## OXFORD CAMBRIDGE AND RSA EXAMINATIONS

## Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
4752
Concepts for Advanced Mathematics (C2)
Wednesday 12 JANUARY 2005 Afternoon 1 hour 30 minutes
Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- There is an insert for use in Question 11.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72 .


## Section A (36 marks)

1 Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $y=x^{6}+\sqrt{x}$.

2 Find $\int\left(x^{3}+\frac{1}{x^{3}}\right) \mathrm{d} x$.

3 Sketch the graph of $y=\sin x$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.
Solve the equation $\sin x=-0.2$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.

4


Not to
scale

Fig. 4
For triangle ABC shown in Fig. 4, calculate
(i) the length of BC ,
(ii) the area of triangle ABC .

5 The first three terms of a geometric progression are 4, 2, 1 .
Find the twentieth term, expressing your answer as a power of 2.
Find also the sum to infinity of this progression.

6 A sequence is given by

$$
\begin{gathered}
a_{1}=4, \\
a_{r+1}=a_{r}+3
\end{gathered}
$$

Write down the first 4 terms of this sequence.
Find the sum of the first 100 terms of the sequence.

7


Not to scale

Fig. 7
Fig. 7 shows a sector of a circle of radius 5 cm which has angle $\theta$ radians. The sector has area $30 \mathrm{~cm}^{2}$.
(i) Find $\theta$.
(ii) Hence find the perimeter of the sector.

8 (i) Solve the equation $10^{x}=316$.
(ii) Simplify $\log _{a}\left(a^{2}\right)-4 \log _{a}\left(\frac{1}{a}\right)$.

## Section B (36 marks)

9 (i) A tunnel is 100 m long. Its cross-section, shown in Fig. 9.1, is modelled by the curve

$$
y=\frac{1}{4}\left(10 x-x^{2}\right),
$$

where $x$ and $y$ are horizontal and vertical distances in metres.


Figure 9.1
Using this model,
(A) find the greatest height of the tunnel,
(B) explain why $100 \int_{0}^{10} y \mathrm{~d} x$ gives the volume, in cubic metres, of earth removed to make the tunnel. Calculate this volume.
(ii) The roof of the tunnel is re-shaped to allow for larger vehicles. Fig. 9.2 shows the new crosssection.


Not to scale

Fig. 9.2
Use the trapezium rule with 5 strips to estimate the new cross-sectional area.
Hence estimate the volume of earth removed when the tunnel is re-shaped.

10 A curve has equation $y=x^{3}-6 x^{2}+12$.
(i) Use calculus to find the coordinates of the turning points of this curve. Determine also the nature of these turning points.
(ii) Find, in the form $y=m x+c$, the equation of the normal to the curve at the point $(2,-4)$.

11 Answer part (iii) of this question on the insert provided.
A hot drink is made and left to cool. The table shows its temperature at ten-minute intervals after it is made.

| Time (minutes) | 10 | 20 | 30 | 40 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Temperature $\left({ }^{\circ} \mathrm{C}\right)$ | 68 | 53 | 42 | 36 | 31 |

The room temperature is $22^{\circ} \mathrm{C}$. The difference between the temperature of the drink and room temperature at time $t$ minutes is $z^{\circ} \mathrm{C}$. The relationship between $z$ and $t$ is modelled by

$$
z=z_{0} 10^{-k t}
$$

where $z_{0}$ and $k$ are positive constants.
(i) Give a physical interpretation for the constant $z_{0}$.
(ii) Show that $\log _{10} z=-k t+\log _{10} z_{0}$.
(iii) On the insert, complete the table and draw the graph of $\log _{10} z$ against $t$.

Use your graph to estimate the values of $k$ and $z_{0}$.
Hence estimate the temperature of the drink 70 minutes after it is made.

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