity-on-arc network in the insert to represent the precedences.
(ii) Perform a forward pass and a backward pass to find early and late event times. Give the critical activities and the time needed to complete the perm.
(iii) Give the total float time for the activity G.

Activities D, F, H, K and N require a stylist.
Activities A, B, C, E, G, J and M are done by a trainee.
Activities I and L require no-one in attendance.
A stylist and a trainee are to give a perm to a customer.
(iv) Use the chart in the insert to show a schedule for the activities, assuming that all activities are started as early as possible.
(v) Which activity would be better started at its latest start time?

5 There is an insert for use in parts (iii) and (iv) of this question.
This question concerns the simulation of cars passing through two sets of pedestrian controlled traffic lights. The time intervals between cars arriving at the first set of lights are distributed according to Table 5.1.

| Time interval (seconds) | 2 | 5 | 15 | 25 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | $\frac{3}{7}$ | $\frac{2}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ |

## Table 5.1

(i) Give an efficient rule for using two-digit random numbers to simulate arrival intervals.
(ii) Use two-digit random numbers from the list below to simulate the arrival times of five cars at the first lights. The first car arrives at the time given by the first arrival interval.

Random numbers: $24,01,99,89,77,19,58,42$

The two sets of traffic lights are 23 seconds driving time apart. Moving cars are always at least 2 seconds apart. If there is a queue at a set of lights, then when the red light ends the first car in the queue moves off immediately, the second car 2 seconds later, the third 2 seconds after that, etc.

In this simple model there is to be no consideration of accelerations or decelerations, and the lights are either red or green.

Table 5.2 shows the times when the lights are red.

| first set <br> of lights | red start time | 14 | 50 | 105 | 155 |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  | red end time | 29 | 65 | 120 | 170 |
| second set <br> of lights | red start time | 10 | 55 | 105 | 150 |
|  | red end time | 25 | 70 | 120 | 165 |

Table 5.2
(iii) Complete the table in the insert to simulate the passage of 10 cars through both sets of traffic lights. Use the arrival times given there.
(iv) Find the mean delay experienced by these cars in passing through each set of lights.
(v) How could the output from this simulation model be made more reliable?

6 A recipe for jam states that the weight of sugar used must be between the weight of fruit used and four thirds of the weight of fruit used. Georgia has 10 kg of fruit available and 11 kg of sugar.
(i) Define two variables and formulate inequalities in those variables to model this information.
(ii) Draw a graph to represent your inequalities.
(iii) Find the vertices of your feasible region and identify the points which would represent the best mix of ingredients under each of the following circumstances.
(A) There is to be as much jam as possible, given that the weight of jam produced is the sum of the weights of the fruit and the sugar.
(B) There is to be as much jam as possible, given that it is to have the lowest possible proportion of sugar.
(C) There is to be as much jam as possible, given that it is to have the highest possible proportion of sugar.
(D) Fruit costs $£ 1$ per kg , sugar costs 50 p per kg and the objective is to produce as much jam as possible within a budget of $£ 15$.

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