OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2615

Statistics 3

Friday **21 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

Registered Charity 1066969
[Turn over

1 The random variable X has the rectangular (continuous uniform) distribution over the interval $1 \leqslant x \leqslant 5$, so that its probability density function is

$$f(x) = \begin{cases} \frac{1}{4} & 1 \leqslant x \leqslant 5, \\ 0 & \text{elsewhere.} \end{cases}$$

The cumulative distribution function (cdf) of X is denoted by $F(x)$.

(i) Write down the values of the cdf of X at $x = 1$ and $x = 5$. [2]

(ii) Obtain $F(x)$, and verify that the values of $F(1)$ and $F(5)$ are correct. [3]

The random variable Y is defined by $Y = 2X^2$.

(iii) The values that Y can take are given by $y_1 \leqslant y \leqslant y_2$. Write down y_1 and y_2. [1]

(iv) Let $G(y)$ denote the cdf of Y. By considering

$$G(y) \equiv P(Y \leqslant y) = P(2X^2 \leqslant y)$$

and using the cdf of X, show that

$$G(y) = \frac{\sqrt{y}}{4\sqrt{2}} - \frac{1}{4}$$

(for $y_1 \leqslant y \leqslant y_2$). [3]

(v) Obtain the probability density function of Y. [2]

(vi) Use $G(y)$ to obtain m_Y, the median of Y. [2]

(vii) Write down m_X, the median of X. Verify that $m_Y = 2m_X^2$. [2]

2 A craftsman makes hand-made souvenirs of two types, A and B. The time taken to make a type A souvenir is a Normally distributed random variable with mean 34 minutes and standard deviation 2.6 minutes. Independently, the time taken to make a type B souvenir is a Normally distributed random variable with mean 39 minutes and standard deviation 4.0 minutes.

(i) Find the probability that it takes more than 30 minutes to make a type A souvenir. [2]

(ii) Find the probability that a type A souvenir takes more time to make than a type B souvenir. [4]

(iii) The souvenirs are packed in boxes containing 3 of each type. Find the probability that the total time to make the 6 souvenirs in such a box exceeds 210 minutes. (Assume that the souvenirs are chosen randomly and independently.) [3]

The craftsman undertakes a training course to improve his skill at making type A souvenirs. Afterwards, a random sample of 8 times taken to make type A souvenirs is as follows (in minutes).

34.9 31.8 26.1 29.9 31.4 33.3 29.1 27.9

(iv) Assuming that the underlying standard deviation has not changed, provide a two-sided 95% confidence interval for the true mean time to make a type A souvenir after the training course. Interpret this confidence interval carefully. [6]

3 Researchers at an industrial company are studying new apparatus for delivering a carefully controlled volume of a certain chemical at a particular stage in the production process. The volume delivered is specified to be 21 ml. Small variations from this can be tolerated, but the researchers need to determine whether the volume delivered is as specified on average.

The volumes delivered are carefully measured on 10 occasions and found to be as follows (in ml).

21.0 21.6 19.8 22.9 22.0 20.9 22.5 21.4 21.8 20.6

(i) State the appropriate null and alternative hypotheses for the usual t test that the researchers might use. [2]

(ii) State two conditions necessary for the correct use of the t test. [2]

(iii) Carry out the test, using a 5% significance level. [7]

(iv) Provide a two-sided 99% confidence interval for the true mean volume delivered. [4]

[Turn over

4 Psychologists are developing a new index of overall intelligence for 11-year-old children. It is assumed that the index is Normally distributed over the whole underlying population and that the standard deviation of this distribution is 12. If the index has been created correctly, its mean over the population should be 50.

(i) The index is measured for a random sample of 100 11-year-old children. It is found that the sample mean value is 47.8. Test the hypothesis that the true mean of the index is 50, against the alternative that it is not 50, at the 1% level of significance. Discuss briefly whether the assumption that the underlying distribution is Normal is necessary for this test to be valid. [8]

(ii) If the assumption of underlying Normality with mean 50 and standard deviation 12 is correct, then

- 6.68% of the population will have an index value less than 32,
- 43.32% of the population will have an index value between 32 and 50,
- 43.32% of the population will have an index value between 50 and 68,
- 6.68% of the population will have an index value greater than 68.

In the random sample of 100 11-year-old children, it is found that the numbers in these categories are 12, 48, 36 and 4 respectively. Use this information to test whether the assumption is reasonable, at the 10% significance level. [7]

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