OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2605

Pure Mathematics 5

Wednesday **12 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 **(a)** The equation $8x^4 + 16x^3 + 1 = 0$ has roots α, β, γ and δ.

Use a suitable substitution to find a quartic equation with integer coefficients which has roots

$$8\alpha^3, 8\beta^3, 8\gamma^3 \text{ and } 8\delta^3.$$ [5]

(b) When the polynomial $f(x) = x^5 + kx^4 + mx^3 + 7x - 2$ is divided by $(x - 2)$, the remainder is 12.

When $f(x)$ is divided by $(x + 1)$, the remainder is 3.

(i) Find k and m. [4]

(ii) Find the remainder when $f(x)$ is divided by $(x - 2)(x + 1)$. [4]

(iii) Show that $f'(-1) = -30$. [2]

(iv) Find the remainder when $f(x)$ is divided by $(x + 1)^2$. [5]

2 **(a)** Find the exact value of $\displaystyle\int_{2.5}^{7.5} \frac{1}{4x^2 + 75}\,dx$. [5]

(b) **(i)** Starting from $\cosh x = \frac{1}{2}(e^x + e^{-x})$, show that $\cosh 2x = 2\cosh^2 x - 1$. [3]

(ii) Show that the two stationary points on the curve $y = 7\sinh x - \sinh 2x$ have y-coordinates $3\sqrt{3}$ and $-3\sqrt{3}$. [7]

(iii) Show that $\displaystyle\int_{0}^{\ln 3} (7\sinh x - \sinh 2x)\,dx = \frac{26}{9}$. [5]

3 In this question, $z = \cos\theta + j\sin\theta$ where θ is real.

(a) By considering z^5, express $\tan 5\theta$ in terms of $\tan\theta$. [6]

(b) **(i)** Write $z^n + \dfrac{1}{z^n}$ and $z^n - \dfrac{1}{z^n}$ in simplified trigonometric form. [3]

(ii) By considering $\left[\left(z - \dfrac{1}{z}\right)\left(z + \dfrac{1}{z}\right)\right]^3$, show that $\sin^3\theta\cos^3\theta = \frac{3}{32}\sin 2\theta - \frac{1}{32}\sin 6\theta$. [6]

(iii) Hence find the first two non-zero terms of the Maclaurin series for $\sin^3\theta\cos^3\theta$. [3]

(iv) Given that θ^7 and higher powers may be neglected, show that

$$\sin^3\theta\cos^3\theta \approx \theta^3\cos 2\theta.$$ [2]

4 **(a)** A curve has polar equation $r = k\sin 4\theta$, for $0 \leqslant \theta \leqslant \pi$, where k is a positive constant.

(i) Sketch the curve, using a continuous line for sections where $r > 0$, and a broken line for sections where $r < 0$. [3]

(ii) Find the area of one loop of the curve. [5]

(b) $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The tangents to the hyperbola at P_1 and P_2 meet at the point $Q(r, s)$.

(i) Find (in terms of a, b, x_1 and y_1) the gradient of the hyperbola at P_1, and hence show that the equation of the tangent at P_1 is $\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1$. [5]

(ii) Show that P_1 and P_2 lie on the line with equation $\frac{rx}{a^2} - \frac{sy}{b^2} = 1$. [3]

(iii) Given that Q lies on a directrix of the hyperbola, show that the line P_1P_2 passes through a focus. [4]

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