OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2604

Pure Mathematics 4

Wednesday **12 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 A curve has equation $y = \dfrac{18x - 5x^2}{x^2 - 36}$.

(i) Write down the equations of the three asymptotes. [3]

(ii) Find $\dfrac{dy}{dx}$. Hence find the coordinates of the stationary points. [6]

(iii) Sketch the curve. [5]

(iv) On a separate diagram, sketch the curve with equation $y = \left|\dfrac{18x - 5x^2}{x^2 - 36}\right|$. [3]

(v) State the values of k for which the equation $\left|\dfrac{18x - 5x^2}{x^2 - 36}\right| = k$ has exactly three distinct real solutions. [3]

2 **(a)** Find the sum of the series

$$(1 \times 7) + (3 \times 11) + (5 \times 15) + \ldots + (2n - 1)(4n + 3),$$

giving your answer in a fully factorised form. [6]

(b) Solve the inequality $\dfrac{x}{x - 1} < \dfrac{x - 1}{x}$. [6]

(c) Express $\dfrac{9r + 14}{r(r + 1)(r + 2)}$ in partial fractions, and hence find the sum of the first n terms of the series

$$\frac{23}{1 \times 2 \times 3} + \frac{32}{2 \times 3 \times 4} + \frac{41}{3 \times 4 \times 5} + \ldots .$$ [8]

3 Throughout this question, $\alpha = 3 + 2\mathrm{j}$.

(a) **(i)** Find α^2 and α^3. [3]

(ii) Given that α is a root of the equation $2x^3 + px^2 + 20x + q = 0$, where p and q are real numbers,

(A) find p and q, [5]

(B) find the other two roots of the cubic equation. [4]

(b) **(i)** Find $|\alpha|$ and $\arg \alpha$. [2]

(ii) On an Argand diagram, shade the region corresponding to complex numbers z for which

$$|z - \alpha| \leqslant 2.$$ [2]

(iii) Given that $|z - \alpha| \leqslant 2$, find

(A) the minimum possible value of $|z|$,

(B) the maximum possible value of $|z|$,

(C) the maximum possible value of $\arg z$. [4]

4 **(a)** Given that $\mathbf{M} = \begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}$, prove by induction that $\mathbf{M}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 1+3n \end{pmatrix}$, where n is a positive integer. [7]

(b) **(i)** Find the vector product $(2\mathbf{i} - 9\mathbf{j} - 8\mathbf{k}) \times (5\mathbf{i} + 10\mathbf{j} + 6\mathbf{k})$. [2]

(ii) Find the equation of the line of intersection of the two planes

$$\begin{aligned} 2x - 9y - 8z &= 48, \\ 5x + 10y + 6z &= -10. \end{aligned}$$ [3]

(iii) Given that $\begin{pmatrix} 2 & -9 & -8 \\ 5 & 10 & 6 \\ 2 & 1 & k \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 48 \\ -10 \\ 13 \end{pmatrix}$, and $k \neq 0$, express x, y and z in terms of k. [6]

(iv) Describe geometrically how the following three planes intersect.

$$\begin{aligned} 2x - 9y - 8z &= 48 \\ 5x + 10y + 6z &= -10 \\ 2x + y \quad &= 13 \end{aligned}$$ [2]

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