OCR

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2623/1

Numerical Methods

Friday **21 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 **(i)** Use the formula

$$n! \approx e^{-n}(n+\tfrac{1}{2})^n \sqrt{2n+1} \qquad (1)$$

to find approximate values for 10!, 20! and 40!

Find the relative errors in these values. [6]

(ii) An alternative formula is

$$n! \approx e^{\frac{1}{12}} e^{-n}(n+\tfrac{1}{2})^n \sqrt{2n+1}\,. \qquad (2)$$

Obtain the relative errors in the values of 10!, 20! and 40! as given by this formula.

Identify *two* respects in which formula (2) appears to be better than formula (1). [5]

(iii) Explain why $n!$ for $n \geqslant 70$ cannot be evaluated on a standard calculator.

Use formula (1) to estimate the value of $\dfrac{80!}{40!}$, showing your working. [4]

2 **(i)** Show that the equation $x^3 = 3^x$ has a root in the interval $(2, 2.8)$.

Apply the bisection process three times to obtain a shorter interval containing this root. Give the best estimate of the root at this stage and state its maximum possible error.

Determine how many further applications of bisection would be needed to reduce the maximum possible error to less than 0.001. [12]

(ii) Show that the bisection method fails if the initial interval is taken as $(2, 3.2)$. Explain why this happens. [3]

3 In this question you are required to estimate gradients on the curve $y = 2^{\sqrt{x}}$.

(i) Use the forward difference method with $h = 0.2, 0.1, 0.05$ to find a sequence of estimates A_1, A_2, A_3 of the gradient when $x = 3.5$.

Show that $A_3 - A_2 \approx \frac{1}{2}(A_2 - A_1)$. Hence obtain by extrapolation an improved estimate of this gradient, giving your answer to an appropriate degree of accuracy. [6]

(ii) Use the central difference method with $h = 0.2, 0.1, 0.05$ to find a sequence of estimates B_1, B_2, B_3 of the gradient when $x = 4.5$.

By considering $B_3 - B_2$ and $B_2 - B_1$, obtain an improved estimate of this gradient, giving your answer to an appropriate degree of accuracy. [7]

(iii) Comment briefly on the relative merits of the forward difference method and the central difference method. [2]

4 The function $g(x)$ has values as given in the table.

x	2.0	2.5	3.0	3.5	4.0
$g(x)$	0.91	1.14	1.40	1.74	2.31

It is required to estimate I, the integral of $g(x)$ for the given range of x.

(i) Assume in this part of the question that the data in the table are exact.

Obtain two estimates of I using Simpson's rule. Extrapolate to obtain a further estimate of I. Give a value for I to the accuracy that appears to be justified. [7]

(ii) Now suppose that the values of $g(x)$ in the table are correct to 2 decimal places but that the values of x are exact.

Determine the maximum possible error in the two Simpson's rule estimates and state the implications for the extrapolated value. [4]

(iii) Finally, suppose that the x values are correct to 1 decimal place.

Explain what effect this has on the range of integration and hence on the value of I. [4]

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